7-28-2008

Absorption of Finite-Sized Microperforated Panels with Finite Flexural Stiffness at Normal Incidence

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28 July 2008

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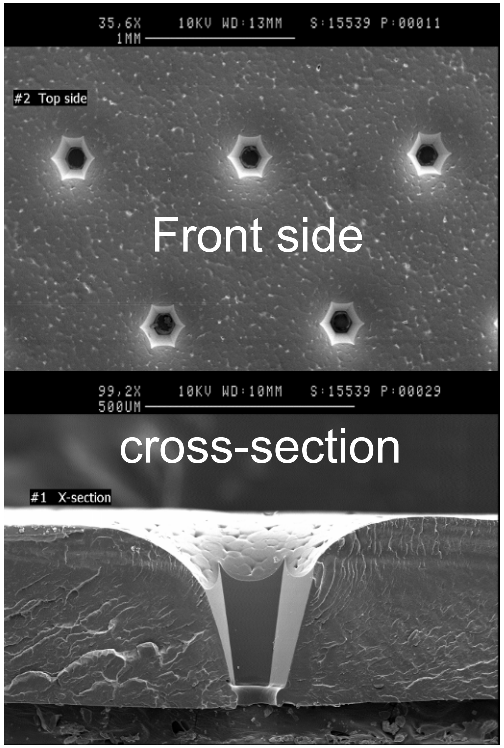
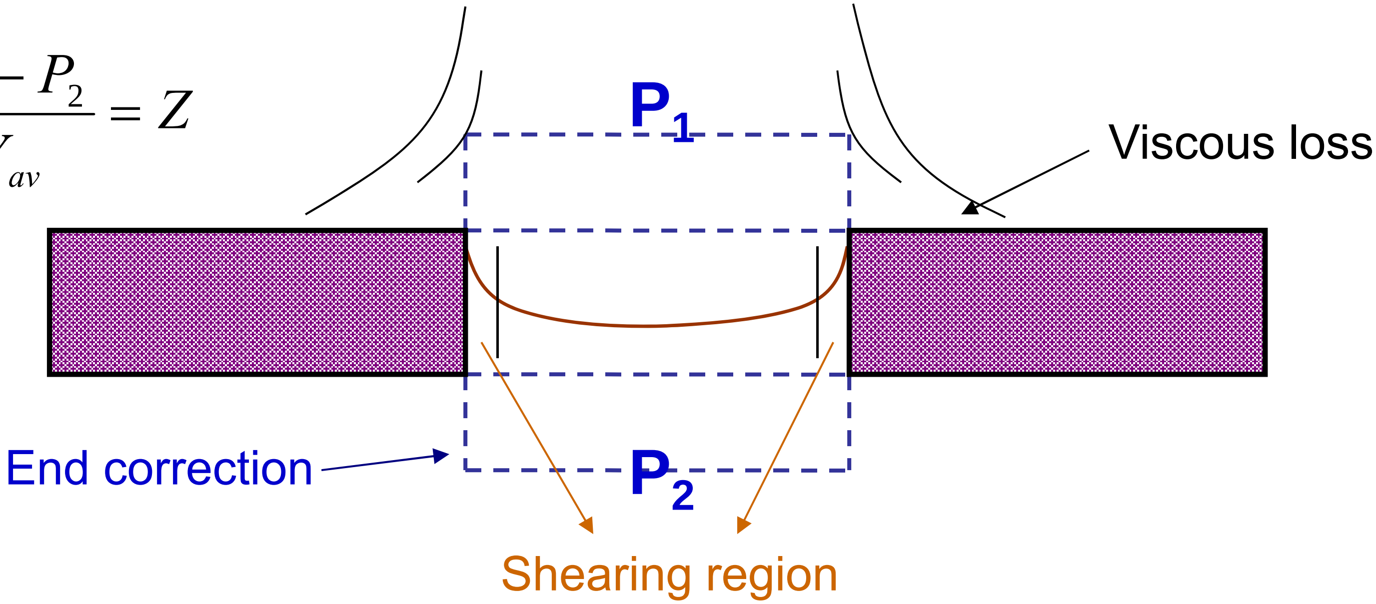
Micro-perforated panel



- Advantages over fibrous material
  - stiffness (self supporting), robust, weatherproof
- Energy dissipates when sound moves through small holes
  - d=0.1~0.9 mm, t=0.1~2 mm, N=10^3~10^6 per m^2

$$\frac{P_1 - P_2}{V_{av}} = Z$$



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Objectives

- Build models to predict the performance of micro-perforated samples in various conditions and validate the models with measurements
  - Finite-sized sample in duct with normal incidence plane wave
    - Two boundary conditions
      - Clamped-Clamped-Clamped-Clamped
      - S-S-S-S
    - Mode shapes
    - Energy dissipation

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Maa model

- Maa model:

The panel is assumed to be rigid

Perforation constant : $k = r_0\sqrt{\frac{\rho_0\omega}{\eta}} = d\sqrt{\frac{\rho_0\omega}{4\eta}}$

Transfer Impedance

$$z_t = \frac{Z_1}{\Omega_s \rho_0 c} = r + jx_m = r + j\omega m$$

$$r = \frac{32\eta t}{\sigma\rho_0 c d^2}k_r = \frac{32\eta t}{\sigma\rho_0 c d^2}\left(\sqrt{1+\frac{x^2}{32}} + \sqrt{\frac{2kd}{8t}}\right)$$

$$\omega m = \frac{\omega t}{\Omega_s c}k_m = \frac{\omega t}{\Omega_s c}\left\{1 + \frac{1}{\sqrt{9+\frac{k^2}{2}}} + 0.85\frac{d}{t}\right\}$$

d: diameter of hole

η: viscosity of air (17.9e-6)

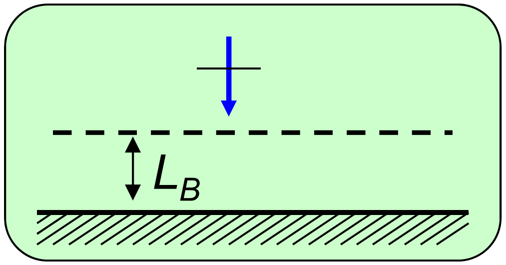
σ: porosity

t: hole depth

L_B: depth of backing space

Normal incidence Absorption coefficient

$$\alpha_n = \frac{4r}{(1+r)^2 + (\omega m - \cot(\frac{\omega L_B}{c}))^2}$$



Flow resistance (R_t) : $\frac{32\eta t}{\sigma d^2}\left(\sqrt{1+\frac{x^2}{32}} + \sqrt{\frac{2kd}{8t}}\right)$, End correction ($\delta$): $\delta = \frac{1}{2}\left\{\frac{t}{\sqrt{9+\frac{k^2}{2}}} + 0.85d\right\}$

R_t and δ are used in finite-sized models

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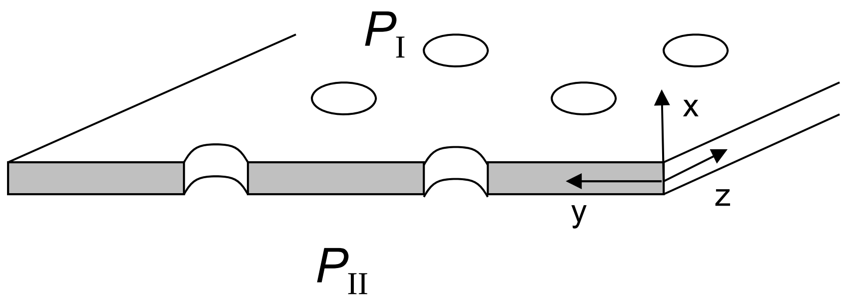
Equations of motion and velocity conditions

Volume velocity continuity at x=0

$$\begin{cases} -\dfrac{1}{j\omega\rho_o}\dfrac{\partial p_I}{\partial x}\Big|_{x=0} = (1-\Omega)\dfrac{\partial d_s}{\partial t} + \Omega\dfrac{\partial d_f}{\partial t} \\ -\dfrac{1}{j\omega\rho_o}\dfrac{\partial p_{II}}{\partial x}\Big|_{x=0} = (1-\Omega)\dfrac{\partial d_s}{\partial t} + \Omega\dfrac{\partial d_f}{\partial t} \end{cases}$$



Force equilibrium at x=0

Solid

$$p_I - p_{II} + R_t\dfrac{\Omega^2}{1-\Omega}\dfrac{\partial(d_f-d_s)}{\partial t} = D\nabla^4 d_s - T\nabla^2 d_s + \rho_s\dfrac{\partial^2 d_s}{\partial t^2}$$

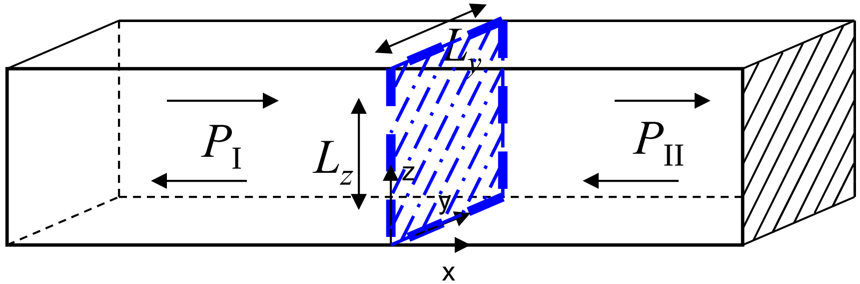
Fluid

$$p_I - p_{II} - R_t\Omega\dfrac{\partial(d_f-d_s)}{\partial t} = j\omega\rho_o h'\dfrac{\partial d_f}{\partial t}$$

- p_I: Pressure at source side
- p_{II}: Pressure behind the panel
- d_s: Displacement of solid part
- d_f: Displacement of fluid part
- ρ_s: Membrane mass per unit area
- R_f: Flow resistance
- D: Flexural stiffness
- T: Tension
- h': Effective thickness
- Ω: Porosity

Multi-mode solution in duct and membrane

- 3-dimensional model



Only symmetric modes exist

Sound pressure in each region

$$P_I = e^{-jkx} + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} B_{mn}\cos(k_{2m}z)\cos(k_{2n}y)e^{jk_{x2m2n}x}$$

$$P_{II} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_{mn}\cos(k_{2m}z)\cos(k_{2n}y)\left(e^{-jk_{x2mn}x} + e^{jk_{x2mn}(x-2L)}\right)$$

$$k_{2m} = \frac{2m\pi}{L_z} \quad k_{2n} = \frac{2n\pi}{L_y}$$

(m, n=0,1,2,…)

$$k_{x_{2m2n}} = \sqrt{k^2 - k_{2m}^2 - k_{2n}^2} \quad (k>k_{2n}+k_{2m})$$

$$= -j\sqrt{k^2 - k_{2m}^2 - k_{2n}^2} \quad (k<k_{2n}+k_{2m})$$

Displacement of membrane

for simply supported BC

Solid part $d_s = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\sin(k_{2m-1}z)\sin(k_{2n-1}y)$

Fluid part $d_f = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} F_{mn}\cos(k_{2m}z)\cos(k_{2n}y)$

for clamped BC

Solid part $d_s = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{mn}\{\cos(k_{2m}z)-1\}\{\cos(k_{2n}y)-1\}$

Fluid part $d_f = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} F_{mn}\cos(k_{2m}z)\cos(k_{2n}y)$

$$k_{2m-1} = \frac{(2m-1)\pi}{L_z}, \quad k_{2n-1} = \frac{(2n-1)\pi}{L_y} \quad (m, n=1,2,\ldots)$$

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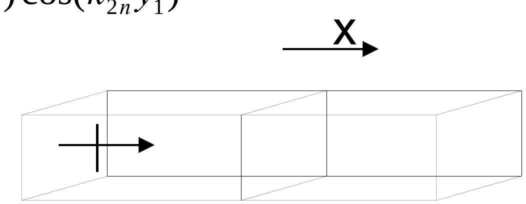
(Modeling) Application of boundary conditions

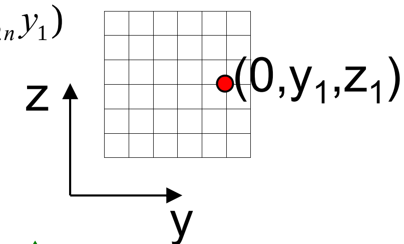
① $$-\frac{1}{j\omega\rho_o}\frac{\partial p_{\mathrm{I}}}{\partial x}\bigg|_{x=0} = (1-\Omega)\frac{\partial d_s}{\partial t}+\Omega\frac{\partial d_f}{\partial t}$$

→ $$j(1-\Omega)\omega^2\rho_0\sum\sum A_{mn}\sin(k_{2m-1}z_1)\sin(k_{2n-1}y_1)+j\Omega\omega^2\rho_0\sum\sum F_{mn}\cos(k_{2m}z_1)\cos(k_{2n}y_1)$$
$$+\sum\sum B_{mn}k_{x_{2m2n}}\cos(k_{2m}z_1)\cos(k_{2n}y_1)=k$$



② $$-\frac{1}{j\omega\rho_o}\frac{\partial p_{\mathrm{II}}}{\partial x}\bigg|_{x=0} = (1-\Omega)\frac{\partial d_s}{\partial t}+\Omega\frac{\partial d_f}{\partial t}$$

→ $$j(1-\Omega)\omega^2\rho_0\sum\sum A_{mn}\sin(k_{2m-1}z_1)\sin(k_{2n-1}y_1)+j\Omega\omega^2\rho_0\sum\sum F_{mn}\cos(k_{2m}z_1)\cos(k_{2n}y_1)$$
$$-\sum\sum C_{mn}k_{x_{2m2n}}\cos(k_{2m}z_1)\cos(k_{2n}y_1)=0$$



③ $$p_{\mathrm{I}}-p_{\mathrm{II}}+R_t\frac{\Omega^2}{1-\Omega}\frac{\partial(d_f-d_s)}{\partial t}=D\nabla^4 d_s-T\nabla^2 d_s+\rho_s\frac{\partial^2 d_s}{\partial t^2}$$

→ $$\sum\sum A_{mn}\sin(k_{2m}z_1)\sin(k_{2n}y_1)\left\{D(k_{2m-1}^2+k_{2n-1}^2)^2+T(k_{2m-1}^2+k_{2n-1}^2)+j\omega R_f\frac{\Omega^2}{1-\Omega}-\omega^2\rho_s\right\}-\sum\sum B_{mn}\cos(k_{2m}z_1)\cos(k_{2n}y_1)$$
$$+\sum\sum C_{mn}\cos(k_{2m}z_1)\cos(k_{2n}y_1)-R_f\frac{\Omega^2}{1-\Omega}(j\omega)\sum\sum F_{mn}\cos(k_{2m}z_1)\cos(k_{2n}y_1)=1$$

④ $$p_{\mathrm{I}}-p_{\mathrm{II}}-R_t\Omega\frac{\partial(d_f-d_s)}{\partial t}=j\omega\rho_o h'\frac{\partial d_f}{\partial t}$$

→ $$j\omega\Omega R_f\sum A_{mn}\sin(k_{2m-1}z_1)\sin(k_{2n-1}y_1)+\sum\sum F_{mn}\cos(k_{2m}z_1)\cos(k_{2n}y_1)\left(\omega^2\rho_0 h'-R_f\Omega j\omega\right)+\sum\sum B_{mn}\cos(k_{2m}z_1)\cos(k_{2n}y_1)$$
$$-\sum\sum C_{mn}\cos(k_{2m}z_1)\cos(k_{2n}y_1)=-1$$

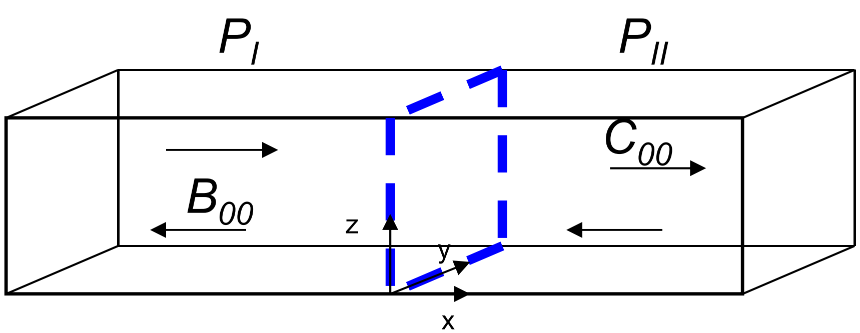
Building a matrix from BC's

$$X \times \begin{bmatrix} A_{mn} \\ F_{mn} \\ B_{mn} \\ C_{mn} \end{bmatrix} = Y \quad \Rightarrow \quad \begin{bmatrix} A_{mn} \\ F_{mn} \\ B_{mn} \\ C_{mn} \end{bmatrix} = X^{-1} \times Y$$

Solve for B_{00} and C_{00}

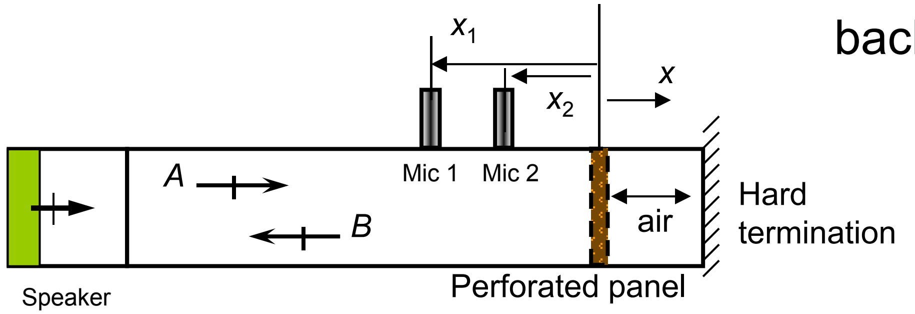
B_{00}: reflection coefficient $\Rightarrow$ $\alpha_n = 1 - |B_{00}|^2$

C_{00} : transmission coefficient $\Rightarrow$ $TL = 20\log_{10} \frac{1}{|C_{00}|}$



B_{00} and C_{00} are amplitudes of the plane waves

Absorption coefficient (ASTM E1050)



Using finite air backing space

Normal Incidence Absorption coefficient

1. Sound pressures

$$P_1 = \left(Ae^{-jkx_1} + Be^{jkx_1}\right)e^{j\omega t}$$
$$P_2 = \left(Ae^{-jkx_2} + Be^{jkx_2}\right)e^{j\omega t}$$

2. Measuring transfer function

$$H_{21} = \frac{Ae^{-jkx_2} + Be^{jkx_2}}{Ae^{-jkx_1} + Be^{jkx_1}}$$
$$H_{21} = \frac{e^{-jkx_2} + \mathrm{Re}^{jkx_2}}{e^{-jkx_1} + \mathrm{Re}^{jkx_1}}$$

3. Solve for R

$$R = \frac{-H_{21}e^{-jkx_1} + e^{-jkx_2}}{H_{21}e^{jkx_1} - e^{jkx_2}}$$

4. Absorption coefficient

$$\alpha = 1 - |R|^2$$

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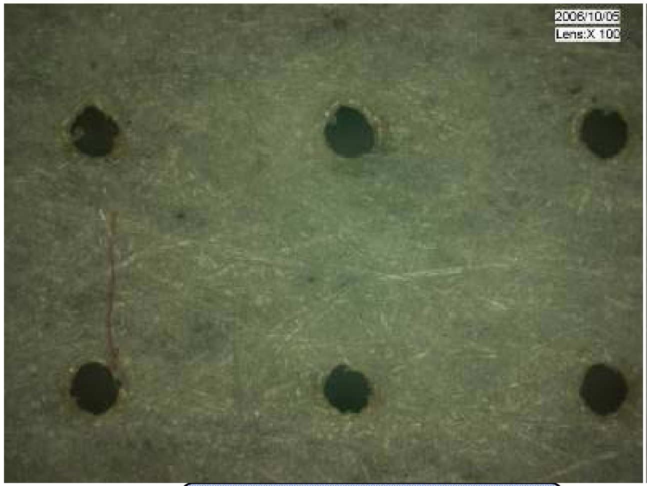
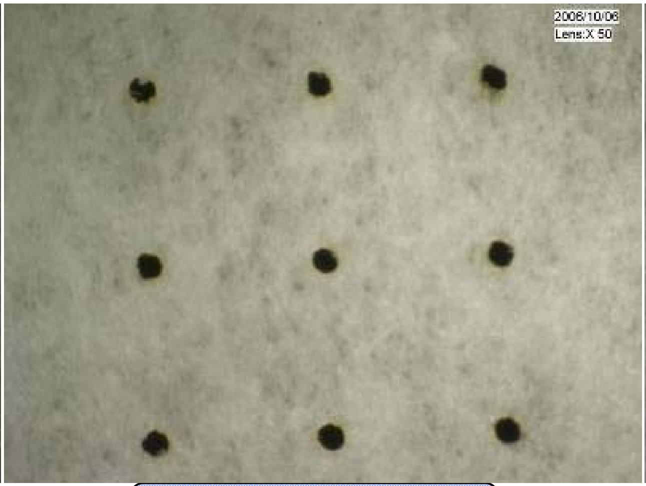
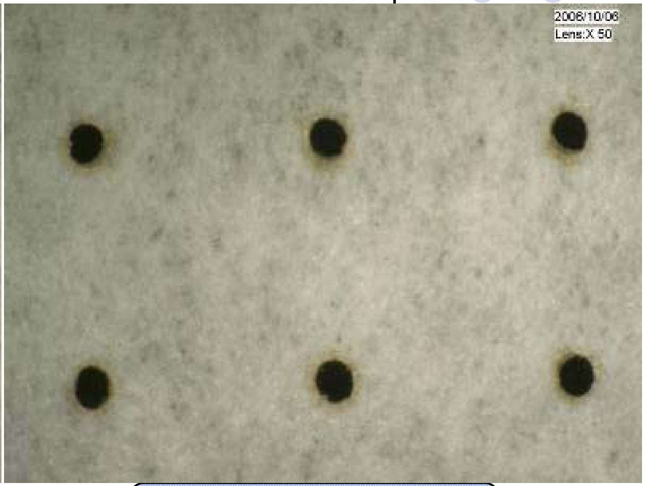
Measurement of Plexiglass samples



Sample 1 (x100)

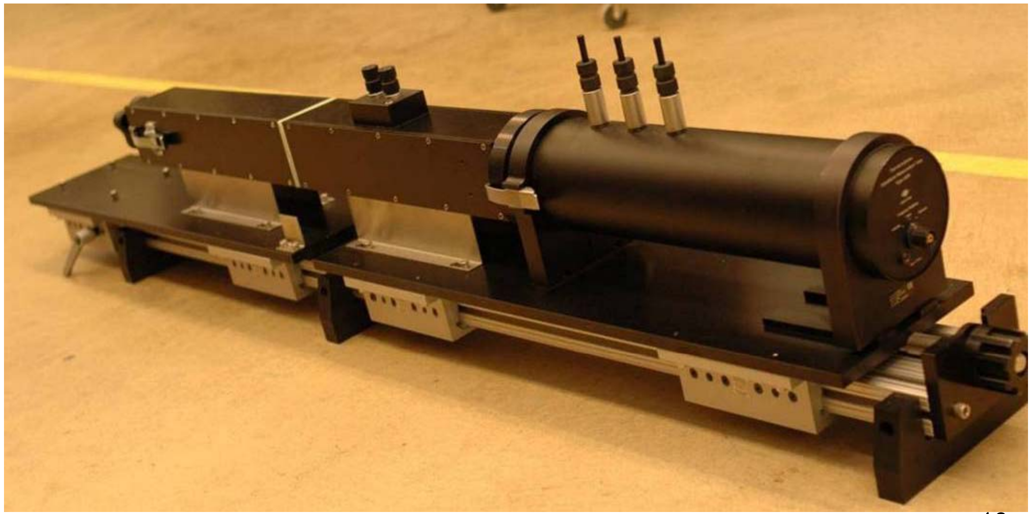


Sample 2 (x50)

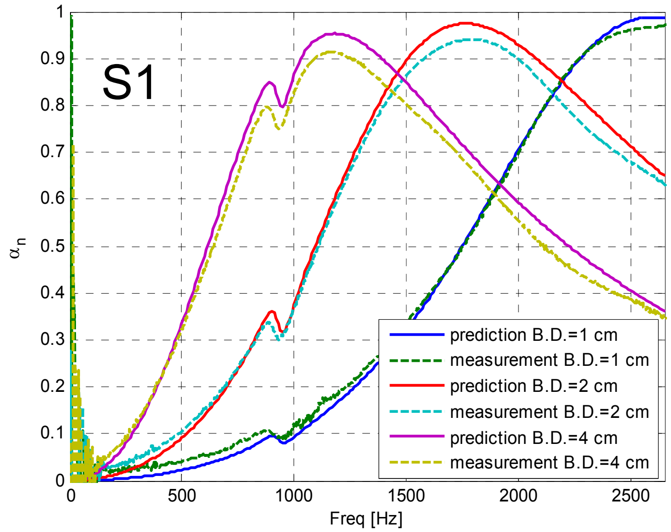


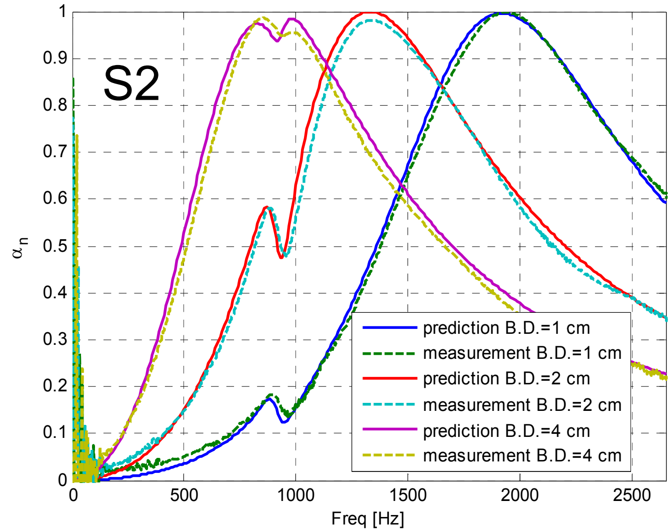
Sample 3 (x50)

	d (nominal) [mm]	*t* [mm]	ρ_s [kg/m^2]	N
Sample 1	0.254	1.588	1.584	722500
Sample 2	0.2667	1.588	1.627	291600
Sample 3	0.4064	1.588	1.631	160000

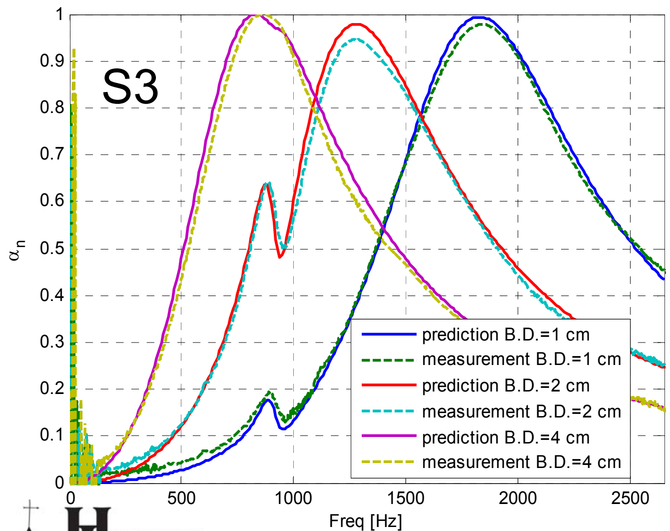


Comparison between predictions and measurements





Good agreement is shown between measurements and predictions by using adjusted hole size



	d (nominal) [mm]	*d* (adjusted) [mm]	*t* [mm]	*D* / loss factor [N·m^2]	*T* [N]	ρ_s [kg/m^2]	*N*
S 1	0.254	0.305	1.588	0.7/ 0.07	0	1.584	722500
S 2	0.2667	0.35	1.588	0.7/ 0.07	0	1.627	291600
S 3	0.4064	0.45	1.588	0.7/ 0.07	0	1.631	160000

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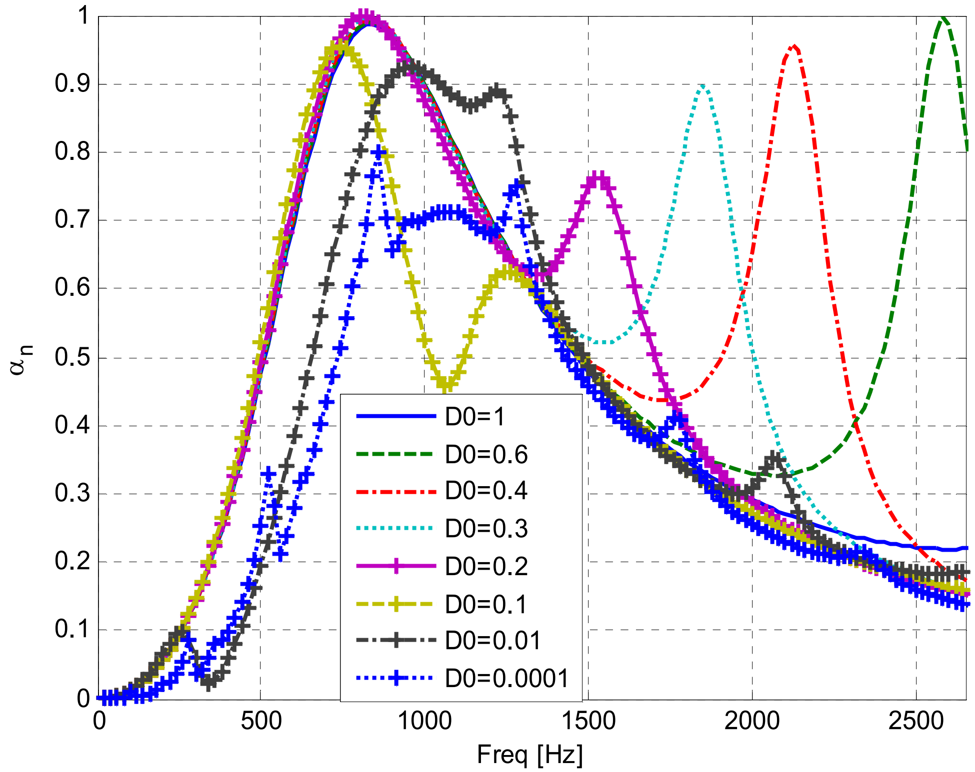
Flexural stiffness effect



Depending on the flexural stiffness, the absorption performance can be enhanced with a proper loss factor

d [mm]	*t* [mm]	*D* [N·m^2]	loss factor in *D*	*T* [N]	Mass/area [kg/m^2]	*N*	Size [mm]
0.45	1.588	1, 0.6, 0.4, 0.3, 0.2, 0.1, 0.01, 0.0001	0.05	0	0.1631	160000	63.5 x 63.5

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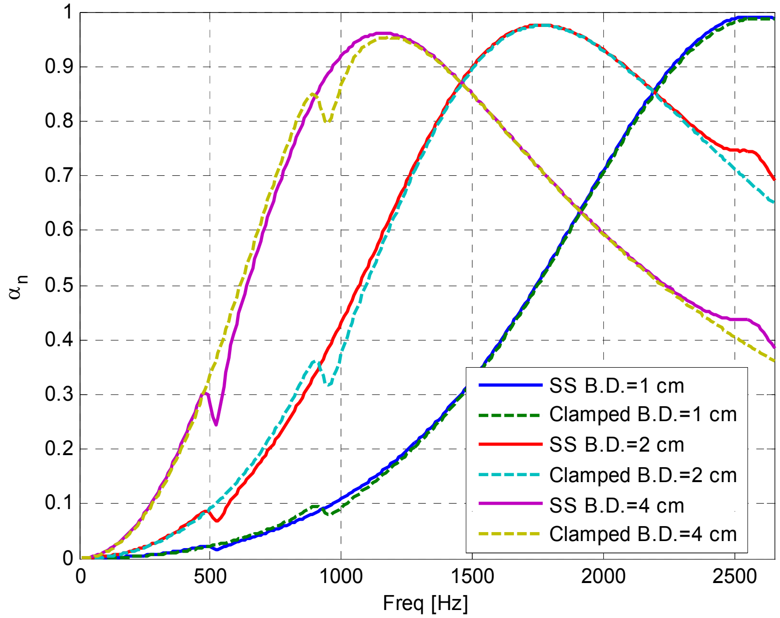
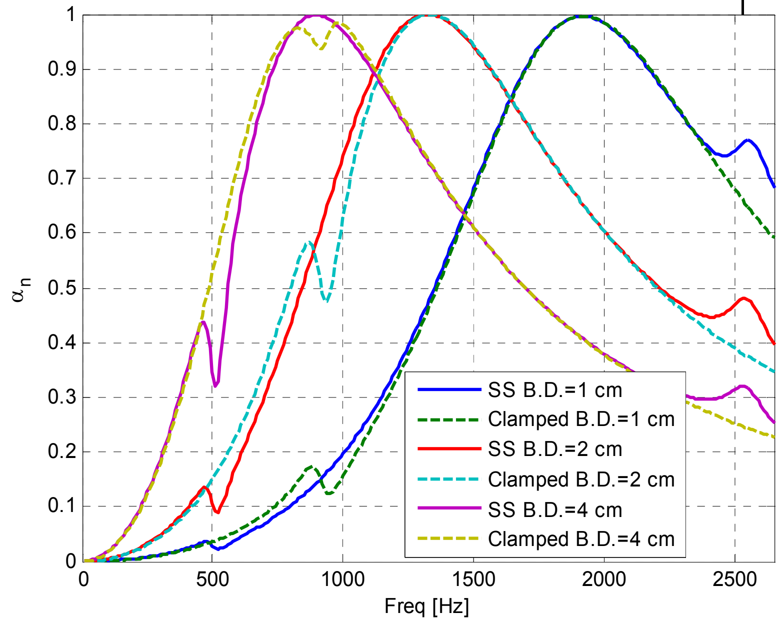
Boundary condition effect



Plexiglass Sample 1



Plexiglass Sample 2

Although same material properties were used, very different flexural natural frequency is shown depending on the boundary condition.

Simply supported boundary condition has lower natural frequency

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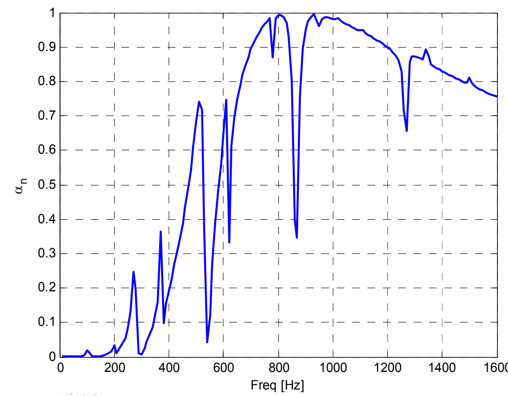
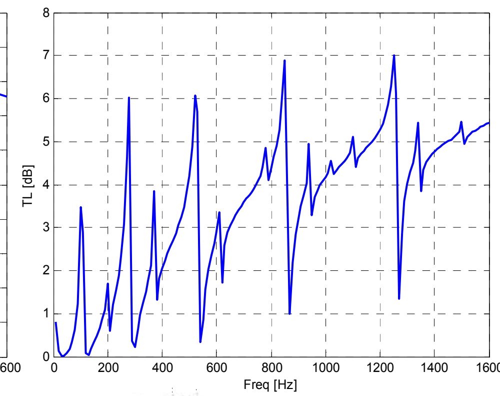
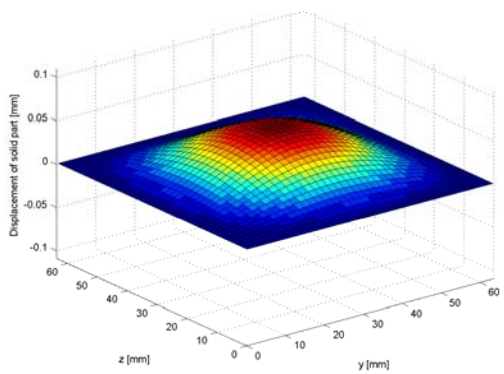
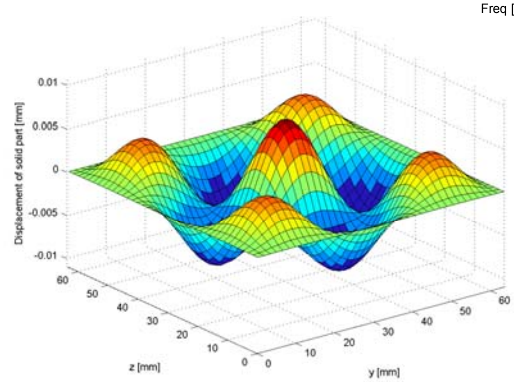
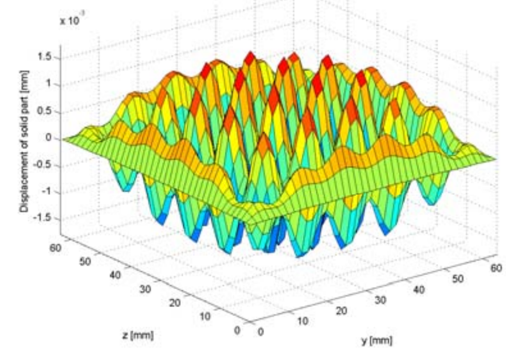
3D Clamped-Clamped Displacements of fluid and solid parts







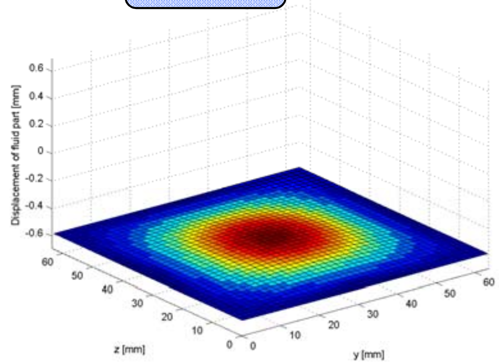


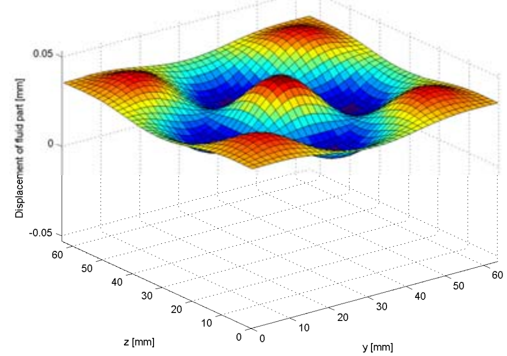


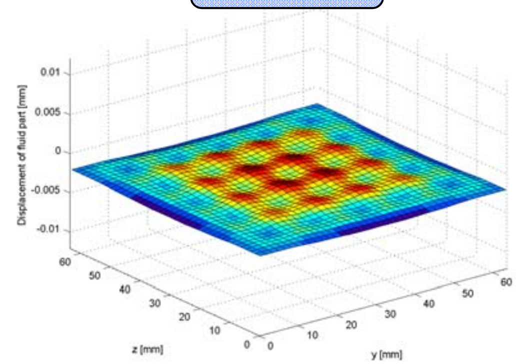
10 Hz

200 Hz

1250 Hz







d [mm]	*t* [mm]	*D* [N·m^2]	loss factor	*T* [N]	ρ_s [kg/m^2]	*N*	Size [mm]
0.15	0.4	0.0001	0.001	0	0.1631	756589	63.5 x 63.5

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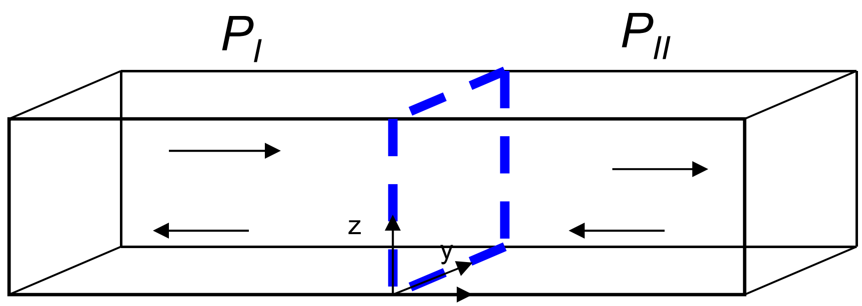
Energy dissipation

$$W_{\mathrm{I}} = \frac{1}{2}\mathrm{Re}\left\{\iint P_{\mathrm{I}} u_{\mathrm{I}}^{*} dydz\right\}$$



$$W_{\mathrm{II}} = \frac{1}{2}\mathrm{Re}\left\{\iint P_{\mathrm{II}} u_{\mathrm{II}}^{*} dydz\right\}$$

From sound fields

$$W_{\mathrm{E.D.\_field}} = W_{\mathrm{I}} - W_{\mathrm{II}}$$

$$W_{E.D.\_solid} = \frac{1-\Omega}{2}\mathrm{Re}\left\{\iint\left(D\nabla^4 d_s - T\nabla^2 d_s + \rho_s \frac{\partial^2 d_s}{\partial t^2} - R_t \frac{\Omega^2}{1-\Omega}\frac{\partial(d_f - d_s)}{\partial t}\right)\left(\frac{\partial ds}{\partial t}\right)^{*} dydz\right\}$$

$$W_{E.D.\_fluid} = \frac{\Omega}{2}\mathrm{Re}\left\{\iint\left(R_t\Omega\frac{\partial(d_f - d_s)}{\partial t} + j\omega\rho_0 h'\frac{\partial d_f}{\partial t}\right)\left(\frac{\partial df}{\partial t}\right)^{*} dydz\right\}$$

From displacements of solid and fluid parts

$$W_{\mathrm{E.D.\_disp.}} = W_{\mathrm{E.D.\_solid}} + W_{\mathrm{E.D.\_fluid.}}$$

$$W_{\mathrm{E.D.\_field}} = W_{\mathrm{E.D.\_disp.}}$$

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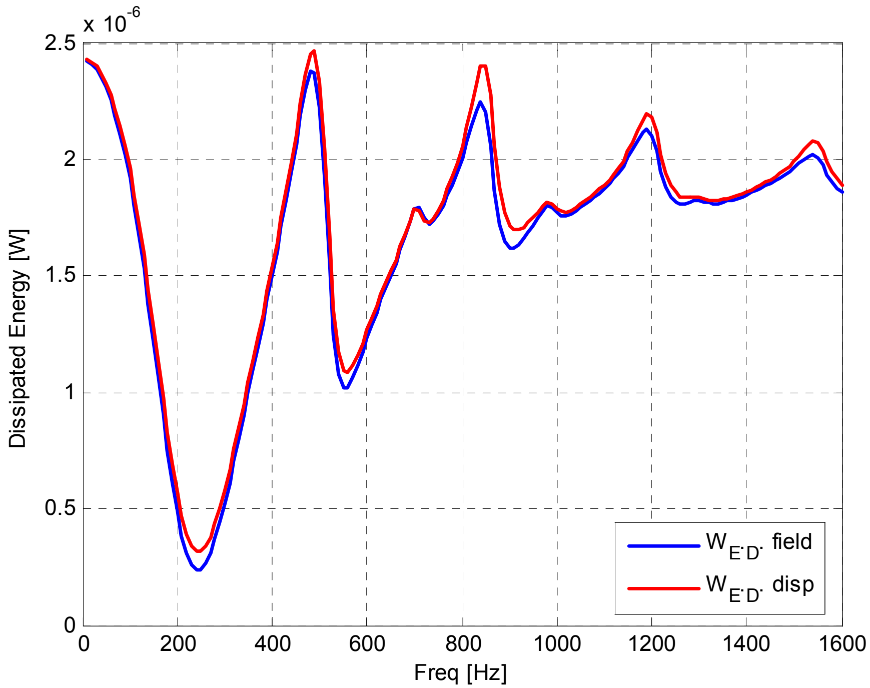
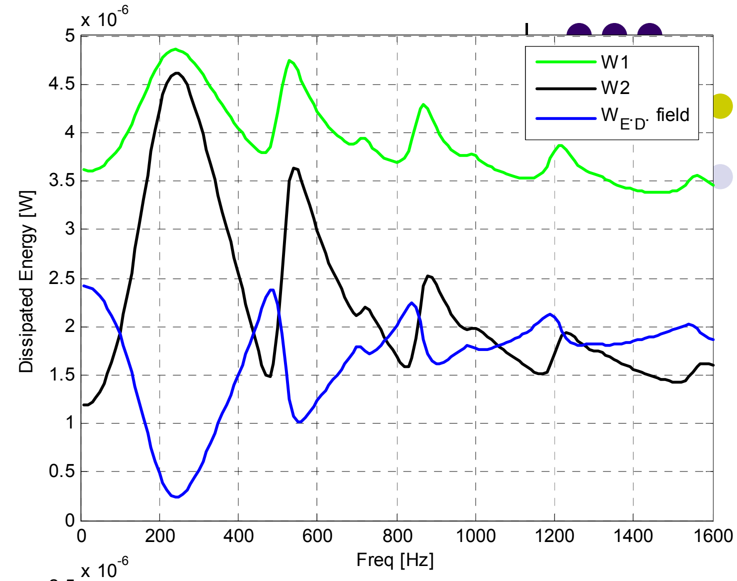
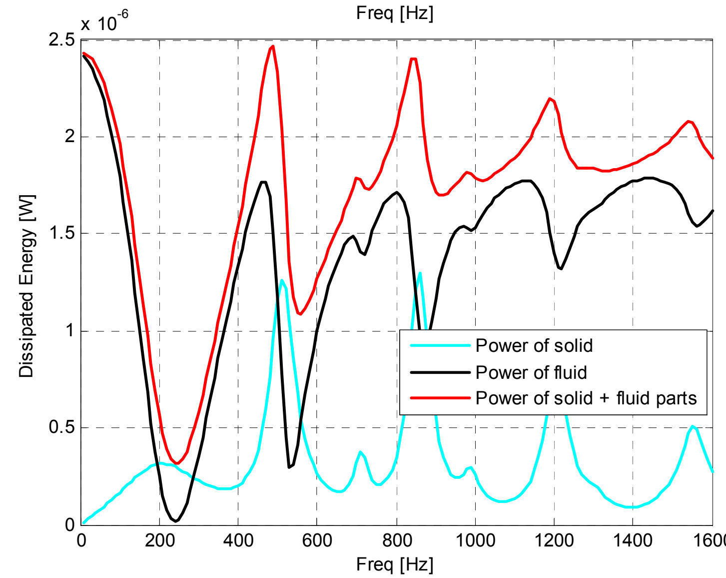
Energy dissipation







d [mm]	t [mm]	Number of holes per unit area	Mass/area [kg/m²]	Tension / loss factor	Flexural Stiffness/ loss factor	Panel size
0.15	0.4	756,589	0.174	85.41/0.05	0	63.5 x 63.5 mm

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Conclusions

- Found that flexural vibration of finite micro-perforated panels can enhance absorption
- Developed initial model that accounts for main features of flexural resonances and their interaction with viscous dissipation in holes
- Verified this results with normal incidence absorption coefficient measurement
- Energy is dissipated both by viscous flow through holes and by flexural losses in the solid