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Nearfield Acoustical Holography for the Visualization of Interior Sound Fields

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Nearfield Acoustical Holography

- Measure acoustic pressure on two-dimensional surface

- Determine entire acoustic field in three-dimensional region
Free-Field Algorithm

- Assume pressure satisfies wave equation

\[ \nabla^2 P(x, y, z) + k^2 P(x, y, z) = 0 \]

- Solution is (free-field boundary condition)

\[
P(x, y, z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} A(k_x, k_y) e^{ik_x x} e^{ik_y y} e^{ik_z z} dk_x dk_y \\
= \mathcal{F}^{-1}[A(k_x, k_y) e^{ik_z z}]
\]

where

\[
k_z = \sqrt{k^2 - k_x^2 - k_y^2}
\]

- Coefficients, in terms of pressure at \( z = z' \), are

\[
A(k_x, k_y) = \frac{1}{e^{ik_z z'}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} P(x', y', z') e^{ik_x x'} e^{ik_y y'} dx' dy' \\
= \frac{1}{e^{ik_z z'}} \mathcal{F}[P(x, y, z')]
\]
-Total algorithm is then

\[ P(x, y, z) = F^{-1} \left[ F \left[ P(x, y, z') e^{jkz'} \right] e^{jkz} \right] \]

-Velocity components, from Euler's equation, are

\[ U_\eta(x, y, z) = \frac{-j}{\rho_0 ck} \frac{\partial P(x, y, z)}{\partial \eta} = \frac{1}{\rho_0 c} F^{-1} \left[ \frac{k_\eta}{k} F \left[ P(x, y, z') \right] e^{jkz} \right] \]

-Intensity is

\[ I(x, y, z) = \frac{1}{2} \text{Re} \left[ P(x, y, z) U^*(x, y, z) \right] \]
\[ Q(x, y, z) = \frac{1}{2} \text{Im} \left[ P(x, y, z) U^*(x, y, z) \right] \]
Applying NAH in Interiors

- NAH was developed for anechoic environments
- Interiors contain reflecting surfaces which produce incoming and outgoing waves
- Measured pressure characterized by acoustic modes
Ceiling Reflections

-Two unknowns, incoming and outgoing waves

\[ P(x, y, z) = F^{-1}[A_{mn}e^{ikz} + B_{mn}e^{-ikz}] \]
- Measure pressure on two surfaces, $z = z_1$ and $z = z_2$

Reflect

--- projection plane

--- measurement plane 2

--- measurement plane 1

--- source plane

-Coefficients are then solutions to

$$
\begin{bmatrix}
e^{jkz_1} & e^{-jkz_1} \\
e^{jkz_2} & e^{-jkz_2}
\end{bmatrix}
\begin{bmatrix}
A_{mn} \\
B_{mn}
\end{bmatrix} = \begin{bmatrix}
\mathcal{F}[P(x, y, z_1)] \\
\mathcal{F}[P(x, y, z_2)]
\end{bmatrix}
$$
Wall Reflections

-Wall reflections excite acoustic modes, for hard walls

\[ P(x, y, z) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} \cos \frac{m\pi x}{l_x} \cos \frac{n\pi y}{l_y} e^{jk_z z} \]

\[ = F^{-1}[A_{mn} e^{jk_z z} ] \]

where

\[ k_z = \sqrt{k^2 - \left(\frac{m\pi}{l_x}\right)^2 - \left(\frac{n\pi}{l_y}\right)^2} \]

-Coefficients, in terms of pressure at \( z = z' \), are

\[ A_{mn} = \frac{1}{e^{jk_z z'}} \int_0^{l_x} \int_0^{l_y} \bar{P}(x, y, z) \cos \frac{m\pi x}{l_x} \cos \frac{n\pi y}{l_y} \, dx \, dy \]

\[ = \frac{1}{e^{jk_z z'}} F[\bar{P}(x, y, z)] \]

-Barred quantities are even extensions
Experimental Setup
Experimental Results

Verification - 126 Hz

Pressure - z=0.075 m

Pressure - z=0.102 m

Velocity - Measured

Velocity - Predicted
Experimental Results

Verification - 236 Hz

Pressure - $z=0.075$ m

Pressure - $z=0.102$ m

Velocity - Measured

Velocity - Predicted
Source Location

Intensity - 80 Hz

Active Intensity

Reactive Intensity

Intensity - 294 Hz

Active Intensity

Reactive Intensity
Source Location

Pressure - 160 Hz

\[ z = 0.076 \text{ m} \quad \text{and} \quad z = 0.102 \text{ m} \]

Intensity - 160 Hz

Active Intensity

Reactive Intensity
Source Location

Pressure - 480 Hz

$z=0.076 \text{ m} \quad z=0.102 \text{ m}$

Intensity - 480 Hz

Active Intensity

Reactive Intensity
Rodeo Excitation Location
Rodeo Test

Interior View

View of Reference Microphone and Source Location
Rodeo Results

Pressure - \( z=0.02 \text{ m} \)

Pressure - \( z=0.072 \text{ m} \)

Reactive Intensity on Floor Pan
Conclusions

- Free-field NAH techniques can be modified for interior sound fields
  - wall reflections require extended pressure distribution
  - ceiling reflections require two surface measurements

- Interior NAH methods can be used to accurately determine sound field and locate sources
  - point sources highly resolved, even at acoustic resonance
  - mechanical point forces visible when plate modal density sufficiently high