Color Algebras

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A *color algebra* refers to a system for computing sums and products of colors, analogous to additive and subtractive color mixtures. The difficulty addressed here is the fact that, because of metamerism, we cannot know with certainty the spectrum that produced a particular color solely on the basis of sensory data. Knowledge of the spectrum is not required to compute additive mixure of colors, but is critical for "subtractive" (multiplicative) mixture. Therefore, we cannot predict with certainty the multiplicative interactions between colors based solely on sensory data. There are two potential applications of a color algebra: first, to aid modeling phenomena of human visual perception, such as color constancy and transparency; and, second, to provide better models of the interactions of lights and surfaces for computer graphics rendering.

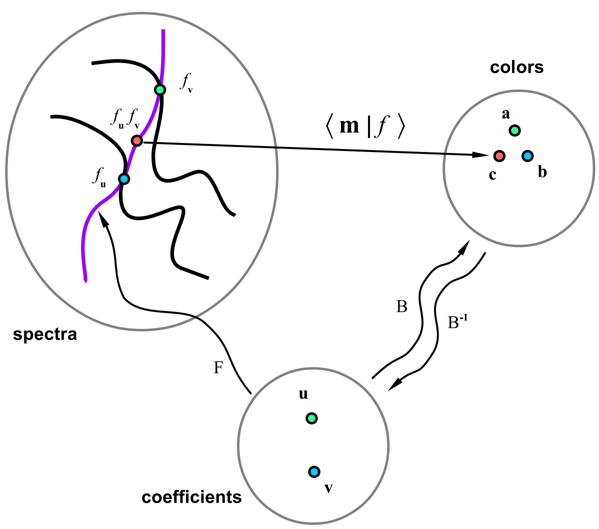

Figure 1: Cartoon illustrating the calculation of the color product.

We consider a plausible subset of possible color algebras based on *spectral manifolds*. Each color is associated with a member of a three-dimensional manifold of spectra. The color product is defined as the color of the spectrum of the wavelength-wise product of the spectra associated with the two input colors. We solve this difficulty by *assuming* a particular spectrum to associate with each color. This idea is illustrated in figure 1, which depicts the infinite dimensional space of spectral distributions, represented by the oval on the left of the figure, while the three dimensional space of color percepts is represented by the circle on the right. Two sample colors are indicated, along with the corresponding loci of metameric spectra (shown as black squiggles). The manifold of spectra is parameterized by coefficients, and is indicated by the purple squiggle.

A good solution is provided by the three dimensional space of functions that are sinusoidal in the log domain:

$$\log(f(\lambda)) = \alpha \cos\left(2\pi(\lambda - \lambda_0)\right) + \beta,$$
$$f(\lambda) = ke^{\alpha \cos(2\pi(\lambda-\lambda_0))}, \qquad k = e^{\beta}. \tag{1}$$

This function is a generalization of the Von Mises distribution (circular normal) (Batschelet, 1981), and has three parameters: center wavelength (hue) λ, expressed in normalized units in which the visual range is spanned by [0,1]; α, the modulation amplitude controlling the saturation; and a scaling parameter k that controls overall intensity.

This model produces spectra which are closer to natural spectra than those produced by the ubiquitous RGB model used universally in computer graphics. Figure 2 shows metameric spectra for yellow and cyan produced by the RGB and von Mises models, along with the corresponding product spectra. While the predictions have roughly the same green hue, it can be seen that the intensity predicted by the von Mises model is significantly lower.

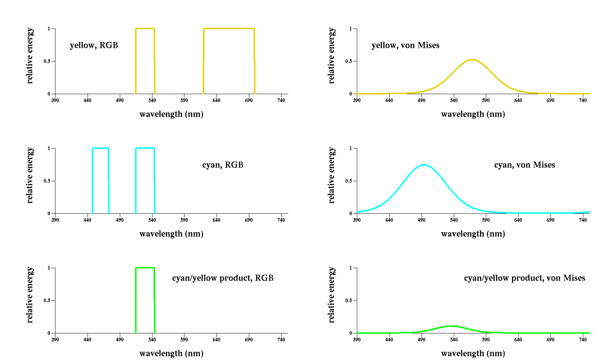

Figure 2: Comparison of yellow/cyan color product, computed with the RGB model (left) and von Mises model (right).

References

Batschelet, E. (1981). *Circular statistics in biology.* Academic Press.

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