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Influence of Boundary Conditions on the Prediction Accuracy of a Biot-based Poroelastic Model for Melamine Foam

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Influence of Boundary Conditions on the Prediction Accuracy of a Biot-based Poroelastic Model for Melamine Foam

Research Assistant:  Ryan Schultz

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But Why?

More Accurate, More Versatile Models

More Accurate, More Confident Predictions

Improved Noise Control Treatment Design

Lower Cost

Lower Weight

Better Performance
Poroelastic Materials

- Poroelastic Materials:
  - 2 phases: solid frame saturated with fluid
  - Pore cells: closed, open, partially reticulated

- Melamine Foam
  - Open Cell
  - Good acoustic, fire, thermal properties
Boundary Conditions

Facing Conditions

No Facing

Loose Facing

Glued Facing

![Diagram showing different facing conditions](image-url)
Boundary Conditions

Mounting Conditions

Gap

1. Foam [1, 2]
2. Air [5]
3. Wall

Fixed

1. Foam [1, 2]
2. Wall

$p_1, v_1$
$p_2, v_2$
$p_3, v_3$
Measurement: Apparatus

2-Mic Absorption

4-Mic Transmission Loss
Measurement: Sample Fit

Die Cut

Throw Away

Loose

Tight

Trim Edge

Sample Appropriately Sized

Ideal

Tight

Loose

Transmission Loss, [dB]

Frequency, [Hz]

As-Cut
1st Trim
2nd Trim
3rd Trim

Measurement for different sample fits with respect to frequency and transmission loss.
Absorption Measurement Results

**Gap Mounting**

- 25 mm – 12 trials
- 38 mm – 12 trials

**Fixed Mounting**

- 25 mm – 12 trials
- 38 mm – 12 trials
## The Biot Theory: Terminology

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Unit</th>
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<tr>
<td>$\phi$</td>
<td>Porosity</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Flow Resistivity</td>
<td>Ns/m$^4$</td>
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<td>$\alpha_\infty$</td>
<td>Tortuosity</td>
<td>-</td>
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<tr>
<td>$N$</td>
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<td>$\nu$</td>
<td>Poisson’s Ratio</td>
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<td>$\eta$</td>
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<tr>
<td>$\Lambda$</td>
<td>Viscous Characteristic Length</td>
<td>$\mu$m</td>
</tr>
<tr>
<td>$\Lambda'$</td>
<td>Thermal Characteristic Length</td>
<td>$\mu$m</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>Bulk Density</td>
<td>kg/m$^3$</td>
</tr>
</tbody>
</table>
The Biot Theory

**Stress-Strain Relations**

\[
\sigma_x = P e_x + Q \epsilon_x \\
\sigma = R \epsilon_x + Q e_x
\]

**Dynamic Relations**

\[
\frac{\partial \sigma_x}{\partial x} = \rho_1 \frac{\partial^2 u_x}{\partial t^2} + \rho_a \frac{\partial^2 (u_x - U_x)}{\partial t^2} + b \frac{\partial}{\partial t} (u_x - U_x) \\
\frac{\partial s}{\partial x} = \rho_a \frac{\partial^2 (U_x - u_x)}{\partial t^2} + b \frac{\partial}{\partial t} (U_x - u_x)
\]

\[
Q e_x + R \epsilon_x = -\omega^2(\rho_{12} u_x + \rho_{22} U_x) \\
P e_x + Q \epsilon_x = -\omega^2(\rho_{11} u_x + \rho_{12} U_x)
\]

\[
\nabla^4 e_x + A_1 \nabla^2 e_x + A_2 e_x = 0
\]

\[
e^{-jkr} e^{j\omega t} \quad \Rightarrow \quad k_{1,2}^2 = \frac{1}{2} \left( A_1 \pm \sqrt{A_1^2 - 4A_2} \right)
\]
Solution of Field Variables: Example

Front Condition: No Facing

Mounting Condition: Fixed

Solution for Field Variables:

- Insert expressions for field variables into constraint relations for Front & Rear surfaces and Mounting condition.
- Number of equations = Number of unknown field variables + Reflection coefficient
- Solve the linear system

\[
\begin{align*}
(1 - \phi)p_{ext} &= p_1 \\
\phi p_{ext} &= p_2 \\
v_{ext} &= (1 - \phi)v_1 + \phi v_2
\end{align*}
\]

\[
v_1 = 0 \\
v_2 = 0
\]

\[
\begin{pmatrix} -\phi & -f_1 & f_1 & -f_2 & f_2 \\ -f_1 & -g_1 & g_1 & -g_2 & g_2 \\ 1/Z_c & d_1 & d_1 & d_2 & d_2 \\ 0 & W_1 & W_1^{-1} & W_2 & W_2^{-1} \\ 0 & b_1 W_1 & b_1 W_1^{-1} & b_2 W_2 & b_2 W_2^{-1} \end{pmatrix}
\begin{pmatrix} R \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} =
\begin{pmatrix} (1 - \phi) \\ \phi \\ 1/Z_c \\ 0 \\ 0 \end{pmatrix}
\]
Intensity in the Porous Material

For Plane Harmonic Waves:

Acoustic Pressure

\[ p = Ae^{-jk_0x} + Be^{jk_0x} \]

Particle Velocity

\[ v = \frac{1}{\rho c} \left( Ae^{-jk_0x} - Be^{jk_0x} \right) \]

Intensity*

\[ I = \frac{1}{2} \text{Re}\{pv^*\} \]

*time-averaged rate of energy transmission through a unit area
Intensity

\[ I_{foam} = I_{solid} + I_{fluid} \]

\[ I_{foam} = I_{k_1} + I_{k_2} + I_{coupled} \]

\[ I_{plane\ wave} = \frac{1}{2} Re\{pv^*\} \]

\[ I_{foam} = \frac{1}{2} Re\{p_1v_1^*\} + \frac{1}{2} Re\{p_2v_2^*\} \]
Intensity: By Phases

◆ No Facing + Gap

◆ Glued Facing + Fixed
Intensity: By Wave Type

**No Facing + Gap**

- 250 Hz
- 500 Hz
- 1000 Hz
- 2000 Hz

**Glued Facing + Fixed**

- 250 Hz
- 500 Hz
- 1000 Hz
- 2000 Hz

*Note: The diagrams show the relative intensity in W/m² as a function of position in mm at different frequencies.*
Predictions vs. Measurements

No Facing + Gap

Absorption Coefficient

0 0.2 0.4 0.6 0.8 1

0 400 800 1200 1600

Frequency, [Hz]

Prediction Line
Measurement Line

Loose Facing + Gap

Absorption Coefficient

0 0.2 0.4 0.6 0.8 1

0 400 800 1200 1600

Frequency, [Hz]

Prediction Line
Measurement Line

Glued Facing + Gap

Absorption Coefficient

0 0.2 0.4 0.6 0.8 1

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Frequency, [Hz]

Prediction Line
Measurement Line

Glued Facing + Fixed

Absorption Coefficient

0 0.2 0.4 0.6 0.8 1

0 400 800 1200 1600

Frequency, [Hz]

Prediction Line
Measurement Line
Predictions vs. Measurements

When Intensity in the Air is Significant, Agreement is Good

When Intensity in the Frame is Most Significant, Agreement is Poor
Helmholtz Resonator Effect

Mechanical Impedance

Mass

Stiffness

Total Acoustic Impedance

$$z_m = R_r + j(\omega m - s/\omega)$$

$$m = \rho_0 SL'$$

$$s = \rho_0 c_0^2 S^2 / V$$

$$z = 1/(1/z_H + 1/z_f)$$
Helmholtz Resonator Effect

Combined Foam + Helmholtz Resonator System is Similar to Measured System
Helmholtz Resonator Effect

But is it really due to edge gaps?

Measured Glued Facing + Fixed with Edge Sealed
Summary

- Measured acoustical properties of melamine foam using two- and four-microphone standing wave tube techniques
- Developed a 1-dimensional formulation of the Biot theory for wave propagation in poroelastic materials
- Explored the intensity distribution among the two phases & two wave types
  - Intensity distribution is dependent on the imposed boundary conditions
  - Frame-borne wave type decays much more slowly than airborne wave type
  - Exchange between wave types is more significant at higher frequencies
- Compared model predictions with measurements
- Determined that model-measurement agreement appears to be dependent on the boundary conditions applied to the foam sample
  - Agreement appears to deteriorate when Frame plays a larger role
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Thank you!

Questions?
Measurement: Sample Fit

Ideally:

Sample = Layer of infinite lateral extent

Reality:

Sample either over- or under-sized
- Constrained
- Leaky
## Field Variables

### Frame

| Particle Velocity | $v_1 = j\omega(C_1 e^{jk_1x} + C_2 e^{-jk_1x} + C_3 e^{jk_2x} + C_4 e^{-jk_2x})$ |
| Acoustic Pressure | $p_1 = -f_1 C_1 e^{jk_1x} + f_1 C_2 e^{-jk_1x} - f_2 C_3 e^{jk_2x} + f_2 C_4 e^{-jk_2x}$ |

### Air

| Particle Velocity | $v_2 = j\omega(b_1 C_1 e^{jk_1x} + b_1 C_2 e^{-jk_1x} + b_2 C_3 e^{jk_2x} + b_2 C_4 e^{-jk_2x})$ |
| Acoustic Pressure | $p_2 = -g_1 C_1 e^{jk_1x} + g_1 C_2 e^{-jk_1x} - g_2 C_3 e^{jk_2x} + g_2 C_4 e^{-jk_2x}$ |
Field Variables

Particle Velocity

Frame-borne Wave Type

\[ v_1 = j \omega (C_1 e^{j k_1 x} + C_2 e^{-j k_1 x} + C_3 e^{j k_2 x} + C_4 e^{-j k_2 x}) \]

\[ p_1 = -f C_1 e^{j k_1 x} + f C_2 e^{-j k_1 x} + f C_3 e^{j k_2 x} + f C_4 e^{-j k_2 x} \]

Acoustic Pressure

Airborne Wave Type

\[ v_2 = j \omega (b_1 C_1 e^{j k_1 x} + b_1 C_2 e^{-j k_1 x} + b_2 C_3 e^{j k_2 x} + b_2 C_4 e^{-j k_2 x}) \]

\[ p_2 = -g_1 C_1 e^{j k_1 x} + g_1 C_2 e^{-j k_1 x} - g_2 C_3 e^{j k_2 x} + g_2 C_4 e^{-j k_2 x} \]
We have expressions for the acoustic velocity and pressure in each phase.

### Frame

- **Particle Velocity**
  
  \[ v_1 = j\omega(C_1 e^{jk_1 x} + C_2 e^{-jk_1 x} + C_3 e^{jk_2 x} + C_4 e^{-jk_2 x}) \]

- **Acoustic Pressure**
  
  \[ p_1 = -f_1 C_1 e^{jk_1 x} + f_1 C_2 e^{-jk_1 x} - f_2 C_3 e^{jk_2 x} + f_2 C_4 e^{-jk_2 x} \]

### Air

- **Particle Velocity**
  
  \[ v_2 = j\omega(b_1 C_1 e^{jk_1 x} + b_1 C_2 e^{-jk_1 x} + b_2 C_3 e^{jk_2 x} + b_2 C_4 e^{-jk_2 x}) \]

- **Acoustic Pressure**
  
  \[ p_2 = -g_1 C_1 e^{jk_1 x} + g_1 C_2 e^{-jk_1 x} - g_2 C_3 e^{jk_2 x} + g_2 C_4 e^{-jk_2 x} \]
Intensity

Apply the field variable equations, expand and separate terms by wave type

Frame-borne
- Solid
  \[ I_{\text{frame } k_1} = \frac{1}{2} \text{Re}\{ (j\omega f_1) [C_1 C_1^* e^{-2\beta_1 x} + C_1 C_2^* e^{j2\alpha_1 x} - C_2 C_2^* e^{2\beta_1 x} - C_1^* C_2 e^{-j2\alpha_1 x}] \} \]

- Fluid
  \[ I_{\text{pore } k_1} = \frac{1}{2} \text{Re}\{ (j\omega g_1 b_1^*) [C_1 C_1^* e^{-2\beta_1 x} + C_1 C_2^* e^{j2\alpha_1 x} - C_2 C_2^* e^{2\beta_1 x} - C_1^* C_2 e^{-j2\alpha_1 x}] \} \]

Airborne
- Solid
  \[ I_{\text{frame } k_2} = \frac{1}{2} \text{Re}\{ (j\omega f_2) [C_3 C_3^* e^{-2\beta_2 x} + C_3 C_4^* e^{j2\alpha_2 x} - C_4 C_4^* e^{2\beta_2 x} - C_3^* C_4 e^{-j2\alpha_2 x}] \} \]

- Fluid
  \[ I_{\text{pore } k_2} = \frac{1}{2} \text{Re}\{ (j\omega g_2 b_2^*) [C_3 C_3^* e^{-2\beta_2 x} + C_3 C_4^* e^{j2\alpha_2 x} - C_4 C_4^* e^{2\beta_2 x} - C_3^* C_4 e^{-j2\alpha_2 x}] \} \]

Coupled
- Solid
  \[ I_{\text{frame coupled}} = \frac{1}{2} \text{Re}\{ j\omega f_1 X_1 + j\omega f_2 X_2 \} \]

- Fluid
  \[ I_{\text{pore coupled}} = \frac{1}{2} \text{Re}\{ j\omega g_1 b_2^* X_1 + j\omega g_2 b_1^* X_2 \} \]

\[ X_1 = C_1 C_3^* e^{jx(k_1-k_2^*)} + C_1 C_4^* e^{jx(k_1+k_2^*)} - C_2 C_3^* e^{-jx(k_1+k_2^*)} - C_2 C_4^* e^{-jx(k_1-k_2^*)} \]

\[ X_2 = C_1^* C_3 e^{jx(k_2-k_1^*)} + C_2^* C_3 e^{jx(k_2+k_1^*)} - C_1^* C_4 e^{-jx(k_2+k_1^*)} - C_2^* C_4 e^{-jx(k_2-k_1^*)} \]
TL Measurement Results

25 mm – 12 trials

F: No Facing
R: No Facing

F: Loose Facing
R: No Facing

F: Glued Facing
R: No Facing
Parameter Estimation

- Uncertain of parameter values:
  - Porosity
  - Flow Resistivity
  - Tortuosity
  - Characteristic Length
  - Shear Modulus
  - Loss Factor

- Allow these to take a range of values & predict absorption coefficients

- Find the parameter set that best predicts the measured absorption coefficient
Parameter Estimation

- Uncertain of parameter values:
  - Porosity
  - Flow Resistivity
  - Tortuosity
  - Characteristic Length
  - Shear Modulus
  - Loss Factor

- Allow these to take a range of values & predict absorption coefficients

- Find the parameter set that best predicts the measured absorption coefficient
**Key Assumptions:**

- Plane Waves Only
- Layer of Infinite Lateral Extent
- Finite Sample

**Sound Field:**

\[ P = A e^{-j k_0 x} + B e^{j k_0 x} \]

Unknows

\[ P = e^{-j k_0 x} + R e^{j k_0 x} \]

\[ v = \frac{1}{\rho c} \left( e^{-j k_0 x} - R e^{j k_0 x} \right) \]

**Pressure at 2 Points:**

\[ P_1 = A e^{j k_0 (L_1 + s_1)} + B e^{-j k_0 (L_1 + s_1)} \]

\[ P_2 = A e^{j k_0 L_1} + B e^{-j k_0 L_1} \]

**Acoustical Properties:**

\[ R = B/A \]

\[ \alpha = 1 - |R|^2 \]
Poroelastic Material Modeling

◆ Zwikker & Kosten:
  - Extension of Kirchhoff’s theory for cylindrical pore sections
  - Found there are two waves that propagate in an elastic porous material
  - Elastic frame, normal incidence
  - Rosin, Lauriks et al., Bolton

◆ Biot:
  - Generalized theory using 3-D continuum mechanics, allows for 3-D wave propagation by 2 dilatational and 1 shear wave
  - Widely applied: soils, foams
  - Described & modified by: Allard and Atalla, Bolton et al.

◆ Modifications of Biot Theory:
  - Modifications of Coupling Terms:
    - Bolton, Johnson et al., Pride et al., Wilson, Kino
    - Bolton, Champoux-Allard, Lafarge, Kino
**Mass & Elastic Coefficients**

**Elastic Coefficients**

\[ N = G \]
\[ P = A + 2G + K_f (1 - \phi)^2 / \phi \]
\[ Q = (1 - \phi) K_f \]
\[ R = \phi K_f \]

**Mass Coefficients**

\[ \rho_{11} = \rho_1 + \rho_a + b / j\omega \]
\[ \rho_{12} = -\rho_a - b / j\omega \]
\[ \rho_{22} = \phi \rho_0 + \rho_a + b / j\omega \]

\[ \frac{\partial \sigma_x}{\partial x} = \rho_1 \frac{\partial^2 u_x}{\partial t^2} + \rho_a \frac{\partial^2}{\partial t^2}(u_x - U_x) + b \frac{\partial}{\partial t}(u_x - U_x) \]

**Inertial Coupling Term**

\[ \rho_a = \phi \rho_0 (\alpha_\infty - 1) \]

**Viscous Coupling Term**

\[ b = \sigma \phi^2 G(w) / j\omega \]
Boundary Condition Constraints

No Facing

$$v_{\text{ext}} = (1 - \phi)v_1 + \phi v_2$$

$$(1 - \phi)p_{\text{ext}} = p_1$$

$$\phi p_{\text{ext}} = p_2$$

Loose Film Facing

$$v_{\text{side1}} = v_{\text{film}}$$

$$v_{\text{side2}} = v_{\text{film}}$$

$$p_{\text{side1}} - p_{\text{side2}} = M_{s1} \frac{d}{dt} v_{\text{film}}$$

Glued Film Facing

$$v_{\text{ext}} = v_{\text{film}}$$

$$v_1 = v_{\text{film}}$$

$$v_2 = v_{\text{film}}$$

$$p_{\text{ext}} - p_1 - p_2 = M_{s2} \frac{d}{dt} v_{\text{film}}$$
Sensitivity to Model Inputs

No Facing + Gap Configuration

Key Findings:
- Sample Depth
- Porosity
- Flow Resistivity
- Thermal Characteristic Length
- Bulk Density

Glued Facing + Fixed Configuration

Key Findings:
- Facing Area Density
- Porosity
- Bulk Density
- Shear Modulus
- Poisson’s Ratio
- Loss Factor