A color algebra refers to a system for computing sums and products of colors, analogous to additive and subtractive color mixtures. This is straightforward when we are dealing with the physical spectral data describing lights and surface reflectances, but less so when the data are projected to a three-dimensional color space. Here we explore the family of algebras that result from associating each color with a member of a three-dimensional manifold of spectra. This association can be used to construct a color product, defined as the color of the spectrum of the wavelength-wise product of the spectra associated with the two input colors. The choice of the spectral manifold determines the behavior of the resulting system, and certain special subspaces allow computational efficiencies. We examine the performance of various systems for describing the behavior of sets of natural spectra, and compare the results to human color constancy data.

1. INTRODUCTION

Color refers to a psychological construct that is distinct from, but related to, the spectral composition of light reaching the eye. The situations in which color appearance is not well-predicted on the basis of the spectral distribution can be crudely lumped into a couple of categories: spatial interactions between different regions of the visual field, such as simultaneous contrast \[1\] and assimilation \[2\]; and context effects in which an observer “discounts the illuminant” to perceive the color associated with a particular surface reflectance, known as color constancy. In this paper, we will ignore spatial interactions, but will propose a framework in which color constancy and other phenomena can be considered. We begin with an abbreviated description of basic colorimetry; various texts are available for readers desiring a more thorough review of this topic \[3\]. Colorimetry is excellent at predicting when two spectral distributions will have the same appearance (metamerism), but is less good at predicting exactly what the common appearance will be.

In practice, colors are often represented by three numbers known as the tristimulus values. These are obtained by computing the integral of the product of the spectral distribution with each of three color matching functions:

\[ t^i = \int_{\lambda_1}^{\lambda_2} m^i(\lambda) f(\lambda) d\lambda, \quad i \in \{1, 2, 3\} \]  

Note that in writing \( t^i \) we do not mean exponentiation, but are adopting the convention used in physics of using upper indices for coefficients and lower indices for basis vectors. The color matching functions \( m^i(\lambda) \) are assumed to be a linear transformation of the photoreceptor spectral sensitivities. The limits of integration are often taken to be 400 and 700 nanometers, the nominal range of the visible spectrum. While we have written the basis functions \( f_i(\lambda) \) as continuous functions of wavelength, in practice they will be sampled at some finite interval. For the sake of concreteness, for the remainder of this paper we will take the color matching functions to be those corresponding to the CIE 1931 Standard Colorimetric Observer \[4\]; for calculations...