Effect of Fins on Transient Behavior of Cross-Flow Air-Liquid Heat Exchangers

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EFFECT OF FINS ON TRANSIENT BEHAVIOR OF CROSS-FLOW AIR-LIQUID HEAT EXCHANGERS

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Abstract

Prediction of time response of heat exchangers, particularly finned tube liquid-air heat exchangers is of practical importance in improving control systems of refrigeration plants. In this study the well-known finite volume method is used to predict time response of a cross-flow circumferentially finned tube heat exchanger to step change of liquid temperature, in order to determine the effective values of coefficients of an approximate response function, which is based on a lumped parameter model. Experiments carried out show good agreement and verify the method used to take the effect of fins into consideration. Although adaptive control systems are being used more extensively, range of parameters of these systems should be fixed by means of the above mentioned method.
Because of its contribution to the quality of performance and to economy, improvement of control systems is gaining ever higher importance in air conditioning, refrigeration and other fields of industry. In control of temperature, the major component is the final heat exchanger. To use full capacity of the heat exchanger, control of the inlet temperature of the primary fluid is generally the preferred one. It has been shown [1], [2] that in air conditioning this method is advantageous in comparison to controlling steam pressure or water flow rate.

Recent advances in adaptive and self tuning control systems have improved the performance of control systems considerably. However at early stages of designing such systems and for their stability analysis thermal response of the main components, especially that of the final heat exchanger should be known at least approximately. This paper aims prediction of thermal response of heat exchangers with emphasis on cross flow air-liquid heat exchangers and the role of fins.

Gas to gas cross flow heat exchangers have been treated theoretically in a recent paper [3], also there is a study directed to the thermal performance of fins [4]. However, need exists for a readily applicable method of predicting thermal behavior of cross flow liquid-air heat exchangers with fins.

An approximate response function is being used in many fields of control engineering and in air conditioning control, to represent the response $y(t)$:

$$y(t) = Y_0 \left[ 1 - e^{-(t-d)/t^*} \right] u(t-d). \quad (1)$$

$Y_0$ is the gain $= K_8 \cdot X_0$ whereby $X_0$ is the amplitude of excitation and $K_8$ is the gain factor of the system. $d$ is the time lag and $t^*$ is the time constant of the system. $u(t-d)$ is the unit step function.

This very basic equation can be traced back to quasi-lumped model of a heat exchanger with negligible mass of walls (Figure 1). Assuming linear variation of temperature of both fluids between inlet and exit and effective rate of
heat transfer proportional to the difference of average temperatures on both sides, one can write

\[
\frac{d\theta_h}{dt} = \hat{C}_h (T_{hi} - T_{he}) - U (T_h - T_c) \quad (2.a)
\]

\[
C \frac{dT_c}{dt} = \hat{C}_c (T_{ci} - T_{ce}) + U (T_h - T_c) \quad (2.b)
\]

in this equation \( \hat{C}_h, \hat{C}_c \) indicate the heat capacity of the contents and \( \dot{C}_h', \dot{C}_c \) indicate the heat capacity flow rate, i.e. \( m (\text{mass flow rate}) \times c (\text{specific heat}) \) of hot and cold fluids respectively. \( U \) is the over-all heat transmittance of the heat exchanger. Temperatures are taken as difference from their steady state values \( (T_{ho}', T_{co}) \) prevailing before the excitement; by writing the same equations once for actual case and once for steady state and subtracting the second one from the first one, Eqn. (2) will be seen. At a deviation from steady state, e.g. as the hot fluid undergoes a temperature jump at inlet, exit temperature of both fluids will start to approach their new steady state values. Indicating the ratio of the above temperatures to step change of excitement by \( \Theta = T/\Delta \) and taking as characteristic time \( \hat{C}_h'/\hat{C}_h \) (or in general \( C_{\min}/C_{\min} \)) one obtains those equations in dimensionless form with the dimensionless coefficients:

\[
N = U/\hat{C}_h \quad \text{Number of transfer units}
\]

\[
\hat{C}_r = \dot{C}_c'/\dot{C}_h \quad \text{Ratio of heat capacity flows}
\]

\[
C_r = C_c/C_h \quad \text{Ratio of heat capacity contents and}
\]

\[
\tau = t \hat{C}_h'/\hat{C}_h \quad \text{Dimensionless time}
\]

Since the flow conditions of both fluids are fixed (for hot fluid \( h \), for cold fluid \( c \), assuming linear change between inlet and exit, one can use the relations \( \theta_h = (1 + \theta_h) / 2 \) and \( \theta_c = (\Theta + \theta_{ce}) / 2 \) to eliminate \( \theta_{he} \) and \( \theta_{ce} \) from the dimensionless form of these equations which simplify to:

\[
\frac{d\theta_h}{d\Theta} = 2(1-\theta_h) + N(\theta_c - \theta_h) \quad (3.a)
\]

\[
\frac{d\theta_c}{d\Theta} = N(\theta_h - \theta_c) + 2C_r \theta_c \quad (3.b)
\]

Although one can easily give their complete solution the simplification of these equations by neglecting internal energy change at gas side is usual and one substitutes second equation in the first one which allows quick integration:

\[
\theta_h = K'(1-e^{-\frac{6 - 6\Theta}{K}}) \quad (4.a)
\]

\[
\theta_c = K'' (1-e^{-\frac{6 - 6\Theta}{K}}) \quad (4.b)
\]

In this integration the initial condition has been taken as
at \( \mathcal{Z}=\mathcal{Z}_d \) \( \theta_h=0 \), which is more realistic in case of cross flow then in other cases. The coefficients of Equation (4.a, b) can be expressed as

\[
K = \frac{N-2\mathcal{Z}_r}{2(N-2\mathcal{Z}_r-N\mathcal{C}_r)} \quad K' = \frac{N-2\mathcal{Z}_r}{N-2\mathcal{Z}_r-N\mathcal{C}_r} \quad \text{and} \quad K'' = \frac{N}{N-2\mathcal{Z}_r-N\mathcal{C}_r}
\]

For exit temperature of the cold and hot fluids one can write according to the assumption of linear variation (Fig 1.b)

\[
\theta_{ce} = 2K''(1-e^{-\mathcal{Z}}) \quad (5.a)
\]

\[
\theta_{he} = 2K'(1-e^{-\mathcal{Z}}) - 1 \quad (5.b)
\]

This very simple model can not clarify the hot side exit temperature variation at the early phase. This phase can be considered as the period during which the quarter of the capacity is filled. However this is not accurate and further refinement should be directed also in other directions.

These equations can be written like Eqn. (1) in dimensional form which will result in, for the hot fluid:

\[
d = \frac{C_h}{4\mathcal{C}_h} \quad (6.a)
\]

\[
t^* = K \frac{C_h}{C_h} = \frac{U-2\mathcal{C}_c}{2(U-2\mathcal{C}_c-U\mathcal{C}_c/C_c)} \cdot \frac{C_h}{C_c} \quad (6.b)
\]

\[
y = \frac{N-2\mathcal{Z}_r-N\mathcal{C}_r}{2(N-2\mathcal{Z}_r-N\mathcal{C}_r)} (T_{hi} - T_{ho}) \quad (6.c)
\]

**OTHER MODELS**

Although there have been recent publications on theoretical aspects of thermal response of heat exchangers they are of analytical character\[3,4\]. Müller has taken single fins into consideration and developed transfer function for its response. Also his measurements, carried out on a fin, confirmed his theory. However measurements in air shows great discrepancy. Apparently his theory is good at micro level but not at macro level.

From the point of view of heat exchanger designer results of computations of London and his coworkers are more applicable. To come closer to practice, they have taken into consideration the variation of the driving force of heat transfer along the heat transfer surface and also the heat capacity of the wall. By solving the differential equations analytically or by finite difference method or by means of experiments on electric/mechanical analogy, they have produced charts in terms of the above mentioned dimensionless parameters and two more parameters, which consider the ratio of the resistances on both sides of the wall and heat capacity of the wall:

\[
C_w = \frac{C_w}{C_{min}}
\]
Furthermore the heat capacity content of both sides are compared by a dwell time ratio

$$R^*= R_{on \ min \ side}/R_{on \ max \ side}$$

Because of the large number of variables, they restrict some of them in the charts.

Because of considerable importance of finned tube liquid-gas heat exchangers an approximate model for their investigation has been developed by the authors. Figure 2 shows the thermal model (b) and its electrical analog (c). The heat transfer characteristics of each row and each type of bends are different (Figure 2a). Dividing each of n tubes into three parts (Figure 2b) and at each part specifying four thermal capacities: Hot fluid (H), wall (W), fin (F) and finally air (A) at constant temperature, one can determine time-wise variation. A crucial point in estimating $R_{W-F}$ is the assumption that it is equal to the loss of transmittance due to the difference of fin efficiency from one. To develop a computer code for this purpose finite volume technique is being used. Detailed results will be published elsewhere.

DISCUSSION AND CONCLUSIONS

The above mentioned models are being studied at present. Also an experimental set up has been built (Figure 3). Preliminary results show that time lag of the water exit temperature is smaller than the time lag of air temperature (Figure 4). Table 1 and 2 give the data for the experiments mentioned in Figure 4. Also calculated values of the coefficients are included.

REFERENCES


Table 1 Operation data of two experiments

<table>
<thead>
<tr>
<th>Operation data</th>
<th>Exp.1</th>
<th>Exp.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air flow rate, m³/s</td>
<td>0.161</td>
<td>0.160</td>
</tr>
<tr>
<td>Water flow rate, m³/s</td>
<td>0.325</td>
<td>0.278</td>
</tr>
<tr>
<td>Uniform temperature before the experiment, °C</td>
<td>14.9</td>
<td>16.6</td>
</tr>
<tr>
<td>Temperature jump of inlet water, °C</td>
<td>56.9</td>
<td>48.6</td>
</tr>
</tbody>
</table>

Table 2 Derived characteristics

<table>
<thead>
<tr>
<th>Derived characteristics</th>
<th>Exp.1</th>
<th>Exp.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_h$ kJ/Ks</td>
<td>1.360</td>
<td>1.163</td>
</tr>
<tr>
<td>$C_c$ kJ/Ks</td>
<td>0.195</td>
<td>0.195</td>
</tr>
<tr>
<td>$U_{tot}$ kJ/Ks</td>
<td>137.16</td>
<td>135.76</td>
</tr>
</tbody>
</table>

Other derived characteristics:

- $C_h = 7.72$ kJ/K
- $C_c = 0.072$ kJ/K
- $U_{wi} = U_{wo} = 3.549$
- $U_{wi} = 0.140$

FIGURE 1. Simplest quasi-lumped model and linear temperature variation.
FIGURE 2. Lumped parameter model for computer simulation.

FIGURE 3. a) Experimental set up, b) heat exchanger which is tested and c) Geometry of tubes and fins.
FIGURE 4. Variation of exit temperatures $T_h$ and $T_c$ during the initial phase of experiments 1 and 2.