LABORATORY MODELING OF THERMAL STRUCTURE IN STAGNANT WATER

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ABSTRACT

Thermal stratification induced by solar radiation (insolation) and cooling of thermally stratified and unstratified water from the surface has been modeled in a laboratory to gain improved understanding of temperature structure in lakes, reservoirs, and ponds. Understanding of internal physical processes in stagnant waters is required in order to develop predictive models for complex hydraulic systems on a steady and unsteady basis. Such models are needed by water pollution control agencies, electric utilities, and those concerned with water quality and ecology.

Experimentally, solar radiation was simulated by tungsten filament lamps in parabolic reflectors of known spectral radiation characteristics. The unsteady temperature distribution resulting from radiant heating of pure water in a glass-walled test cell was measured with a Mach-Zehnder interferometer. An analytical model appropriate for the laboratory arrangement has been developed to predict the thermal stratification induced by radiation. Comparison between measured and predicted temperature profiles showed good agreement thus verifying the radiative and total energy transport models.

Cooling of an initially uniform and thermally stratified layer of water was studied by observing the interference fringe patterns and by measuring interferometrically the temperature distribution. The buoyancy induced flow and temperature structure was quite complex with the detailed observations and discussion of results given in the body of the report. A model was developed to predict the dynamics of the convective (mixed) layer, and the predictions were found to be in good agreement with the experimental data. However, the temperature structure in a thermally stratified layer of water which was being cooled from the surface could not be modeled adequately on the basis of the simple turbulence and thermal energy models employed.
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LIST OF SYMBOLS

c  specific heat
D  diffusion coefficient of water vapor in air
e  mean kinetic energy of turbulence
F  radiative flux
$F_0$  surface absorbed radiative flux defined by Eq. (11)
g  mass transfer coefficient of acceleration due to gravity
H  volumetric rate of internal absorption of radiation defined by Eq. (10) or heat flux defined by Eq. (25)
h  convective heat transfer coefficient or convective (mixed)
layer thickness
$h$  effective heat transfer coefficient defined by Eq. (11)
$h_{fg}$  latent heat of vaporization
$k$  thermal conductivity
$L$  depth of the layer of water
$p$  partial pressure of water vapor in air
$q$  heat flux
$T$  temperature
$T_a$  effective ambient temperature
$T'$  fluctuating temperature
$t$  time
$w'$  fluctuating vertical velocity component
$x$  distance measured from the water surface
$\alpha$  thermal diffusivity, k/pc
$\alpha_t$  turbulent diffusivity
$\beta$  thermal expansion coefficient
$\gamma$  temperature gradient defined in Eq. (21)
$\Delta$  defined by Eq. (25)
$\delta$  thickness of the surface skin layer
$\varepsilon$  rate of dissipation of turbulent energy or emissivity
$\kappa$  absorption coefficient or factor defined in Eq. (30)
\( \lambda \) wavelength
\( \rho \) density or reflectivity of air-water interface
\( \sigma \) Stefan-Boltzmann constant
\( \tau \) transmissivity of air-water interface

Subscripts

a ambient air
b bottom
inc incident
m mixed mean
o surface
\( 0 \) initial temperature
s top of the stable layer
1. INTRODUCTION

1.1 Statement of Problem

The thermal structure and energy transfer in natural bodies of water such as lakes, reservoirs, and ponds is of interest for many reasons. The vertical temperature (density) variations in a waterbody have important effects on chemical and physical properties, dissolved oxygen, biological oxygen demand, chemical oxygen demand, aquatic life, and ecological balance as well as mixing processes in water. The thermal structure influences the plant and animal life and affects the status of the water resources. Data on thermal structure often provide the basic information that leads to better understanding of complex hydraulic phenomena. Such data are invaluable as inputs to those concerned with water quality and ecological studies and as a verification of water quality models. Also, water temperatures must be monitored by regulatory agencies responsible for determining if the temperature standards that have been designed to protect aquatic life from adverse thermal impacts are being met.

Concern with water quality, thermal pollution, aquatic life, eutrophication, persistence of DDT and other nonionic synthetic organic compounds as well as chemicals in lakes, reservoirs, and ponds has generated a need to gain understanding of the thermal structure in stagnant waters. Models for predicting thermal structure in waters are needed by electric utilities and regulatory agencies as well as provide valuable information to those concerned with ecology and water quality. The research needs in the area of modeling of the thermal structure in waters has been discussed in some detail by Tichenor and Cawley (1969) and Parker and Krenkel (1969), and there is no need to summarize their recommendations. Unfortunately, the physical processes governing the internal dynamics of impounded water-bodies is not completely understood, and the prediction of biological and ecological phenomena as well as dispersion of thermal effluents cannot proceed until sufficient knowledge of fluid dynamics and energy transfer in waters has been gained.
The principal difficulties in modeling thermal structure in natural waters is lack of knowledge of internal physical processes such as mixing and energy transfer. Several energy transfer processes must be taken into account simultaneously, and the sources of energy are not obvious. An array of mixing phenomena which can be relevant in large waterbodies has been reviewed (Turner, 1973). However, pertinent quantitative data are lacking to understanding detailed physical processes of stratification mixing and restratification in thermally stratified natural bodies of water heated by solar irradiation. For example, the thermal stratification is governed by an energy balance involving insolation, heat losses by convection, evaporation (or condensation), and radiation from the water surface, molecular and/or turbulent diffusion and advective inflows and outflows, if any. The thermal stratification, through the density, has a profound influence on the flow patterns in a lake, reservoir, or pond since vertical motions (mixing) are inhibited in density stratified waterbodies. Such internal mixing processes are important, for example, not only in the transport of oxygen or any other substance introduced into the water but also in establishing the thermal structure in the water.

The diurnal, seasonal, and spatial variations in the temperature profiles in lakes, reservoirs, and oceans has long been recognized and investigated by limnologists and oceanographers. A review of these experimental field and empirical analytical studies is available (Ryan, 1972; Turner, 1973) and there is no need to review these surveys. Few laboratory studies have been reported because researchers believed that it was impossible to simulate realistically natural phenomena in a laboratory and then to transfer and apply the results to the field. However, the same can be said about field studies. Data and empirical correlations which typically result from such studies are applicable only to the specific waterbodies and meteorological conditions investigated, and the range of validity and applicability of the correlations are not known (Benedict, Anderson, and Yandell, 1973). Application of the results to situations different from those studied involves considerable risk if the phenomena is not completely understood.

It appears that fundamental laboratory and analytical studies must be performed to acquire the necessary data and knowledge. The research described in this report is intended to fill some gaps in the available
knowledge and seek to develop basic understanding and modeling capability of energy transport in stagnant water. To this end, experiments in a laboratory where the conditions can be controlled would be desirable. The laboratory experiments can be modeled analytically to verify their validity or to check available empirical correlations. The results obtained and understanding gained would then be valuable in developing much needed deterministic models for predicting the hydraulic and thermal characteristics of complex systems on both macro and micro scales (Tichenor and Cawley, 1969; Parker and Krenkel, 1969). This is the eventual goal of the research.

1.2 Scope of Study

The complexity of the energy transport and thermal stratification processes in natural waters and the difficulty of their realistic experimental and analytical modeling are fully recognized. Effects that cannot be accurately simulated in a laboratory or influences that make it impossible to describe the relationships mathematically and which lead to equations that cannot be solved are excluded from the consideration. Consequently, laboratory models cannot adequately simulate the conditions existing in natural waterbodies. However, this should not discourage attempts to gain knowledge of the natural physical processes in waters. The basic phenomena must be clearly understood before analytical models can be constructed and used intelligently in predictions.

As the title makes clear, the present study is limited to experiments and analyses of energy transfer in a laboratory model of stagnant water where experimental conditions can be carefully controlled. The research is relevant to:

1. Environmental impact of heated water discharges,
2. Development of improved mathematical models used to describe the hydraulic and thermal characteristics of complex systems,
3. Interpretation of field data,
4. Dynamics of eutrophication and the dynamics and persistence of DDT as well as other nonionic synthetic compounds in aquatic ecosystems, and
5. Water resources as a science by a better understanding of the internal mixing and energy transport processes.
The diagnostic techniques and procedures employed are discussed in Chapter 2. Mathematical analyses modeling the experimental arrangement are described in some detail in Chapter 3. Experimental results for several different simulations and comparison of data with theoretical predictions are presented in Chapter 4. The conclusions and recommendations are summarized in Chapter 5.
2. EXPERIMENTAL RESEARCH PROCEDURES

2.1 General Considerations

To attain the objectives of the research program, data on the internal temperature field in water under different heating and/or cooling conditions are needed. However, there are difficulties in measuring the temperature distribution in water using probes (thermocouples, thermistors, and other sensors) because probes of any kind disturb and distort the temperature and radiation fields and, if not carefully executed, yield erroneous results. To overcome these difficulties the temperature distribution data are obtained using accurate and efficient optical (interferometric) techniques. The advantages of measuring steady and unsteady temperature fields are well known in the engineering literature (Hauf and Grigull, 1970).

The uniqueness of the research is the use of a Mach-Zehnder interferometer for optical measurement of unsteady temperature and flow visualization in a test cell filled with water. The interferometer does not appear to have been employed previously in studies concerned with water resources, and therefore was adopted as a diagnostic tool. The Mach-Zehnder interferometer is an ideal instrument to study two-dimensional heat transfer and fluid flow phenomena. If it is possible to align the axis of symmetry of the phenomena to be studied along the optical axis, then each fringe shift can be directly related to a change in the index of refraction of the test fluid. Since the instrument senses differences in the refractive index of water, it requires no physical contact of a foreign object such as thermocouples or thermistors in the fluid, yields an instantaneous temperature field in a layer of water and does not disturb or distort the temperature, flow, or radiation fields. For this reason, Interferometry is then considered the best method for obtaining accurate quantitative temperature data on an unsteady or steady basis (Hauf and Grigull, 1970) while also being useful for qualitative visualization through the temperature field of stability and mixing.
2.2 Test Apparatus

A Mach-Zehnder interferometer was the center of the test apparatus which is shown schematically in Figure 1. Using an interferometer, the instantaneous two-dimensional temperature distribution can be recorded without disturbing the field. The interferometer used in the study was of typical rectangular design with 25 cm diameter optics. A He-Ne laser served as the light source and a system of lenses with a 25 cm diameter parabolic mirror produced a collimated beam.

The solar irradiation (insolation) was modeled using tungsten filament lamps in parabolic reflectors with known spectral radiance characteristics. The reflectors were designed to provide a collimated radiation beam within a 5 cm wide by 25 cm long rectangular region.

To simulate cooling of water from the surface in some experiments, a heat sink was used. The sink was designed to fit into the test cell and was installed (from about 0.1 to 2 cm) above the water surface but not in direct contact with it. The heat sink was made from a copper block in which a labyrinth passage was machined in order to increase the uniformity of temperature and a coolant (ethylene glycol) was circulated through the passage. The coolant temperature was controlled using a Lauda Ultra Kryomat.

2.3 Test Cell

A rectangular test cell was placed in one leg of the Mach-Zehnder interferometer, see Figure 1. The inside dimensions of the cell were 10 cm along the optical path, 25 cm wide, and 40 cm deep. The optical glass windows, 2.5 cm thick, acted as walls. To assure that the windows were parallel, the sides and bottoms of the cell were machined together to 10 cm (± 0.013 cm) which assured a uniform width. Aluminum frames were placed over the outside of the windows but supplied only enough force to counter the hydrostatic pressure. Silicone rubber cement was used to seal the edges of the test cell.

Urethane insulation, 5 cm thick, was placed along the side walls, and 2.5 cm thick sections covered the optical windows. The insulation on the windows could be easily removed to take photographs of the interference fringe patterns. Rubber isolation pads were placed under the legs of the
large metal test stand and the interferometer, the experiments were performed late at night when the traffic was light, and all motors were turned off in the laboratory to eliminate vibration problems and obtain quality interferograms.

2.4 Test Procedure

The test cell was cleaned and then filled with distilled water, covered, and left undisturbed for some time to eliminate all convective currents which are normally present and to attain a uniform room temperature. The procedures in the heating and cooling experiments were different and are described here.

In thermal stratification tests a thin sheet metal shield was placed approximately 30 cm above the test cell and covered the entire interferometer leg in which the test cell rested. A hole in the shield allowed only water to be heated while the shield protected other components from radiant heating and reduced free convection within the interferometer leg. After it had been determined that convective currents in the test cell were absent, the heating of the water with the radiant heaters simulating the
the solar irradiation was initiated. The interference patterns were photographed with 35 mm cameras at discrete times and simultaneously the thermocouple emf data were recorded with a digital voltmeter while the heating continued to a predetermined temperature level.

Cooling experiments with two types of initial temperature distributions were performed: (1) a uniform-temperature layer, and (2) a thermally stratified layer of water. The latter experiments were intended to simulate the thermal stratification conditions produced by solar radiation in lakes, reservoirs, and ponds. After the heating (stratification) was terminated, the water was either allowed to cool freely by convection, evaporation, and radiation heat exchange with the ambient air and by radiation with surroundings or was cooled by placing a cold cooper sink above the water but not in direct contact with it. At prescribed intervals, the interference patterns were photographed with a 35 mm camera and thermocouple emf readings simultaneously recorded.

2.5 Data Reduction

The position of the interference fringes was measured using a measuring microscope that was accurate to ±0.01 mm which corresponded to an actual distance of approximately ±0.012 mm. Subsequently, the interferograms were interpreted using the accurate relation between index of refraction and temperature data of Tilton and Taylor (1938) to obtain temperature profiles. The reference temperature needed for interpreting the interferograms was measured with a thermocouple: Type K (Chromel-Alumel), 30 gauge. A multi-channel, integrating voltmeter, with both printout and visual output, recorded the millivolt potential of the thermocouples. Before and after each experiment, distances between water surface temperature and the reference thermocouple was measured with a cathometer to determine the position of the reference thermocouple relative to the water surface.
3. THERMAL STRATIFICATION OF WATER BY SOLAR RADIATION

The importance of solar radiation in the formation of temperature structure in water bodies has been recognized for some time (Ruttner, 1963). Solar radiation is the principal natural heat load in waters; however, confusion still exists as to how to properly account for it. Bachman and Goldman (1963) and Goldman and Carter (1963), for example, presented some field measurements which indicated that the effects of direct solar radiation heating were much greater than had been previously considered. Subsequently, a number of mathematical models for simulating the diurnal as well as seasonal heating and cooling of deep water impoundments has been developed. A recent review of pertinent literature is available (Ou, et al., 1973). All of the models are one-dimensional (vertical transport only). Some investigators have considered the radiation to be absorbed at the water surface (i.e., opaque) and others treated the water as being semitransparent but ignored the spectral nature of insolation. For example, Dake and Harleman (1969) carried out laboratory experiments and detailed calculations of thermal structure formation using a model in which, except for the water layers, heat was transferred through water by combined conduction and radiation. Water motion was considered to play no part in determining the temperature structure. However, the theory of Dake and Harleman may be limited in its applicability because it does not consider the spectral nature of radiation and neglects the effects of scattering in the water. Other investigators have approximated the volumetric rate of internal absorption of radiation in the water by a single (Ou, et al., 1973) or three (Foster, 1971) term exponentials.

The only heating process of any consequence in the surface layers of water is caused by solar radiation. The ultraviolet and infrared parts of the incoming radiation are largely absorbed in the top meter of the surface, while the visible part penetrates more deeply and carries significant energy to depths on the order of tens of meters (Viskanta and Toor, 1972). Therefore, the modeling of radiative energy transfer by ignoring the spectral nature of absorption of radiation in the water, i.e., assuming that the
water is gray and has a wavelength independent absorption coefficient, is open to question and needs to be carefully examined. Laboratory studies modeling thermal stratification induced by solar radiation, where experimental conditions can be reasonably well controlled, are practically nonexistent. The notable exception is the work of Dake and Harleman (1969). The existing radiant and total energy transfer models in stagnant water have not been verified experimentally.

The purpose of the work described in this part of the report is to gain understanding of energy transfer during thermal stratification of water induced by solar radiation and to lay groundwork for modeling of more complex hydraulic systems. To this end, thermal stratification of water has been simulated in a laboratory under carefully controlled conditions to obtain the needed data. As a complement to the experimental program, a parallel analytical study was conducted. The internal energy transfer processes of molecular conduction and radiation as well as heat dissipation and evaporation from the water surface are modeled for the laboratory arrangement. The mathematical model, although crude in some respects, serves to illustrate the basic physical approach and is adequate for many problems. The experimental data are then used to evaluate the validity of the radiant and the unsteady total energy transfer models in stagnant water. Only some typical results are included in the report and reference is made to the published literature (Snider and Viskanta, 1974, 1975) for more extensive results.
3.1 Analysis of Thermal Stratification of Water Induced by Radiation

3.1.1 Physical Model and Assumptions

Consider a lake, reservoir, or pond of such large extent that the energy transfer is essentially one-dimensional, i.e., the temperature varies in the vertical direction only. Any inflows and outflows into the body of water are neglected and with only slight winds, the water is assumed to be stagnant allowing for neglect of advective energy transfer. The water is assumed to be heated by solar radiation and/or convection. Due to the diurnal and seasonal variations in incident solar radiation, atmospheric and meteorological conditions, the heating depends on the time of the day, month, and the season; however, in the present analysis the incident solar flux is taken as constant. The convective heat and mass transfer coefficients at the water surface depend on the surface temperature but are assumed uncoupled from atmospheric and meteorological conditions. The variation of these parameters with time can readily be included for specific situations if data are available. The physical model, although crude in this respect, serves to illustrate the basic physical approach.

Within the range of temperatures encountered, the variation of density, specific heat, thermal conductivity, spectral absorption and scattering coefficients of water is slight and can be justifiably neglected. Using the above assumptions the unsteady energy equation governing the temperature distribution becomes

\[ \rho c \frac{\partial T}{\partial t} = - \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} + F \right) \]  \hspace{1cm} (1)

The first term on the right-hand side of this equation represents molecular diffusion and the second, radiative transfer. The radiative transfer contribution is evaluated explicitly in the following subsection.

At the air-water interface, energy is transferred by convection, evaporation or condensation, and radiation. The instantaneous energy balance at the interface can be expressed as

\[ k \left. \frac{\partial T}{\partial x} \right|_{x=0} = q_0 = h(T_0 - T_a) + h_f g(p_0 - p_a) + \varepsilon_0 \sigma(T_0^4 - T^4) \]  \hspace{1cm} (2)
The first term on the right-hand side of Eq. (2) accounts for convection, the second for evaporation or condensation, and the third for radiation transfer in the thermal part (infrared) of the spectral region where water is effectively opaque to radiation. In writing Eq. (2) the spectrum of the radiation incident on the water surface, as is customary in problems of a geophysical nature, has been divided into two bands: (1) the solar ($0 < \lambda < \lambda_{s}$), and (2) the thermal (infrared) ($\lambda_{s} \leq \lambda \leq \infty$) bands. This separation is not only desirable from the computational point of view but is also realistic physically because water is semitransparent in the visible and the near infrared parts of the spectrum. However, in the infrared part of the spectrum ($\lambda > 2 \mu m$) the water is effectively opaque (Viskanta and Toor, 1972) and radiative transfer can be considered to be a surface rather than a volumetric phenomenon. An energy balance at the bottom results in

$$-k \frac{\partial T}{\partial x} \bigg|_{x=L} = q_{b} = q_{c} + \int_{0}^{\lambda_{s}} \alpha_{b\lambda} F_{b\lambda}(L) d\lambda$$

(3)

where $q_{c}$ represents the conductive heat transfer into the soil and the second term the absorption of transmitted solar radiation. The emission of radiation by the soil has been neglected. The temperature distribution in the water initially is taken as

$$T(x, 0) = T_{o}(x)$$

(4)

In the laboratory experiments and in natural waters the initial temperature distribution is considered to be known.

3.1.2 Radiative Transfer

Analysis of radiative energy transfer within waters has been presented by Viskanta and Toor (1972, 1973) using concepts of radiative transfer theory, and the details are omitted here. In the model, a specified solar radiation field is incident on a plane layer of water of finite depth $L$. A fraction of the incident flux is reflected, and the remaining fraction is transmitted across the interface at $x = 0$ into the water. The air-water interface is assumed to be optically smooth, and hence reflection and
transmission can be predicted from Fresnel's equations of classical electromagnetic theory. As the radiation propagates into the water, it is absorbed and scattered until it reaches the bottom at \( x = L \), where it is absorbed and diffusely reflected. The radiation reflected from the bottom then propagates through the water until it reaches the water-air interface where part of the incident radiation is transmitted into the air; the remaining part is specularly reflected down into the water. Volumetric emission of radiation by water is small compared to absorption and can therefore be neglected. For temperature levels which occur in natural waters, emitted radiation is in the infrared part of the spectrum where water is relatively opaque and this emission by water is considered to be a surface phenomenon. This has already been accounted for in Eq. (2).

Separating the incident solar radiant flux into collimated, \( F_{c\lambda}^0 \), and diffuse, \( F_{d\lambda}^0 \), components, using the forward approximation for multiple scattering (Viskanta and Toor, 1973), and assuming that the bottom soil is a gray, diffuse reflector, the total radiative flux can be expressed as

\[
F(x) = \int_0^\infty \left\{ F_{c\lambda}^0 \tau_{\lambda}(\mu^0) F_{d\lambda}^0 e^{-\kappa_{\lambda} x/\mu} + F_{d\lambda}^0 T_3(\kappa_{\lambda} x) + F_{b\lambda} R_3(\kappa_{\lambda} (L+x)) - F_{b\lambda} E_3(\kappa_{\lambda} (L-x)) \right\} d\lambda
\]  

where the spectral radiative flux leaving the bottom is given by

\[
F_{c\lambda} = 2p_{b\lambda} \left[ F_{c\lambda}^0 \tau_{\lambda}(\mu^0) e^{-\kappa_{\lambda} L/\mu} + F_{d\lambda}^0 T_3(\kappa_{\lambda} L) \right] / \left[ 1 - 2p_{b\lambda} R_3(2\kappa_{\lambda} L) \right]
\]  

The exponential, transmission, and reflection integral functions, \( E_n(x) \), \( T_n(x) \), and \( R_n(x) \), respectively, are defined as

\[
E_n(x) = \int_0^1 e^{-x/\mu} \mu^{n-1} d\mu
\]  

\[
T_n(x) = n^2 \int_{\mu_C}^1 \tau(\mu) e^{-x/\mu} \mu^{n-2} d\mu
\]  

\[
R_n(x) = \int_0^1 \rho(\mu) e^{-x/\mu} \mu^{n-2} d\mu
\]
The direction cosine $\mu$ inside the water is related to the direction cosine $\mu^0$ outside the water by Snell's law of refraction, and the critical angle $\Theta_c$ is given by $\Theta_c = \sin^{-1}(1/n_A)$. The first and second terms on the right-hand side of Eq. (5) represent attenuation of the transmitted collimated and diffuse components of the incident solar flux, respectively. The third term represents multiple internal reflections, and the fourth term represents the contribution to the flux due to the diffuse reflection from the bottom.

The total volumetric rate of internal absorption of solar radiant energy $H(x)$, i.e., the divergence of the radiative flux, is obtained by differentiating Eq. (5) and there results

$$- \frac{\partial F}{\partial x} = H(x) = \int_0^\infty 2\kappa_\lambda \left\{ I_2(\mu^0/\mu) \tau_\lambda(\mu^0) F_{c\lambda} e^{-\kappa_\lambda x/\mu} + F^0 \tau_2(\kappa_\lambda x) + F_{b\lambda} R_2[\kappa_\lambda (L+x)] + F_{b\lambda} E_2[\kappa_\lambda (L-x)] \right\} d\lambda$$

(10)

Hence, if the incident flux, spectral absorption coefficient, and index of refraction of water and its impurities, water surface, and bottom reflectivities and the depth are known, the local radiative flux and the rate of internal absorption of radiant energy in water can be predicted.

3.2 Prediction of Solar Heating

Before the temperature distribution during thermal stratification of water by radiation can be predicted, the internal volumetric absorption rate of radiant energy must be determined. Extensive results for both spectral and total local volumetric absorption rate and the flux are given elsewhere (Viskanta and Toor, 1972 and 1973) for shallow and deep waters heated by solar radiation; therefore, the results presented here are only for the laboratory heating arrangement. In order to establish the usefulness of the analytical model, radiant heating rate and flux in the water were predicted for several operating conditions of the tungsten filament lamps simulating the solar heating of the water in the test cell. Different internal and surface heating rates were obtained by operating the radiant heater at three different temperatures of 2550 K, 2880 K, and 3250 K. The spectral extinction coefficient of distilled water given by Viskanta and
Toor (1972) was used. The reflectivities of the air-water and water-test cell bottom (Plexiglas) interfaces were estimated from Fresnel's equations (Siegel and Howell, 1972). Since it was of interest to accurately predict the total local volumetric absorption rate (H) and the total radiative flux (F) these quantities were evaluated over the spectral range of interest, 0.2 \mu m < \lambda < 7 \mu m, using approximately 60 bands.

Water is relatively transparent in the spectral region from about 0.3 to 2.0 \mu m and opaque in the rest of the spectrum. The lamps in the radiant heater, operating at a lower temperature, emit proportionally more energy in the infrared than when operating at a higher temperature and thus produce greater surface heating. A comparison of lamps operating at 2550 K and 3250 K which have emission similar to Planck's function at those temperatures indicates that about 36% of the energy from the lamp at the lower temperature is in the range \lambda > 2.0 \mu m while only 23% of the energy of the lamp at the higher temperature is in that part of the spectrum where water is a strong absorber. As compared to the solar flux, which has an energy distribution similar to Planck's function at about 5800 K, the tungsten filament lamps operating at a lower temperature produce proportionately greater surface heating and less internal absorption.

Figures 2 and 3 illustrate the normalized internal heating rates and fluxes, respectively, corresponding to the experimental conditions for the lamps at the three heater temperatures. The results for solar incident flux based on the spectral distribution of Johnson (1954) are also shown in the figures for the purposes of comparison. Results are given in Figure 2 for the limiting cases of a perfectly reflecting bottom \rho_b = 1 and an absorbing bottom \rho_b = 0 which approximates the experimental test cell. It is interesting to note for solar irradiation the large amount of energy transmitted to the bottom and then absorbed (or reflected back out of the water for \rho_b = 1). Some investigators have employed a single (Dake and Harleman, 1969; Ou, et al., 1973) term exponential approximation for the radiant heating rate H(x), but as seen from Figure 2, a single term exponential cannot approximate the volumetric radiant energy absorption rate or the radiative flux accurately throughout the layer.

The extent to which the reflectivity of the bottom affects the radiative transfer in water is illustrated in Figure 4 where the volumetric absorption rate of radiation is compared for three different reflectivities,
Figure 2. Variation of the internal volumetric absorption rate of radiation with depth, $L = 38.1$ cm; lamps at 3250 K, $H(0) = 7.73(10)^3$ kW/m$^3$; lamps at 2880 K, $H(0) = 2.22 (10)^4$ kW/m$^3$; lamps at 2520 K, $H(0) = 1.56 (10)^5$ kW/m$^3$; and solar insolation (Johnson, 1954), $H(0) = 2.74 (10)^3$ kW/m$^3$.

Figure 3. Variation of total radiant flux with depth, nonreflecting bottom, $L = 38.1$ cm; lamps at 3250 K, $F(0) = 1.60$ kW/m$^2$; lamps at 2880 K, $F(0) = 1.27$ kW/m$^2$; lamps at 2520 K, $F(0) = 0.686$ kW/m$^2$; solar insolation (Johnson, 1954), $F(0) = 1.39$ kW/m$^2$. 
i.e., \( \rho_b = 0, 0.1, \) and 1.0. It is seen that little difference exists between the results for reflectivities of 0 and 0.1. It is important to note that near the bottom, the volumetric radiant energy absorption rate is small and approximating the bottom as a black body would not appreciably affect the prediction of the rate. For the estimated reflectivity of the Plexiglas bottom, \( \rho_b \approx 0.005 \), the results are essentially the same as for \( \rho_b = 0 \).

Figure 5 shows a comparison of the internal volumetric absorption rates of radiation for single- and three-term exponential approximations to the accurately computed radiant heating rate for water considering the entire absorption spectrum. It is clear from the figure that a single-term exponential cannot accurately approximate either the radiant heating rate \( H \) or the radiant flux \( F \). Near the surface, additional terms are needed for an accurate representation of \( H \) and \( F \).

3.3 Method of Solution

Since the boundary condition at the air-water interface is nonlinear and the internal volumetric rate of radiant energy absorption \( H(x) \) cannot be accurately approximated by a simple analytical function, a closed form solution of Eq. (1) is not possible. An exact solution of the energy Eq. (1) with the general boundary and initial conditions imposed can be obtained (Crosbie and Viskanta, 1968). However, first a solution of a nonlinear Volterra integral equation for the water surface temperature is required, and then it is necessary to evaluate a convolution integral for the temperature distribution. Instead, an approximate solution based on linearization of the boundary condition, Eq. (2), and a finite difference solution is presented.

3.3.1 Approximate Analytical Solution

The advantage of linearization is that a closed form approximate analytical solution is possible. If, in addition to the linearization of the interface condition Eq. (2), the local volumetric rate of absorption of radiant heat, \( H(x) \), is approximated by a series of exponentials, an exact analytical solution of the energy Eq. (1) can be obtained (Snider and Viskanta, 1974). Approximation of the total energy flux at the interface
Figure 4. Effect of the bottom reflectance on the volumetric radiant energy absorption rate, \( L = 38.1 \text{ cm} \); single lamp at 3250 K.

Figure 5. Comparison of exact, single, and three-exponential term approximations of the radiant flux and internal radiant energy absorption rate for a single lamp at 3250 K: For a three-term approximation, \( H = 30.9 \exp(-164x) + 3.4 \exp(-19.1x) + 0.206 \exp(-0.035x) \text{ kW/m}^3 \), where \( x \) is in m and \( F_{\text{surf}} = 278 \text{ W/m}^2 \). For a single-term approximation, \( H = 2.1 \exp(-9.95x) \), where \( x \) is in m and \( F_{\text{surf}} = 285 \text{ W/m}^2 \).
by a linear relation between water surface temperature $T_0$ and ambient air temperature $T_a$ is reasonable for small temperature differences, $(T_0 - T_a)$. The boundary condition at the interface $(x = 0)$ can then be approximated as

$$k \frac{dT}{dx} \bigg|_{x=0} = \tilde{h}[T_0(t) - T_a] - F_0$$  \hspace{1cm} (11)

where $\tilde{h}$ is an effective linearized heat exchange coefficient. The surface absorbed flux $F_0$ is defined as

$$F_0 = \int_{\lambda_{\text{opaque}}}^\infty \alpha_{\lambda} F_{\text{inc},\lambda} d\lambda$$  \hspace{1cm} (12)

where $\alpha_{\lambda}$ is the spectral absorptivity of the water surface and $F_{\text{inc},\lambda}$ is the spectral incident flux. The integration indicated in Eq. (12) is performed over that part of the spectrum where water may be considered opaque to radiation, i.e., $\lambda > 2 \mu m$.

The volumetric rate of absorption of radiant energy in water can accurately be approximated by a sum of exponentials such that

$$H(x) \approx \sum_{i=1}^{m} B_i e^{-\kappa_i x}$$  \hspace{1cm} (13)

where $B_i$ and $\kappa_i$ are appropriately determined constants. The accuracy with which Eq. (13) can approximate the predicted variation of $H$ with depth $x$ will be discussed later. A fair approximation of $H$ to within a half centimeter of the surface can be obtained with only a few terms in Eq. (13), see Figure 5. However, more terms are needed to approximate $H$ up to the surface. It should be emphasized that this approximation was made solely for the purpose of facilitating a closed form analytical solution of the energy equation. We note that other investigators who have accounted for internal radiant heating of water approximated $H$ by a single-exponential term (Dake and Harleman, 1969) or three exponential terms (Foster, 1972) by matching in situ experimental transmission data.

Since the sum of exponentials does not accurately account for radiation absorbed near the air-water interface, a surface absorbed flux $F_{\text{surf}}$ was incorporated into the analysis, see Eq. (11). The flux included not
only radiant energy flux in the opaque region of the water spectrum, $F_0$, but other radiant energy that was absorbed at or near the surface and was not completely accounted for in the exponential approximation. To determine the absorbed flux at the surface, the integrated sum of exponential approximating $H$ was compared to the rigorously predicted radiative flux $F$ considering the entire absorption spectrum of water, and the difference between the two was considered to be energy absorbed at the surface (see Figure 5). Of this surface absorbed flux $F_{\text{surf}}$, for the three-term approximation at the radiant heater lamp of 3250 K, 85% results from the radiant heat flux incident on the water surface and absorbed in the opaque part of the spectrum, and 15% represents the inadequacy of the exponential approximation for the heating rate.

The solution of the energy Eq. (1) with an exponential series approximation for the rate of volumetric absorption of radiant energy $H$, Eq. (13), the boundary conditions, Eq. (11), and an adiabatic bottom and a uniform initial temperature can be obtained using the Laplace transform method. The solution is (Snider and Viskanta, 1974).

\[
T(x,t) = T_a + \sum_{i=1}^{m} \left\{ B_i \left[ \kappa_i \left( \frac{\tilde{h}}{k} \right) \right] / k \right\} \left[ \exp(\kappa_i x + \kappa_i^2 \alpha t) \right. \\
e \operatorname{erfc}(\kappa_i \sqrt{\alpha t} + x / 2 \sqrt{\alpha t}) / 2 \kappa_i^2 \left[ \kappa_i \left( \frac{\tilde{h}}{k} \right) \right] - \exp(-\kappa_i x + \kappa_i^2 \alpha t) \\
+ \operatorname{erfc}(\kappa_i \sqrt{\alpha t} + x / 2 \sqrt{\alpha t}) / 2 \kappa_i^2 \left[ \kappa_i \left( \frac{\tilde{h}}{k} \right) \right] + \exp(\tilde{h}/k x + (\tilde{h}/k)^2 \alpha t) \\
+ \operatorname{erfc}(\tilde{h}/k \sqrt{\alpha t} + x / 2 \sqrt{\alpha t}) / (\tilde{h}/k)^2 \left( \frac{\tilde{h}}{k} \right) - \kappa_i^2 \right] + \operatorname{erfc}(x / 2 \sqrt{\alpha t}) / \\
\kappa_i^2 \left( \frac{\tilde{h}}{k} \right) \right\} + \left[ (T_0 - T_a) - (F_0 / \tilde{h}) \right] \left\{ \operatorname{erfc}(x / 2 \sqrt{\alpha t}) - \exp(\tilde{h}/k x + (\tilde{h}/k)^2 \alpha t) \right\} \left\{ \operatorname{erfc}(\tilde{h}/k \sqrt{\alpha t} + x / 2 \sqrt{\alpha t}) \right\} + \sum_{i=1}^{m} \left( B_i / \kappa_i^2 \right) \exp(-\kappa_i x) \\
\left[ \exp(\kappa_i^2 \alpha t) - 1 \right] \\
(14)
\]

where $\alpha$ is $k / \rho c$. 
3.3.2 Numerical Solution

Since the boundary conditions are nonlinear and the internal radiant energy absorption rate, \( H(x) \), is not a simple analytical expression, a closed form analytical solution is not possible. Numerical integration of Eq. (1) with the boundary and initial conditions, Eqs. (2), (3), and (4) was obtained using an explicit finite difference method (Carnahan, et al., 1969).

Using a central difference approximation, the energy Eq. (1), within the water (any interior node) can be approximated as

\[
T_{i,n+1} = \frac{\alpha \Delta t}{(\Delta x)^2} \left[ T_{i-1,n} - 2T_{i,n} + T_{i+1,n} \right] - \left( \frac{\alpha \Delta t}{k} \right) H_{i,n} + T_{i,n}
\]

where \( i \) denotes spatial nodes and \( n \) denotes the time. The finite difference energy equations for the surface (1) and bottom (M) nodes becomes

\[
T_{1,n+1} = \frac{2\alpha \Delta t}{(\Delta x)^2} \left[ T_{2,n} - T_{1,n} - q_{b,n}(\Delta x/k) \right] - \left( \frac{\alpha \Delta t}{k} \right) H_{1,n} + T_{1,n}
\]

and

\[
T_{M,n+1} = \frac{2\alpha \Delta t}{(\Delta x)^2} \left[ T_{M-1,n} - T_{M,n} + q_{b,n}(\Delta x/k) \right] - \left( \frac{\alpha \Delta t}{k} \right) H_{M,n} + T_{M,n}
\]

The boundary conditions, Eqs. (2) and (3), have been incorporated in the energy balances at the surface and bottom nodes, Eqs. (16) and (17), respectively. The term \( q_{b} \) represents the heat flux at the water surface as given by the right-hand side of Eq. (2), and \( q_{b} \) denotes the heat flux at the bottom as expressed by the right-hand side of Eq. (3). A uniform grid size was used throughout the depth of the layer of water. Different grid sizes were considered and the results obtained were checked for stability and convergence. Comparison of the results obtained using the explicit finite difference method with those based on exact analytical solutions for limiting cases yielded excellent agreement. The results reported here are for a grid size of 0.2 mm.

In the numerical calculations, the incident radiative flux was separated into bands where water is either effectively opaque or semitransparent to radiation. In the band where water is semitransparent, the rate of volumetric
radiant energy absorption was predicted from Eq. (10). In the opaque bands the solar energy absorbed in the first half finite difference node was considered absorbed at the surface \((x = 0)\). The radiative flux absorbed at the surface \(F_0\) was computed once and included in the boundary condition at the air-water interface by subtracting \(F_0\) from the right-hand side of Eq. (2). From the computational point of view this is desirable because, as seen from Figure 2, \(H\) decreases by a few orders of magnitude in the immediate vicinity of the interface requiring very small grid size for stability and accurate temperature prediction. It should be noted, however, that with the finite difference algorithm employed this procedure is equivalent to an internal volumetric absorption rate at the center of the first node.

The heat loss by convection and evaporation from the water surface could not be accurately specified from basic principles for the experimental test arrangement. Presently, no analysis or correlation is available in the literature for predicting the energy transfer from heated water cooled by free convection, evaporation, and radiation at the air-water interface (Gebhart, 1973). In the absence of more precise knowledge, the turbulent free convection heat transfer coefficient \(h\) was estimated using an empirical relation presented in the literature (McAdams, 1954) for cooling of horizontal heated plates in an unbounded air environment,

\[
h = C(T_0 - T_a)^{1/3}
\]

The constant \(C\) was adjusted from 0.22 to 0.48 in order to improve agreement between analysis and data for a single experimental condition (i.e., given time \(t\)) at a heater lamp temperature of 2880 K. After the initial adjustment to 0.48, the constant was then used in predicting the convective heat transfer coefficient for all other experimental conditions. The adjustment of the empirical constant was required because the water surface was not an unbounded flat plate but was a free surface surrounded by about 13 cm high walls which produced a "chimney" effect, and the fluid was not pure air but a binary air-water mixture. The mass transfer coefficient \(g\) was determined from the analogy between heat and mass transfer (Rohsenow and Choi, 1961),
\[ g = \frac{(h/pc)}{(a/d)}^{2/3} \]  

(19)

It is recognized that this relation is strictly valid for forced and not free convection. Mass transfer tends to reduce density gradients; therefore, the analogy between heat and mass transfer in the present case is not expected to be as simple as for forced flow. The relationship was used, however, because a better correlation appropriate for the physical situation considered in the experiment was not available in the literature. The adjustment of constant C in Eq. (18) has partly compensated for the departure of Eq. (19) from reality.

3.4 Results and Discussion

3.4.1 Comparison of Analytically Predicted and Measured Temperature Structure

Before the temperature distribution could be predicted the linear effective heat exchange coefficient, \( h \), relating energy transfer at the surface (excluding absorbed solar radiation) to the difference between surface and environment temperature was required. There was no means of accurately determining the coefficient \( h \) \textit{a priori} for the experimental arrangement. The appropriate value of \( h \) was determined by matching the predicted temperature distribution (at a given time) with the experimental data for each operating temperature of the lamps. The value of \( h \) was found to be in the range from 24 to 25.5 W/m\(^2\)-C for the three experiments performed (Snider, 1973).

The results of Figure 6 show that the closed form analytical solution agrees reasonably well with experimental data once an appropriate \( h \) is determined. For short time intervals the predicted temperature is in good agreement with the data. For longer times the difference between the predicted and measured temperatures becomes larger, an indication that an approximation of the surface boundary condition using a constant effective heat exchange coefficient \( h \) is not appropriate because of the nonlinear nature of evaporation, convection, and radiation energy transfer at the surface.
Figure 6. Comparison of measured and predicted temperature distributions for lamps at 3250 K, $T_0 = T_a = 25 \, \text{C}$, $F_{surf} = 500 \, \text{W/m}^2$, $h = 23.8 \, \text{W/m}^2\cdot\text{C}$, and $H = 55.6 \exp(-164x) + 6.12 \exp(-19.1x) + 0.370 \exp(-0.033x) \, \text{KW/m}^3$, where $x$ is in m.

During the heating process, the analytical model predicts a reversal of the temperature gradient near the water surface (Figure 6 at $t = 90 \, \text{min}$). Using the interferometer, which allows the surface to be examined closely without disturbing the temperature field, it is observed that the temperature gradient is indeed reversed, see Figure 7. As expected, the phenomenon develops for long heating times when the heat loss from the surface by convection, evaporation, and radiation is greater than the radiant flux absorbed at the surface.

The closed form analytical solution slightly underpredicts the temperature at depths of 5 cm and greater from the surface. The discrepancies away from the surface indicate underprediction of the volumetric absorption
Figure 7. Interferogram showing temperature gradient reversal at the surface at $t = 61$ min, lamp at 3250 K, and $T_o = T_a = 25$ C.

rate of radiant energy. The probable source of the discrepancy is most likely from uncertainty in the optical properties of water, impurities in the water, or inaccuracies in predicting the incident flux. If the incident flux is underestimated, the temperature calculated away from the surface would be too low while the temperature near the surface could be nearly correct. Impurities in the water would be expected to have a negligible effect on the radiative transfer since distilled water was used; however, if the spectral absorption and scattering coefficients of water are in error, particularly in the visible and near infrared part of the spectrum, the internal heating rate will not be correctly predicted. The presently available spectral extinction coefficient data for water from the published literature indicates that discrepancies exist between the data reported by various investigators (Hale and Quarry, 1973). The spectral extinction coefficients recommended by Hale and Quarry are smaller
In some and larger in other parts of the spectrum than those used by Viskanta and Toor (1972).

The temperature distribution in stagnant water is strongly dependent on the volumetric heating rate $H$. In order to determine the accuracy with which $H$ must be approximated to obtain a reasonably good prediction of the temperature distribution, various exponential approximations to $H$ are examined (Snider and Viskanta, 1974). The exponential fits for $H$ in water, heated by the lamps at 3250 K, are:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Exponential Approximations of $H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$H = 55.6 \exp(-164x) + 6.12 \exp(-19.1x) + 0.371 \exp(-0.033x)$</td>
</tr>
<tr>
<td>B</td>
<td>$H = 44.1 \exp(-112x) + 5.00 \exp(-13.7x)$</td>
</tr>
<tr>
<td>C</td>
<td>$H = 44.1 \exp(-112x) + 4.27 \exp(-12.1x) + 0.648 \exp(-0.033x)$</td>
</tr>
<tr>
<td>D</td>
<td>$H = 44.1 \exp(-112x) + 0.738 \exp(-0.033x)$</td>
</tr>
<tr>
<td>E</td>
<td>$H = 3.78 \exp(-0.95x)$</td>
</tr>
</tbody>
</table>

The distance $x$ is in meters and $H$ is in kW/m$^3$. Exponential fits A and B are approximations to $H$ for a nonreflecting bottom (Equations A and E are also compared against the rigorously predicted volumetric rate of radiant energy absorption in Figure 5). Equation C approximates a perfectly reflecting bottom, and Equations D and E are intended to illustrate the use of poor approximations to $H$. The approximations are used for $H$ in Eq. (13) with $h$ held constant and the surface absorption rate adjusted to compensate for the different approximations of internal heating rate. The resulting temperature distributions are presented in Figure 8. Comparison of the results for Equations A and B shows that an accurate approximation of $H$ near the bottom of the test cell is not required for heating times and incident flux distributions used in this study. From this observation, it is concluded that approximating a low reflectance bottom by a black surface is justified.

Comparison of the predicted and measured temperatures shows that for the heating periods and radiant heat sources considered in the laboratory study a two or more term exponential fitted to the internal radiant heating rate together with a surface absorbed flux can be used to adequately
approximate the radiant heating rate in calculating the temperature distribution. A poor fit to the internal heating rate, as given by Equation D, resulted in inadequate temperature distribution. A single-term exponential fit to the heating rate (Equation E) overpredicted the surface temperature because of the large surface heat flux required in the representation to account for the inadequate approximation of H. Using the single exponential, a lower temperature resulted at depths between 1 and 8 cm as would be expected from the underpredicted absorption rate in this region. It is noted that the duration of the experimental tests were not sufficient to evaluate temperature variations at depths far below the surface due to the small heating rates. This heating rate in the lower region of a natural body of water may be important during the annual heating cycle. If this cycle is to be modeled, then approximations such as given by Equations B and E will not provide adequate representations of the heating rate because they underpredicted H(x) at depths greater than 10 cm.
3.4.2 Comparison of Numerically Predicted and Measured Temperature Structure

A comparison of predicted and measured temperature distributions in water heated by thermal radiation is shown in Figures 9 and 10. Absorption of radiation produced strong thermal stratification in the surface layers of the water. Examination of the results reveals good agreement between predictions and data, thus verifying the radiation and total energy transfer models. Since for the laboratory simulation the water layer near the surface absorbed the largest fraction of the energy incident from the lamps, the temperature near the surface is greatest and decreases with depth. The results show that there is little radiant heating of the water near the bottom of the test cell. This finding is consistent with the radiant heating rates illustrated in Figure 2 which show that near the bottom of the cell H has decreased by about four orders of magnitude as compared with the value at the surface. The measured and predicted temperature distributions agreed better when the predictions were based on the finite difference solution of the energy Eq. (1) with exact values of the internal volumetric heating rate H(x) and exact boundary condition, Eq. (2) (Snider and Viskanta, 1975), than when based on the closed form analytical solution Eq. (14).

The greatest radiant heating occurs at or near the interface, as seen in Figure 4. For this reason it is desirable to obtain some insight into the effects of inaccuracies in predicting the surface radiant flux F_s on the internal temperature distribution. Since in the numerical solution of the energy equation the radiant energy absorbed by the layer of water in one-half the grid spacing below the interface was assumed to be absorbed at the surface, the effects of variation about this value was easily made. Figure 11 illustrates the effect of variations in the surface absorbed flux on the temperature distribution. As expected, the difference is greatest near the surface and decreases with depth, and its influence becomes more pronounced with time. It is interesting to note the small effect the variation in surface heat flux has on the predicted temperature a few centimeters below the surface, indicating the importance of the internal volumetric absorption of radiation.

Figures 9 and 10 show that the temperature was underpredicted at depths of 5 cm and greater. The discrepancies in predicted temperatures
Figure 9. Comparison of measured and predicted temperature distributions for lamps at 2520 K, $T_0 = T_a = 25$ C and $L = 38.1$ cm.

Figure 10. Comparison of measured and predicted temperature distributions for lamps at 2880 K, $T_0 = T_a = 25$ C and $L = 28.1$ cm.
Figure 11. Effect of variation in radiant energy considered absorbed at the surface on predicted temperature distribution for lamps at 3250 K, $T_0 = T_a = 25$°C and $L = 28.1$ cm.

away from the interface indicate underprediction of the volumetric absorption rate of radiant energy. The most probable source of the discrepancy is the uncertainty of the spectral absorption coefficient of water. If the spectral absorption and scattering coefficient are in error, particularly in the visible and near infrared parts of the spectrum, the internal heating rate will not be correctly determined. There is uncertainty and considerable discrepancy between the spectral extinction coefficient data for water reported in the literature by various investigators (Hale and Querry, 1973).

3.4.3 Summary of Thermal Stratification Experiments

The relatively good agreement between predictions and experimental data indicates thermal stratification in a layer of water heated by solar radiation can be predicted with confidence. Based on results obtained in this study, the following conclusions can be drawn:

1) The predicted temperature distribution strongly depends on rate of internal volumetric absorption of radiation.
(2) The internal volumetric absorption rate can be correctly predicted if accurate knowledge of the spectral absorption and scattering coefficients for water and its impurities are available and the radiation reflection characteristics at the water surface and bottom are known. A series of three or more exponential terms and use of a surface absorbed radiant flux provides an adequate approximation to the heating rate for predicting the temperature distribution.

(3) To accurately predict the temperature distribution in waters heated by solar radiation, the boundary condition at the water surface must be correctly specified. This means that the convective, latent, and radiative heat transfer processes must be properly modeled.

(4) For a deep layer of water, the bottom reflectance and boundary condition need not be accurately specified to obtain an adequate prediction of the volumetric internal solar heating rate for prediction of the temperature distribution. Of considerable importance is the case of a low reflectance bottom in the visible region of the spectrum (radiation outside the visible region is highly attenuated before reaching the bottom). For this case, the bottom can be considered black which introduces significant simplification in predicting the internal volumetric heating rate and also reduces computation time.

(5) The results show that the approximate analytical solution, Eq. (14), predicts satisfactorily the temperature distribution, except in the immediate vicinity of the water surface, provided the effective heat exchange coefficient \( h \) and the internal rate of radiation absorption \( H \) are properly evaluated. This expression is preferable over a numerical solution.
4. COOLING OF THERMALLY STRATIFIED STAGNANT WATER

Heating and cooling processes in the upper layers of water are quite different in nature. While the main heating process due to solar radiation can be considered primarily as volumetric in nature, the important cooling processes (evaporation, emission of radiation, convection to the atmosphere) are all effectively surface processes. Thus, the natural waters are basically unstable because heat is added below where it is removed. Because the molecular thermal diffusivity of water is so small, this heat must be transferred to the surface by mechanical mixing processes, such as thermally induced buoyancy forces which will cause a convective instability, or else as wind induced turbulence.

The physical situation arising in natural lakes, reservoirs, and ponds is complicated by the fact that solar radiation is time dependent and exhibits a diurnal and a seasonal cycle. Cooling processes are affected by meteorological conditions such as cloud coverage, but only weakly by a diurnal solar cycle. The cooling processes are more nearly constant. Thus, one would intuitively expect that if convection occurs at all, it would take place at night. However, convection may also occur during the daytime because stability of the water column depends on the vertical temperature distribution and the local rates of heating and cooling of the water column. Processes whereby convective motions arising in an unsatable region penetrate into an adjacent stable layer are called "penetrative convection" (Turner, 1973). The problem is important not only for natural lakes, ponds, and reservoirs but also for the upper ocean and the lower atmosphere. A layer of convecting fluid driven by heat transfer across the water (or soil) surface is bounded below (or above) by a stable fluid. As cooling (or heating) is continued, the depth of the convecting layer increases at the expense of the stable region.

Because the convective motion below the water surface is induced by temperature gradients, the vertical temperature structure in the water and the heat dissipation as well as the evaporation rate are of considerable practical interest. The internal free convection processes which arise below the water surface due to thermally induced buoyancy forces are
important not only in the convective dispersive transport of oxygen or any other substances introduced into the waterbody, but also in the internal energy transport for establishing the thermal structure (temperature distribution) in the water as well. The vertical temperature (density) variations in a body of water, in turn, have important effects on chemical and physical properties, dissolved oxygen (DO), biological oxygen demand (BOD), chemical oxygen demand (COD), aquatic life, and ecological balance of the ecosystem.

The convective processes below the water surface are very complex (Turner, 1973). According to available experimental evidence (Dake and Harleman, 1969; Snider and Viskanta, 1974) when a stable, thermally stratified layer of water is cooled from a surface free convection circulation is established a small distance below the surface. The energy transport in the convective layer is by turbulent diffusion and circulation. The physics of the convection processes (including internal energy transport and sources of energy) in a fluid in which the flow is induced by buoyancy and which has time dependent internal heat sources has not been investigated in sufficient detail and is not completely understood. The phenomena occurring are of considerable practical importance because the physical situation arises, for example, when a lake, pond, or reservoir is stratified during the day by solar irradiation (insolation) and after sunset (or even before) is cooled from its surface by convection, evaporation, and radiation.

The objective of this phase of the investigation is to study internal energy transport processes during cooling of thermally stratified water. To this end, carefully controlled laboratory experiments were performed to determine the detailed temperature structure in the surface layers of water and to obtain the needed data. Models are also developed for predicting the dynamics of the mixed layer and the heat dissipation from the surface as well as the turbulent diffusivity in the surface layer of stagnant, stratified water undergoing cooling. The experimental findings are presented and data used to check the validity of the simple theoretical models.
4.1 Thermal Model for Dynamics of the Mixed Layer

4.1.1 Physical Model and Assumptions

The vertical temperature structure in a developing convectively unstable surface layer of water can be represented as in Figure 12. The layer is composed of several regions. Each of these regions is dependent upon different influences and transport processes. First of all, the surface temperature \( T_0 \) is cooler than the temperature of the mixed layer \( T_m \) which is from about 1 to 3 mm below the surface. The region between the surface and the top of the mixed layer is referred to as a "thermal boundary layer." This layer is comprised of a surface "skin layer" which is about 0.2 mm thick and in which energy transport is by molecular diffusion. The lower part of the thermal boundary layer, i.e., between the skin and the mixed layers is a "buffer layer" in which transport of heat by molecular diffusion is of about the same order of magnitude as turbulent diffusion. The results show that this thermal boundary layer is very persistent, is shortly re-established after being distorted, and remains relatively constant in thickness (from about 1 to 3 mm) independent of thermal conditions. The thorough mixing in the free convection layer is buoyancy dominated. Below the mixed layer is an "interfacial entrainment" layer in which cold water is entrained by the warmer fluid. Below this region is the deep stable layer where \( \partial T / \partial x < 0 \) and in which energy transport is by molecular diffusion.

The cooling of stratified water is controlled by the dynamics of the mixed layer, and therefore the dynamics and energy transport in this layer is very important to developing modeling capability. Rigorous analysis of the convective motion and energy transport in the mixed layer is very complex because the equations of motion are nonlinear and the sources of energy are not obvious. Analytical solutions cannot be obtained and numerical solution does not appear warranted since turbulence in buoyancy dominated flow is not completely understood and cannot be modeled. For this reason, a relatively simple thermal model is developed to predict the thickness and the mean temperature of the mixed layer.

A schematic representation of the temperature distribution in the developing convectively unstable surface layers of stratified water is shown in Figure 13. The thermal structure is considered to consist of four layers: surface thermal boundary layer, convective (mixed) layer, interfacial entrainment layer, and stable region. The thin thermal boundary
Figure 12. Typical temperature structure in stratified water cooled from the surface.

Figure 13. Schematic representation of the developing unstable convective layer and the adopted temperature profile.
layer develops rapidly and its thickness is assumed to be constant. Temperature gradients are large in this layer and heat is predominantly transported by molecular and turbulent diffusion. The thorough turbulent mixing in the free convective (mixed) layer below the surface boundary layer is buoyancy dominated and is assumed to produce a temperature profile which is independent of depth. Two of the main factors controlling the development of the convective layer are the total heat transfer rate at the air-water interface and also a turbulent mixing process which occurs at the interface between the well mixed convective layer of water and the nonturbulent water in the stable region below. Laboratory experiments (Dearcroft, et al., 1969) show the unstable-stable interface to be a highly contorted, almost undefinable surface due to the physical overshooting into the stable region of energetic elements which originate in the surface layers and continuously bombard the interface. Within this strongly agitated region, the interfacial entrainment layer, the temperature field indicates marked spatial variation with the net effect being a change across the layer from the free convection layer value to the value of \( T_s \) at the top of the yet undisturbed stable region. The interfacial entrainment layers will vary in depth and character; however, to facilitate the analysis, they are represented here by a step discontinuity, \( \Delta T \), in the temperature profile at \( x = \delta + h \), the bottom of the convective layer.

The simple thermal model described above is very similar to the convectively unstable planetary boundary layer in the atmosphere capped by a stable layer. The essential details concerning the development of the penetrative convection model are reviewed by Plate (1971). More recent contributions and results of the theory are discussed, for example, by Tennekes (1973) and Carson and Smith (1974).

4.1.2 Basic Energy Equation

The layer of water is assumed to be a horizontally homogeneous incompressible fluid obeying the Boussinesq approximation (Phillips, 1966). Viscous heat dissipation and radiation transfer is neglected. Internal absorption of solar radiation, however, can readily be accounted for as shown in the previous subsection. The physical properties are considered constant.
Making use of the approximations introduced above the conservation of energy equation (the First Law of Thermodynamics) for an incompressible fluid leads to the following expression

$$\rho c \left[ \frac{\partial T}{\partial t} + \frac{\partial (\\nu T')}{\partial x} \right] = k \frac{\partial^2 T}{\partial x^2} \quad (20)$$

where \( T \) and \( T' \) are the mean and fluctuating temperature, respectively; \( \nu' \) is the fluctuating vertical velocity component; \( x \) is the depth measured from the surface; \( t \) is time; and \( \rho, c, \) and \( k \) are the density, specific heat at constant pressure, and molecular conductivity of water, respectively. The term \( \rho c (\\nu T') / \partial x \) represents the local divergence of the turbulent flux of heat. It is this term that contributes in a major way to redistribution of the heat exchanged at the air-water interface.

Since the turbulent diffusion term in the energy equation cannot be readily evaluated, the following additional simplifying assumptions are introduced:

1. The change in internal energy of the thermal boundary layer is negligible;
2. The turbulent diffusion predominates over the molecular diffusion in the mixed (convection) layer and the temperature in this region is independent of \( x \);
3. The temperature decreases linearly in the stable region; and
4. The temperature at the bottom of the waterbody is constant.

The above assumptions provide considerable mathematical simplification and allow the mean temperature to be written as

$$T(x, t) = \begin{cases} 
T_0(t) + (x/\delta)[T_m(t) - T_s(t)], & 0 < x < \delta \\
T_m(t), & \delta < x < \delta + h \\
T_s(t) + \gamma(t)[x - (\delta + h)], & x > \delta + h 
\end{cases} \quad (21)$$

With this approximation, the energy Eq. (20) for the convective layer then becomes

$$\frac{dT_m}{dt} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} - \frac{\partial (\\nu T')}{\partial x} \right) = -\frac{1}{\rho c} \frac{\partial H}{\partial x} \quad (22)$$
where

\[ H = \rho c \left[ \frac{\partial T}{\partial t} - \alpha \frac{\partial T}{\partial x} \right] \]  \hspace{1cm} (23)

is the local heat flux at any depth \( x \).

Integrating Eq. (22) with respect to \( x \) from \( x = 0 \) and \( x = \delta + h \) and making use of assumption 1, yields

\[ hpc \frac{dT_m}{dt} = H(0, t) - H(h, t), \hspace{1cm} x < \delta + h \]  \hspace{1cm} (24)

where \( H(0, t) \) is the total heat flux at the water surface \( (x = 0) \), and \( H(h, t) \) is the heat flux at the bottom of the mixed layer \( (x = \delta + h) \).

An additional equation needed to evaluate the heat flux at the bottom of the convection layer can be obtained by making an assumption about the joining of the temperature profile at the "interface" between the stable region and the convective layer. Experimental evidence suggests entrainment of cool water into the mixed layer from the stable region. The relation between the heat flux and the temperature is formulated on the basis of the assumed or ideal temperature structure at the interfacial entrainment layer between the mixed layer and the stable region shown in Figure 13.

It follows from geometry and the assumed temperature profile shown in the figure that

\[ \gamma \Delta = \Delta T = T_s - T_m \]  \hspace{1cm} (25)

which together with Eq. (24) implies that

\[ \rho c h \left[ \frac{d(\gamma h)}{dt} - \frac{d(\gamma \Delta)}{dt} \right] = H(0, t) - H(h, t) \]  \hspace{1cm} (26)

Further, the heat flux at the bottom of the mixed layer can be related to the change in thickness of the convective layer according to the relation

\[ H(h, t) = -\rho c \Delta \frac{d(\gamma h)}{dt} \]  \hspace{1cm} (27)

or

\[ H(h, t) = -\rho c \Delta \frac{dh}{dt} \]  \hspace{1cm} (28)
if \( \gamma \) is assumed to be constant. It is evident from the assumed temperature profile shown in Figure 13 and the definition of \( \Delta T \) in Eq. (25) that

\[
\frac{d\Delta T}{dt} = \frac{d(\gamma h)}{dt} - \frac{dT_m}{dt}
\]  

(29)

This relation simply states that the rate of change of \( \Delta T \) depends on two effects. On the one hand, \( \Delta T \) increases as entrainment into the stable water below the mixed layer progresses, and on the other hand, \( \Delta T \) decreases as the mixed layer cools.

To avoid questionable manipulation of the energy balance, a closure condition is introduced. Following Carson and Smith (1974) it is postulated that the magnitude of the heat transported into mixed layer due to entrainment is proportional to the heat flux at the surface, thus,

\[
H(h,t) = -\kappa(t)H(0,t)
\]

(30)

The proportionality factor \( \kappa(t) \) lies between zero and unity and is taken to be a function of time. This coefficient will be established from the experimental data.

In summary, a model for predicting the growth of the convective layer has been formulated. The analysis is similar to the models developed to predict the height of the convective layer in the atmosphere and can be considered an application of and extension of those analyses (Plate, 1971). The model takes into account both the cooling processes of the convective layer and of entrainment of cold water into the convective layer from the stable region below.

4.1.3 Solution of the Model Equations

Substitution of Eq. (30) into Eq. (24) gives

\[
\frac{dT_m}{dt} = \left[1 + \kappa(t)\right]H(0,t)/\rho c
\]

(31)

Combining Eqs. (24), (28), (29), and (31) yields

\[
\left[(1 + \kappa)/\kappa\right]pc\Delta T \frac{dh}{dt} = \rho ch \left[\frac{d(\gamma h)}{dt} - \frac{d(\Delta T)}{dt}\right]
\]

(32)
If the variation of $\rho$ and $c$ with temperature is neglected and $\gamma$ is assumed constant, the solution of Eq. (32) becomes

$$\Delta T = \gamma h \left[ k / (1 + 2k) \right]$$

(33)

Substitution of this relation into Eq. (28) gives

$$H(h,t) = -\frac{\rho c\gamma h c}{(1+2k)} \frac{dh}{dt}$$

(34)

Now the net heat flux at the water surface is the sum of convective, latent, and radiative energy fluxes and is given by Eq. (2). This expression can be linearized as was done in a previous section and there results

$$k \frac{\partial \Delta T}{\partial x} \bigg|_0 = q_0 = H(0,t) = \tilde{h}(T_0 - \tilde{T}_a)$$

(35)

where $\tilde{h}$ is an effective linearized heat exchange coefficient and $\tilde{T}_a$ is an effective air temperature. Since the temperature profile in the thermal boundary layer is linear, see Figure 13, the surface temperature $T_0$ can be related to the mean temperature $T_m$ of the mixed layer. From the energy balance at the interface, Eq. (35), there results

$$\tilde{h}(T_0 - \tilde{T}_a) = k(T_0 - T_m) / \delta$$

(36)

or

$$T_0 = \left[ \tilde{h}\tilde{T}_a + (k/\delta)T_m \right] / [\tilde{h} + (k/\delta)]$$

(37)

Combining Eqs. (35) and (37) with Eq. (31) gives

$$\rho c\gamma \frac{dT_m}{dt} = [1 + \kappa] (h/\delta)(T_0 - T_m)$$

(38)

Similarly substituting Eqs. (29), (35), and (37) into Eq. (34) yields

$$\rho c\gamma \frac{d(h^2/2)}{dt} = [1 + 2\kappa](T_0 - T_m)(k/\delta)$$

(39)
There is a system of three equations, Eqs. (37) to (39) and three unknowns, $T_m$, $T_c$, and $h$. These equations can be solved simultaneously with the initial conditions

\[
\begin{align*}
T_m &= T_m, & \text{at } t &= t_0 \\
T_0 &= T_{0i}, & \text{at } t &= t_0 \\
\text{and} & \\
h &= h_i, & \text{at } t &= t_0
\end{align*}
\]

(40)

Since the equations are nonlinear, in general, only numerical solutions are possible.

4.1.4 Development of the Mixed Layer

If it is assumed that $\Delta$ is proportional to some constant fraction of the depth of the evolving mixed layer, use of Eq. (27) in Eq. (26) yields

\[
\text{ch} \frac{d(yh)}{dt} = [H(0,t) - 2H(h,t)]
\]

(41)

For $\gamma$ = constant, substituting Eq. (30) into Eq. (41) and integrating results in

\[
h^2 = 2\left\{h(0) + \frac{[(1 + 2\kappa)/\rho_c \gamma]}{1 + 2\kappa}ight\} \int_0^t H(0,\tau) d\tau
\]

(42)

with $h = h(0)$ at $t = 0$. For the special case of $H(0,\tau) = $ constant, Eq. (42) shows that $h \propto \sqrt{t}$. This is in agreement with the result given by Plate (1971).

4.2 Turbulent Diffusivity

Prediction of the temperature distribution within a few millimeters of the water surface requires knowledge of the turbulent diffusivities of momentum and heat. Unfortunately, turbulence in buoyancy dominated flows is not completely understood (Turner, 1973). Therefore, in this section a simple turbulent diffusivity model is developed using as its basis the turbulent kinetic energy equation.
In the absence of mean horizontal flow, the turbulent kinetic energy balance for the laboratory model is given by (Phillips, 1966)

\[ \rho \frac{\partial e}{\partial t} = -\rho \frac{\partial (\overline{w'\epsilon'})}{\partial x} - \overline{w'\rho'}g - \rho \epsilon \]  

(43)

where \( \epsilon = \overline{e^{1/2}} = \frac{1}{2}[\overline{u'^{1/2}} + \overline{v'^{1/2}} + \overline{w'^{1/2}}] \) is the mean kinetic energy of turbulence and \( u', v', \) and \( w' \) are the fluctuating velocity components. In Eq. (43) \( \epsilon' \) and \( \rho' \) are the fluctuations of kinetic energy and density, respectively; \( g \) is the acceleration due to gravity; and \( \epsilon \) is the rate of dissipation of turbulent energy. The first term on the right-hand side of Eq. (43) can be interpreted as diffusion of turbulent kinetic energy, and the second term represents production of turbulent kinetic energy due to buoyancy.

It is estimated that in the vicinity of the air-water interface the rate of change of the kinetic energy term, \( \rho \partial e / \partial t \), and the turbulent diffusion term, \( \rho \partial (\overline{w'\epsilon'}) / \partial x \), are from about one to two orders of magnitude smaller than the remaining two terms in Eq. (43). Following Launder and Spalding (1972), the dissipation rate \( \epsilon \) is expressed as

\[ \epsilon = C_\epsilon e^{3/2} / \lambda \]  

(44)

where \( C_\epsilon \) is a constant (\( C_\epsilon = 0.09 \)) and \( \lambda \) is the mixing length. The production of turbulence by the density gradients can similarly be represented as

\[ -\overline{(w'\rho')g} = \beta g (w'T') = e^{3/2} \left( \rho g \frac{\partial T}{\partial x} \right) \]  

(45)

where \( \rho' \) has been replaced with \( T' \) by use of \( \rho' = \beta \rho dT \) with \( \beta = -(\partial \rho / \partial T) / \rho \). Using the approximations discussed above and substituting Eqs. (44) and (45) into Eq. (43) results in

\[ e^{3/2} \left( \rho g \frac{\partial T}{\partial x} \right) - \rho C_\epsilon e^{3/2} / \lambda = 0 \]  

(46)

This equation simply states that the production of turbulent energy is in a quasi-steady balance with dissipation. Solution of Eq. (46) for \( e \) yields
\[
e = \frac{k^2 \beta \partial T}{C_D \partial x}
\]  

(47)

The turbulent diffusivity of heat, \(\alpha_t\), is generally defined as

\[
\alpha_t = \frac{-\bar{w} \bar{T} \partial T}{\rho c \partial x}
\]  

(48)

If, as postulated by Launder and Spalding (1972), the diffusivity can be related to the mixing length, \(\lambda\), and the mean kinetic energy of turbulence, \(\epsilon\), by the expression

\[
\alpha_t = \lambda \sqrt{\epsilon}
\]  

(49)

then substitution of Eq. (47) into Eq. (49) results in

\[
\alpha_t = \lambda^2 \left(\frac{g \beta \partial T}{C_D \partial x}\right)
\]  

(50)

Using Prandtl's approximation for the mixing length,

\[
\lambda = C_D^{\frac{1}{4}} \kappa x
\]  

(51)

where \(\kappa\) is the von Karman's constant, Eq. (51) becomes

\[
\frac{\alpha_t}{\alpha} = \left(\frac{\kappa x}{\alpha}\right)^2 \left(\frac{g \beta \partial T}{\partial x}\right)^{\frac{1}{2}} = \left[\left(\frac{\kappa x}{\alpha}\right)^2 g \beta (\partial T/\partial x)^{\frac{1}{2}}\right]^{\frac{1}{2}} = \left[Ra_x Pr\right]^{\frac{1}{2}}
\]  

(52)

where \(Pr (= \nu/\alpha)\) is the Prandtl number and

\[
Ra_x = \frac{(\kappa x)^2 g \beta (\partial T/\partial x)}{\nu \alpha}
\]  

(53)

can be regarded as a local Rayleigh number based on an eddy size, \(\kappa x\), as the characteristic distance. Equation (52) predicts that the turbulent diffusivity is proportional to the square root of the local Rayleigh number \(Ra_x\). It also predicts the unrealistic result that \(\alpha_t\) vanishes when the temperature gradient goes to zero. On the contrary, experimental data show that there is effective turbulent transport of heat in the mixed layer where the temperature gradient is very small.
For steady state and the limiting condition when molecular diffusion is negligible in comparison to turbulent diffusion, the energy Eq. (20) reduces to
\[
\left[ Ra \cdot Pr \right]^\frac{1}{2} \frac{dT}{dx} = \text{constant}
\]

If it is assumed that the thermal expansion coefficient \( \beta \) is constant, this equation can be integrated to yield
\[
T = Ax^{-1/3} + B
\]

where \( A \) and \( B \) are constants. Thus, when turbulent diffusion predominates over molecular diffusion, the temperature profile becomes identical to that typically encountered in turbulent free convection flows (Turner, 1973; p. 135).

The above estimate of the turbulent diffusivity in the surface layer of stratified water involves many simplifying assumptions which must await verification using experimental data.

4.3 Results and Discussion

4.3.1 Observations During Cooling of a Uniform Layer of Water

When a uniform layer of water at an ambient temperature was cooled by placing a cold sink above, but not in direct contact with the water surface, the initial temperature distribution was marked by horizontal interference fringe shifts moving downward from the surface (Snider and Viskanta, 1974). Horizontal fringes are characteristic of pure conductive heat transfer. After some time, the bottom few fringes separated from the cooler layer of water near the surface and moved downward into the underlying water, indicating the onset of free convection. As the bottom few fringes moved downward, they became wavy and lost their thermal identity. After the bottom few fringes moved away from the surface, the upper cool layer became temporarily quasi-stable (characterized by stable horizontal fringe shifts) until instability produced more downward moving fluid.
The phenomena observed was different from that reported by other investigators (Townsend, 1959; Sparrow, Husar, and Goldstein, 1970) who have studied heating of water from below. No descending columns of cold fluid ('thermals') which are spaced more or less regularly along the span of the surface and have a characteristic frequency were observed. The phenomenon was intermittent rather than steady. Denser fluid slowly accumulated near the surface and then broke away as an unsteady plume. The site where the plume was produced was not fixed but wandered about the surface. The cooling process can be considered as consisting of two phases: a conductive phase followed by a break-off and mixing phase. During the conductive phase the fluid adjacent to the surface is cooled and as a result, a temperature front moves away from the surface into the fluid. When the thickness of the conduction layer between the surface and the moving front is such that the Rayleigh number exceeds a critical value, the layer becomes unstable and breaks away thereby producing a plume. The mixing and agitation associated with the breakup of the conduction layer restores the fluid near the surface to a more uniform state (Snider and Viskanta, 1974; Figure 4), and the entire process begins again.

The cooling rate had a decisive influence on the generation of plumes. Both the spatial frequency (wave number) of the sites and the temporal frequency of generation increase with increases in the rate of cooling.

The series of interferograms shown in Figure 14 illustrate the interference fringe patterns after the initial transient has passed. After the convective motion had begun, cooler (denser) water was observed to continually descend from the surface layer. The start of the descending water appeared at random locations and no characteristic wave length (wave number) could be defined for the process. The downward moving flow was seen as sheets (Figures 14a, 14b, 14c, and 14d) or as plumes (Figures 14e, 14f, 14g, and 14h) emanating from the surface region. The plumes were often followed by a sheet of moving water which would sometimes connect several plumes. The downward moving water was approximately 0.06 to 0.2 °C cooler than the surrounding bulk mean temperature. The denser water would penetrate from 10 to 25 cm into the underlying region before losing its identity. The water in the test cell a small distance below the surface was well mixed and at a uniform temperature except for the recurring cooler
Figure 14. Interferograms illustrating free convection in an initially uniform layer of water during cooling from the surface, $T_o = 21\,\text{C}$, $T_p = 6\,\text{C}$.
descending water. Near the interface, in the conduction region, a sharp
temperature gradient existed between the surface and the underlying well
mixed region. The layer over which the gradient was in evidence was about
0.5 cm thick and represented about a 1°C difference in temperature between
the surface and underlying region.

As the water cools the conduction layer becomes distorted and individual disturbances grow irregularly and break away from the boundary layer (see Figure 14d and 14e). The fluid parcel rapidly increases in volume by entraining denser fluid. This process happens before the fluid parcels lose their connection with the surface, and the process temporarily denudes a considerable area of the interface of its cold fluid (see Figure 14g). The interferometer yields only a two-dimensional temperature field, and little could be said about the three-dimensional flow processes. For example, part of a vortex formed in the test cell is seen in Figure 14d.

In order to better visualize the flow and obtain quantitative flow data, streak photography (Merzkirch, 1974) or electrochemical (Sparrow, et al., 1970) flow visualization techniques can be used.

4.3.2 Cooling of Thermally Stratified Layer of Water

An initially uniform-temperature layer of water was first thermally
stratified by heating with the radiant lamps and then was cooled. Some
selected photographs of the interference fringe patterns illustrating
the free cooling of initially stratified water are shown in Figure 15 where
the water exchanges heat with colder air and surroundings. The correspond-
ing temperature distribution in the water is given in Figure 16.

Photograph 15a shows that initially there is a thin relatively uniform
temperature region about 1 mm below the water surface with a temperature
gradient reversal near the interface. The results indicate that for the
particular experimental conditions the net heat flux at the surface is
negative (the interface is being cooled), but the bulk of the water is still
being heated by internal absorption of radiation.

*The diagonal fringes at the top of each photograph are due to temperature
differences in the air above the water surface.
Figure 15. Interferograms for cooling of stratified layer of water; heat exchange between the surface and the environment. $T = 22.2 \, \text{C}$.
Figure 16. Temperature distribution during cooling of thermally stratified water; heat exchange between the surface and the environment, \( T_a = 22.2 \text{ C} \).

After the radiant heating has been terminated, the water starts cooling at \( t = 0^+ \) s and continues with consequent development of free convection. The convective motion was unlike that observed in the cooling of the uniform-temperature layer. Instead of cooler water descending into the underlying region the convective flow was confined to a small region near the interface. The mean temperature of the convective layer was higher than that of the underlying stable region, Figure 16. The stable region then resisted downward motion of the fluid and confined the circulation to a layer near the surface. As the time increased the bulk temperature of the well-mixed region decreased, and the convection penetrated deeper into the stable region. As the cooling continued the thickness of the convective layer increased as shown in Photographs 15b, 15c, and 15d. At \( t = 1200 \text{ s} \).
(Figure 15) the thickness reaches about 2.9 cm. The thickness of "the thermal boundary layer," the region in water where most of the temperature drop occurs, is relatively constant. During the experimental run the thickness of this layer increased from about 1 mm at $t = 0$ to about 3.5 mm at $t = 1320$ s. It was also observed that the thicknesses of the thermal boundary layer did not increase monotonically but oscillated, which was particularly apparent after the flow in the mixed layer became turbulent. These fluctuations in the thickness of the thermal boundary layer may be due to penetration (intermittently) of warmer fluid into the layer and then cooling of the layer. The thickness of the mixed layer did not grow continuously after it became turbulent, and there appeared to be a change in scale of the turbulent motion. The mixed layer grew in thickness, the scale of motion changed and then followed another period of growth. After the flow becomes turbulent the rate of growth of the mixed layer into the region of stable temperature gradient is increased significantly and the temperature becomes more uniform. This indicates that the rate of growth of the convective layer depends not only on the heat flux but also on the rate of entrainment of the colder water from the stable region. The magnitude of the heat flux at the surface in the experiment was not constant but decreased with time since it was proportional to the temperature difference between the water surface and the ambient air. As Figure 16 clearly shows, the water surface temperature decreased as the cooling of the water continued.

Photograph 15c shows that the convective flow had a regularly structured pattern. This will also be clearly evident in other photographs to be presented. After a sufficiently long time the water cooled to a low temperature and the mixed layer disappeared. The water below the conduction surface layer was practically uniform and the cooling process was similar to the cooling of an initially uniform-temperature layer. Plumes are shown leaving the interface.

A set of typical photographs of interference fringe patterns for a more intensely stratified layer of water is shown in Figure 17. Initially, the surface temperature was about 22 C higher than the bulk temperature prior to stratification. Because of the large temperature gradients the fringe density was quite high. Figures 17a and 17b clearly show that the
Figure 17. Interferograms illustrating cooling of intensely stratified layer of water; heat exchange between the surface and environment, $T_a = 25$ C.
convection had a regularly structured circulation pattern. The process described was similar to that by Berg, et al. (1966) and called vermiculated rolls. The roll pattern was seen to exist until the convective layer became approximately 2 cm thick, and then as the flow became turbulent, the structure deteriorated. The rolls had a characteristic spacing which increased as the convective region became larger until the roll structure disappeared. The resulting motion after the roll pattern disappeared was characterized by plunging sheets of water which was similar in nature to that described by Spangenberg and Rowland (1961). Plumes of descending cooler water, as observed during cooling of the initially uniform temperature water, were not apparent in the thermally stratified layer. After the convective layer became large and the stable region was completely penetrated by the convective layer, both plumes and sheets of descending colder water were evident leaving the vicinity of the interface.

Some typical photographs of the interference fringe patterns and the temperature distribution for a different cooling experiment are shown in Figures 18 and 19, respectively. Before the cooling began, the water was stratified by placing sheets of glass between the radiant heaters and the test cell in order to "cut off" some of the long wave radiation and to simulate more closely the internal absorption of solar radiation. The water was stratified to greater depths. The interferograms obtained during the cooling of the water show that the convective layer grows at a slower rate because the deeper the stratification, the greater the resistance to the downward motion of the layer.

Some selected photographs illustrating the interference fringe patterns during cooling of water by a copper heat sink placed above, but not in direct contact with the water, are shown in Figure 20. The corresponding temperature distributions are given in Figure 21. About 40 seconds was required to install the heat sink at the top of the cell after the termination of the thermal stratification of water. During this time, the water was cooled by heat exchange with the air and surroundings. The circulation of the coolant through the sink was commenced at \( t = 40 \) s and it took an additional 60 s to overcome the thermal inertia of the heat sink and reach nearly constant plate temperature and heat flux, as shown in Figure 22.
Figure 18. Interferograms illustrating cooling of thermally stratified
Figure 19. Temperature distribution during cooling of thermally stratified water; heat exchange between surface and environment, \( T_a = 17.5 \, ^\circ\text{C}, \, T_0 = 17.5 \, ^\circ\text{C} \).

Figures 20b and 20c show that convective motion had a regular structured circulation (roll) pattern. The roll pattern existed until the convective layer became approximately 1.25 cm thick at \( t = 120 \, \text{s} \). The thermocouple and interference fringe patterns indicated the motion in the convective layer became turbulent at about \( t = 135 \, \text{s} \). The resulting motion at the top of the convective layer after the roll pattern disappeared was characterized by plunging sheets of water (Figure 20e). As the cooling continued, the convective layer grew in thickness and eventually completely disappeared (Figure 20f) as the water temperature became practically uniform below the thermal layer, see Figure 21. After the temperature in the bulk of the water became uniform, cooler water was observed to continually descend from the surface layer. The start of the descending water appeared
Figure 21. Temperature distribution during cooling of thermally stratified water; heat exchange with a cold sink placed above the water surface (see Figure 22 for the sink temperature), $T_0 = 17.5\, ^\circ\text{C}$.

Figure 22. Variation of the plate (sink) temperature for experiment described in Figure 21.
at random locations and no characteristic length (wave number) could be defined for the process. The downward moving flow was seen as plumes (Figure 20f) of cooler water emanating from the surface region. The plumes were often followed by a sheet of descending water which was approximately 0.05 to 0.1°C cooler than the surrounding bulk mean temperature. The denser water would penetrate up to 10 cm into the underlying region before losing its identity.

The interference fringe patterns during cooling of a more highly stratified layer of water are shown in Figure 23. The corresponding temperature distributions are presented in Figure 24. The water was heated for a longer period of time than in the experiment discussed in the preceding paragraph, and this resulted in a more intense stratification. The water was then cooled with a heat sink placed above the surface. The results show that as a consequence of more intense stratification, the flow in the convective layer remained laminar for a longer period. The growth rate of the convective layer was slower during the earlier cooling phase ($t < 300$ s) and from then on was relatively faster than in experiments with less intense initial thermal stratification.

In the surface "skin layer" heat transfer is primarily by molecular conduction, and relatively large temperature gradients are evident as shown in Figures 16, 19, 21, and 24. Turbulent diffusion predominates over molecular diffusion in the lower part of the thermal boundary layer. The nearly isothermal region, the convective (mixed) layer, exhibits very high effective conductivities representative of turbulent overturn. For example, at $t = 420$ s and depth $z = 1.6$ cm, in about the center of the convective layer (Figure 23), the turbulent diffusivity, $\alpha_t$, is estimated to be about twenty-five times larger than $\alpha$, the molecular thermal diffusivity of water. The temperature profile in the stable region below the mixed layer becomes slightly less stable as the cooling continues suggesting the entrainment of colder water into the mixed layer. The heat flux at the top of the stable layer is downward, i.e., heat is being added to the cold water from the mixed layer. This latter effect is small, however, because the molecular conductivity of water is quite small ($k = 0.597$ W/m°C at 20°C).
Figure 25. Selected interferograms illustrating cooling of intensely stratified layer of water; heat exchange with a cold sink placed above the water surface. 

(a) $t = 240 \text{ s}$  
(b) $t = 60 \text{ s}$  
(c) $t = 210 \text{ s}$  
(d) $t = 30 \text{ s}$  
(e) $t = 660 \text{ s}$  
(f) $t = 720 \text{ s}$
Figure 24. Temperature distribution during cooling of thermally stratified water; heat exchange with a sink placed above the water surface, $T_p = 5.5$ C.

4.3.3 Dynamics of the Convective Layer

In the experiments neither the temperature nor the heat flux at the surface of the water were constant. Therefore, Eqs. (37) and (38) could not be solved in closed form so that numerical solutions were obtained instead. The effective heat transfer coefficient $h$ was determined for the experimental conditions. The proportionality factor $k$ which relates the
Figure 27. Comparison of predicted and measured heat fluxes at the water surface; see Figure 25 for conditions.

Figure 28. Comparison between predicted and measured mixed layer thicknesses and temperatures; heat exchange between water and environment. $T_o = 19.5^\circ C$, $T_a = 19.5^\circ C$, $\kappa = 0.3$. 
Figure 25. Effect of parameter $\kappa$ on the mixed layer thickness during cooling of thermally stratified water; heat exchange between the water and environment, $T_o = 17.5 \, ^\circ C$, $T_a = 17.5 \, ^\circ C$.

Figure 26. Effect of parameter $\kappa$ on the mean mixed layer temperature; see Figure 25 for conditions.
heat flux at the water surface to the heat transported into the convective layer due to entrainment, see Eq. (30), was established using experimental data. The factor is not constant throughout the development of the convective layer and varies with time, but in the calculations shown in Figures 25 and 26a mean value of \( \kappa \) was used. The results are found to be rather insensitive to \( \kappa \). Because of the relative insensitivity of the results on \( \kappa \) and the uncertainty of its time dependence, it is not likely to limit the range of applicability of the simple thermal model. The entrainment, however, is essentially a dynamical process and since the model does not treat adequately the dynamics of the interfacial region, this inadequacy may be one of the limitations of the model. In the results to be presented a constant average value of 0.3 was used in the computations. This value is in the range of constants cited in the literature (Carson and Smith, 1974) during the growth of the turbulent convective layer in the atmosphere.

A comparison of the measured and predicted convective layer thickness given in Figure 25 shows that for short times \( (t < 5 \text{ min}) \) the analysis overpredicts and for longer time \( (t > 5 \text{ min}) \) underestimates the convective layer thickness. This is possibly due to the fact that early in the development of the convective layer, the turbulence is not completely developed. The theoretical underestimation of \( T_m \) by less than 10 percent is presumed to be due to inadequate modeling of the entrainment process. A change of \( \kappa \) from 0.3 to 0.2 would bring the measured and predicted temperatures to a better agreement.

The heat flux at the surface, \( H(0,t) \), evaluated from Eq. (35) is compared in Figure 27 with the experimentally determined flux. The results show that the theoretically predicted \( H(0,t) \) is lower than the experimental data for short times and is higher for longer times. This indicates that linearization of the interface energy balance by introducing a constant effective heat transfer coefficient \( h \) is not completely satisfactory. The value of \( h \) is not constant but varies with time because of the nonlinear dependence of the convective, evaporative, and radiative heat transfer at the interface on the water surface temperature.

A comparison of the predicted and measured depth as well as the mean temperature of the convective layer and the surface heat flux are presented in Figures 28 and 29, respectively. The results obtained were under
somewhat different stratification conditions than those already discussed, but they show the same trends. It should be pointed out that a constant value of the thermal boundary layer thickness $\delta$ (= 1.5 mm) was used in the theoretical predictions. The experimental data (see Figures 16, 19, and 20) clearly show that $\delta$ is not constant but increases with time. However, the resistance to heat transfer across the thermal boundary layer is small compared to the effective resistance to heat transfer on the air side of the water, and therefore has relatively little effect on the predicted results of $T_0$, $T_m$, and $h$. 

Figure 29. Comparison of predicted and measured surface heat fluxes, see Figure 28 for conditions.
4.3.4 Evaluation of Turbulent Diffusivity Model

The turbulent diffusivity model appropriate for free convection flow which was developed in Section 4.2 is evaluated by comparing the predicted temperature profiles using Eq. (55) with experimental temperature data during cooling of stratified water. Since the energy equation and the boundary conditions are nonlinear, the solutions of the energy equation, Eq. (20), were obtained numerically using an explicit finite difference method. To check the analytical model, a comparison of the measured temperatures obtained while cooling the stratified water with a cold sink was made. The data at \( t = 120 \) s (see Figure 21) were used as the initial conditions. This time was chosen because the mixed layer became turbulent at about this time. The predicted surface temperatures are seen to agree reasonably well with the measured ones; however, the calculated temperature gradients near the surface are larger than those obtained from experimentally determined profiles. Since the surface temperatures are in reasonably good agreement, this indicates that the turbulent diffusivities predicted in the vicinity of the surface are too low. The results show that as free circulation becomes established (say, at \( t = 180 \) s) it may begin to play a more important role in the transport of heat in the mixed layer. Even with relatively large turbulent diffusivities predicted (\( \alpha_T \approx 12\alpha \) in the vicinity of the center of the mixed layer) the amount of energy transported by molecular and turbulent diffusion is relatively small because the temperature gradients are small. The discrepancy between the results is not surprising since the analysis does not model entrainment of the cold water at the top of the stable region. Also, the free convective circulation in the mixed layer may be sufficiently large to contribute significantly to the vertical transport of energy.

Attempts were made to predict the temperature distribution during the cooling experiment described in Figure 24. However, the comparison was not successful. At \( t = 300 \) s (the temperature data at \( t = 180 \) s were used as an initial condition) the predicted temperatures in the mixed layer were about 0.75 C higher than the data, and at \( t = 600 \) s the predicted temperatures were about 0.25 C lower than the measurements. The cooling rate was too slow for early times and too fast for late times. This poor agreement is attributed to inadequate modeling of the physical situation,
particularly the neglect of free convective circulation in the mixed layer and of the entrainment in the interfacial layer. The ratio of the effective turbulent diffusivities to the molecular diffusivity in the vicinity of the center of the mixed layer were estimated to be about 20, 26, and 35 at \( t = 300, 420, \) and 600 s, respectively. No definite conclusions can be drawn about the validity of the turbulent diffusivity model under free convection conditions, but the energy transport model for intensely thermally stratified water cooled from above is certainly inadequate. Internal physical processes such as circulation in the convection layer and mixing at interface between the layer and the stable region must be accounted for in an analytical model.

4.3.5 Summary of Cooling Experiments

The data obtained in the experiments show complex free convection mixing phenomena in the surface layers of both uniform and stable, thermally stratified water cooled from the surface. During the initial phases of the cooling process the flow is laminar but eventually becomes turbulent. The free convection flow and mixing processes of the type studied in the experiments arise in natural waterbodies and are not only very important in the convective and dispersive transport of oxygen and any other substance introduced into the water, but also in the internal transport for establishing the temperature structure in the water as well. Based on the results obtained, the following conclusions can be drawn:

1. When a thermally stratified layer of water is cooled, a very complex temperature structure results. In the surface skin layer \((x < 0.2 \text{ mm})\), energy transport is by molecular diffusion and there exists a relatively sharp temperature gradient. Turbulent diffusion contributes significantly to the energy transport in the thermal boundary layer \((0.2 < x < 3 \text{ mm})\). In the convective layer, heat transfer is primarily by free convection, circulation, and turbulent diffusion.

2. A simple thermal model has been developed to predict the dynamics of the convective layer. The thickness and mean temperature of the mixed layer as well as the surface heat flux predicted by the
Figure 30. Comparison of predicted and measured temperature profiles during cooling of Intensely stratified layer of water for the experiment described in Figure 21.

model were, in general, within 10 percent of the observed results. However, some inadequacies of the model were noted in the body of the report.

3. The unsteady temperature profiles predicted using the turbulent diffusivity expression in the energy transport model yields reasonable agreement between analysis and data. However, both the diffusivity and energy transport models are not completely satisfactory and need to be improved.
5. CONCLUSIONS AND RECOMMENDATIONS

Laboratory experiments of thermal stratification induced by solar radiation and cooling of thermally stratified water were performed which closely simulate, except for the scale, the energy transport processes in impounded waterbodies. The data obtained and the analytical models constructed have contributed to water resources as a science by a better understanding of the internal physical processes and energy transport phenomena. The original objectives of the research program have been met, but numerous other problem areas needing study have arisen during the course of the investigation. Furthermore, the limitations of this present research present ample opportunity for future research. Two areas which require additional research effort are indicated.

Future laboratory and theoretical modeling studies of radiation induced thermal stratification in waters need to be more in accord with the physical processes occurring in nature. The diurnal heating and cooling cycle, the effects of impurities in the water, inflow and outflow of water, surface waves, and other phenomena occurring in natural water-bodies must be simulated in future modeling efforts.

It is recommended that free convection motion resulting from gravitational forces produced by temperature gradients in the water during the cooling of water from the surface be studied. The flow field in the convective layer should be visualized and quantitative measurements made to gain understanding of the complex circulation and entrainment phenomena. Future experimental modeling of energy transfer during cooling of water from the surface should simulate natural turbulence generated by winds, currents and waves, absorption of solar radiation, meteorological conditions, and the diurnal nature of heat exchange processes at the air-water interface. Theoretical models are needed for predicting turbulent transport occurring in buoyancy driven convection processes and mixing at an interface between unstable and stable regions in the water.
REFERENCES


