

January 2008

# A new field reconstruction method for permanent magnet synchronous machines

A. Khoobroo

B. Fahimi

S. D. Pekarek

Follow this and additional works at: <http://docs.lib.purdue.edu/ecepubs>

---

Khoobroo, A.; Fahimi, B.; and Pekarek, S. D., "A new field reconstruction method for permanent magnet synchronous machines" (2008). *Department of Electrical and Computer Engineering Faculty Publications*. Paper 52.  
<http://dx.doi.org/http://dx.doi.org/10.1109/IECON.2008.4758265>

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact [epubs@purdue.edu](mailto:epubs@purdue.edu) for additional information.

# A New Field Reconstruction Method for Permanent Magnet Synchronous Machines

Amir Khoobroo, *Student member IEEE*  
Power Electronics & Controlled motion  
Lab, #130 Nedderman Hall  
University of Texas at Arlington  
416, S. Yates St., Arlington, TX, 76019  
EML: akhoobroo@uta.edu

Babak Fahimi, *Senior member IEEE*  
Power Electronics & Controlled motion  
Lab, #130 Nedderman Hall  
University of Texas at Arlington  
416, S. Yates St., Arlington, TX, 76019  
EML: Fahimi@uta.edu

Steven D. Pekarek  
Purdue University  
465, Northwestern Ave, W.  
Lafayette, IN 47907  
EML: spekarek@ecn.purdue.edu

**Abstract**-The performance of the permanent magnet synchronous machines depends on the torque ripple and radial forces acting on its shaft and stator frame. There are two major ways to treat the undesirable torque ripples. In the first group of the methods machine design issues is considered. The second group considers optimal excitation of the machine so that the resulting performance is acceptable. In this paper a voltage formulation of the field reconstruction method is derived to predict the machine performance. The voltage source waveforms are used to find the associated 3 phase currents. The currents are used to calculate magnetic field distribution in the middle of the air gap. These quantities are used to predict the performance of the targeted PMSM.

## I. INTRODUCTION

Permanent magnet synchronous machines (PMSM) are widely used in industrial applications. Their relatively high power density, negligible rotor losses, high efficiency, ease of control and being almost maintenance free make them interesting for a variety of high performance applications. The electromagnetic torque of PM is created by the tangential force acting on the surface of the rotor. This tangential force is dependant upon the distribution of the magnetic field in the air gap. The geometry of the machine and the excitation of the stator windings affect the distribution of the magnetic field thereby influencing the torque. The other existing force component in the PMSM is the radial force which can potentially cause the radial vibrations in the stator. As mentioned the torque components are dependant upon the distribution of the magnetic field in the air gap. As a result, the distribution of the field should be altered to optimize the harmful effects of the force components. A wide variety of research have been carried out on this issue [1- 14].

Torque ripple minimization techniques include two areas of research. The first area is to alter machine geometry in the design phase by skewing the rotor and stator, fractional slot pitch winding and introduction of dummy slots [1-3]. Although effective, these methods have their own weaknesses such as reduction of the average torque and also for certain applications the construction of the machine is not economically feasible. The second area includes techniques to modify the stator excitation of the machine [4-14]. Computational methods to optimize the excitation and minimize the torque ripple are investigated in the past. These methods use different optimization algorithms that use field

analysis resulting from finite element analysis. As the finite element procedures are time consuming these methods are not adequate for real time control.

Field reconstruction method is a recent method that improves the process of finding optimal current profiles to minimize torque ripple [10, 11, 13]. This method uses the field created by a single slot along with the field generated by the permanent magnets on the rotor to find the field distribution and electromagnetic force components.

In this paper the field reconstruction method has been improved to be applied to a PMSM drive fed by a voltage source inverter. The practical problems with current source implementation would be eliminated. The field components in the air gap are used to estimate the flux linkages inside the stator teeth. These fluxes are used to estimate the flux linkages of the 3 phases which are necessary to solve machine equations.

## II. FORCE CALCULATION

A 1 hp, 3 phase, 4-pole, 12 slot surface mounted permanent magnet machine is used in this study. The model has been simulated using the commercial finite element package MAGNET from Infolytica©. The Model has been shown in Fig. 1. In this simulation the following assumptions have been made;

- No deformations in the permanent magnets or stator teeth due to the internal forces.
- Concentrated stator windings.
- No end coil effect.

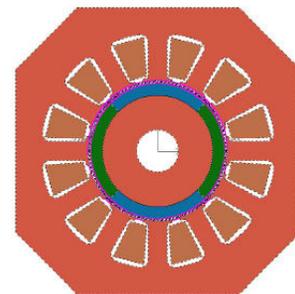


Fig 1: Cross sectional view of the PMSM

There are a variety of ways to calculate the electromechanical force in the electrical machines [15].

Among them Maxwell Stress Tensor (MST) method is chosen here. According to MST the force components densities in the air gap can be calculated using the following formulae:

$$f_t = B_n B_t / \mu_0 \quad (1)$$

$$f_n = (B_n^2 + B_t^2) / 2\mu_0 \quad (2)$$

In which  $B_n$  and  $B_t$  are normal and tangential components of the magnetic flux density. So the force components would be as follows:

$$F_t = \oint_{\Gamma} \vec{f}_t \cdot d\vec{l} \quad (3)$$

$$F_n = \int_0^{2\pi} f_n r d\phi \quad (4)$$

Where,  $r$  is the integration contour. It is obvious that for torque ripple minimization calculations magnetic field components should be known. The MST method is quite effective provided that the FEA solutions would be accurate. In the next section an alternative way of field calculation has been presented.

### III. FIELD RECONSTRUCTION

The conventional FEA methods are time consuming. It is shown in [10] that for an unsaturated PMSM the magnetization curve can be considered to be linear so the superposition rule is applied to the field components as follows:

$$B_t = B_{tpm} + B_{ts} \quad (5)$$

$$B_n = B_{npm} + B_{ns} \quad (6)$$

Where,  $B_{npm}$ ,  $B_{tpm}$ ,  $B_{ns}$  and  $B_{ts}$  denote the normal and tangential field components due to the permanent magnets and stator currents respectively. The resultant magnetic field created by the stator windings is the sum of the field created by each individual stator slot current. The normal and tangential field components due to the stator currents can be written as:

$$B_{ns} = \sum_{k=1}^L B_{nsk} \quad (7)$$

$$B_{ts} = \sum_{k=1}^L B_{tsk} \quad (8)$$

Where  $L$  is the number of stator slots. To evaluate (7) and (8) the local flux densities created by the current in the  $k^{th}$  slot is expressed as follows:

$$B_{tsk}(\phi_s) = I \cdot f_1(\phi_s) \quad (9)$$

$$B_{nsk}(\phi_s) = I \cdot f_2(\phi_s) \quad (10)$$

Where  $f_1$  and  $f_2$  associated with the geometry and  $\phi_s$  is defined as the position on the stator relative to the midpoint of the phase A stator slot in terms of electrical degrees. A single magneto-static FEA is needed to find these basis functions. Having these basis functions for a typical slot say  $1^{st}$  carrying current  $I_0$  (9) and (10) can be rewritten as:

$$B_{tsk} = (I/I_0)B_{ts0}(\phi - k\gamma) \quad (11)$$

$$B_{nsk} = (I/I_0)B_{ns0}(\phi - k\gamma) \quad (12)$$

So by performing a single off-line FEA for a single slot the stator contribution to the field components could be calculated for any normal working condition. In the second step permanent magnet contribution to the field is considered using an FEA off-line analysis for the unexcited stator condition. Having these two components the magnetic field components can be obtained in the middle of the air gap accurately.

The dynamic equations of induction machine can be written as [16]:

$$v_i = r i_i + \frac{d\lambda_i}{dt} \quad i = a, b, c \quad (13)$$

$$T - T_L = J(d\omega/dt) \quad (14)$$

In these equations  $v$  and  $i$  are voltage and currents of the 3 phase stator windings respectively.  $r$  and  $\lambda_i$  are the matrices of the stator winding resistance and phase flux linkages. Also,  $T$  is the generated torque  $T_L$  is the load torque and  $J$  is the moment of inertia. Bases on (13) the 3 phase currents can be written as:

$$i_i = (1/r)(v_i - \frac{d\lambda_i}{dt}) \quad i = a, b, c \quad (15)$$

As a result, the flux linkages should be calculated. Starting from the initial value for currents, the field components would be calculated using field reconstruction. Then  $B_n$  and  $B_t$  in the middle of the airgap are used to calculate the flux in the 12 stator teeth. Having flux in the stator teeth the 3 phase flux linkages would be calculated. The new flux linkages are used in (15) to find new currents. The procedure is summarized in the chart in Fig. 2.

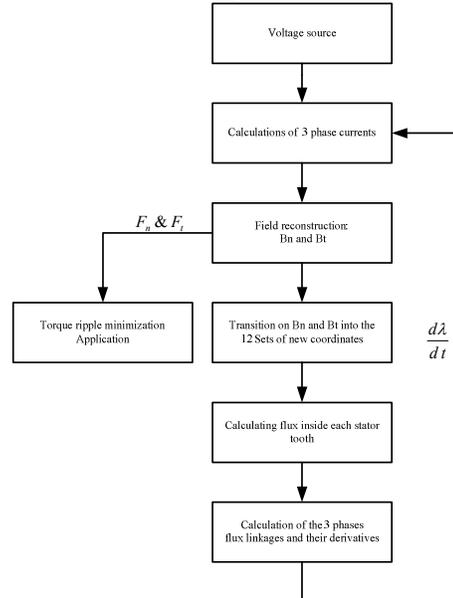


Fig 2: Modeling procedure chart

#### IV. FLUX LINKAGE CALCULATION

Flux linkage calculation has two steps. In the first step using the field components in the middle of the airgap the flux components in the 12 stator teeth would be calculated. In the second step the 3phase flux linkages would be calculated.

##### A. Stator teeth fluxes

The magnetic field distribution in the first quadrant of the model has been shown in Fig. 3. According to this figure almost all the flux lines that exist in the airgap would enter the stator tooth from the top surface. So, the flux in each stator tooth can be calculated using the magnetic fields in the airgap. There would be a slight error in this calculation because of the flux leakage namely some flux lines would enter the stator tooth from the side surfaces instead of top surface. These flux lines are not accounted for in the calculation that causes error.

The field component is projected on the axes passing the middle of each tooth. This can be done using the following equation:

$$B_{proj}(j) = \sum_{i=1}^K \{B_{n,i} \cos(\phi_i - \theta_j) - B_{t,i} \sin(\phi_i - \theta_j)\} \quad (16)$$

Where,  $\phi$  and  $\theta$  are the position of the field components in the airgap and the position of the projection axes in the model respectively. The indices  $i = 1 \dots K$  and  $j = 1 \dots L$  refers to the number of field components solutions in the airgap covering one stator tooth and the respective stator teeth order respectively. Having the normal field components the flux in the airgap which is almost equal to the flux in the stator tooth can be calculated as:

$$\Phi = \iint_S \vec{B}_{proj} \cdot d\vec{S} \quad (17)$$

The above integration is performed on the surface which is concentric to the rotor surface and passes through the stator teeth.

##### B. 3 phase fluxes

In the model there are 4 wires in each winding namely two sets of current carrying windings so the flux can be calculated for one set and then doubled to get the phase flux linkage. Fig. 4 depicts the flux related to, phase A in the first quadrant which is A1-A2 set The flux linkage of this winding is as follows:

$$\lambda_{A1-A2} = N(\Phi_2 + \Phi_3 + \Phi_4) \quad (18)$$

So, Phase A flux linkage is as follows:

$$\lambda_A = 2N(\Phi_2 + \Phi_3 + \Phi_4) \quad (19)$$

Where, N represents the number of conductors in each coil. This equation could be generalized into the following form for a machine with  $q$  stator tooth per pole per phase and  $2P$  magnetic poles ( $P$  represents the number of magnetic pole pairs):

$$\lambda_A = PN * \sum_{k=1}^q \Phi_k \quad (20)$$

The same analysis can be carried out for phases B and C. In

the next section the simulation results based on this analysis have been shown.

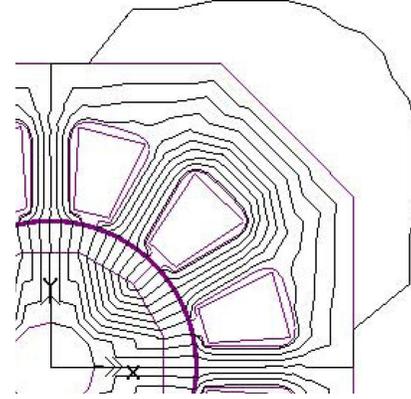


Fig 3: Field distribution in the model

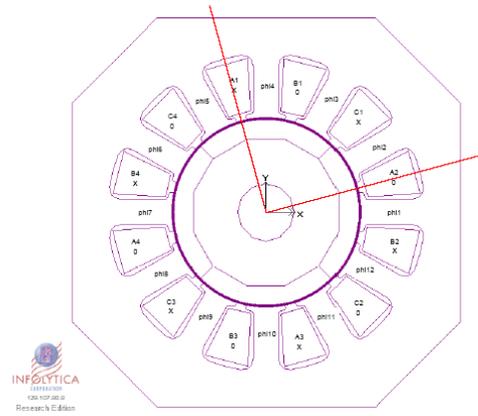


Fig 4: Flux assignment to stator teeth

#### V. RESULTS

This method has been tested on the machine for two cases in the first case the stator windings are open and rotor is rotating at 1000 rpm. The results from MAGNET have been compared to the results of the proposed procedure. It should be noticed that the simulation of this model on a desktop with a 2.8 GHz Pentium® 4 CPU for a period of 0 to 60 milliseconds take almost a full day while the written code based on field reconstruction and flux linkage computation takes less than 2 minutes for the same platform and the results are accurate. The proposed method minimizes the computation costs, which is one of the major drawbacks of the field analysis. Figs. 5 to 7 depict the results of the simulation. Fig. 5 depicts the phase A winding induced voltage by the permanent magnets. The procedure has very good accuracy except for the transient time and this is because of the different initial values used in the proposed procedure and the transient behavior of the machine. Fig. 6 depicts the normal component of the magnetic field in the middle of the airgap from MAGNET compared to the one from field reconstruction method.

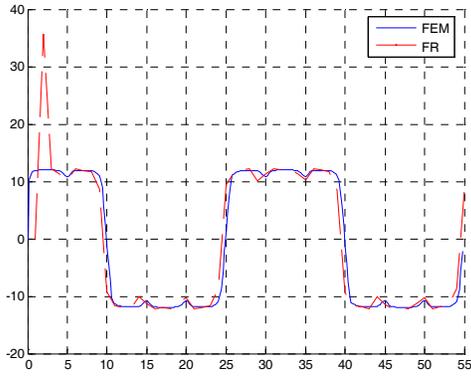


Fig 5: Phase A induced voltage

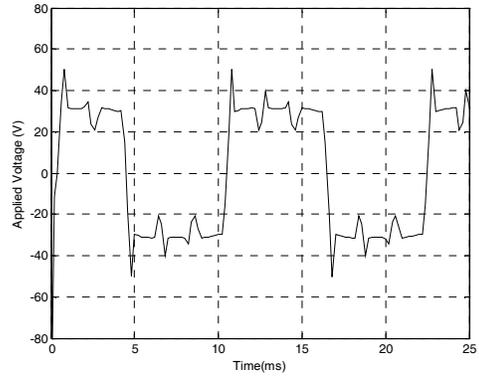


Fig 8: Phase A Applied voltage

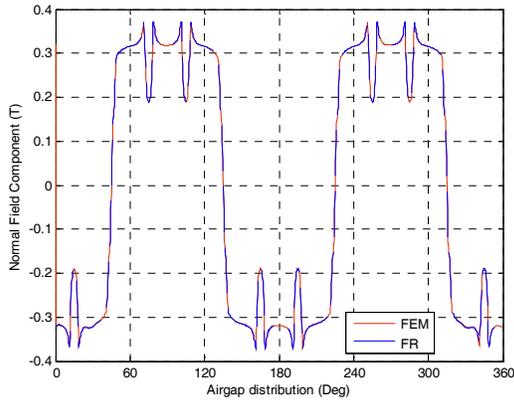


Fig 6: Normal field component comparison FEM vs. FR

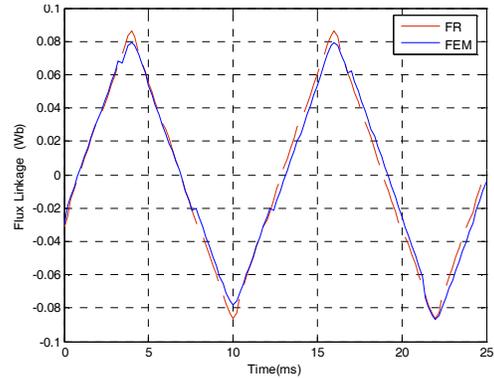


Fig 9: Phase A flux linkages FEM vs. FR

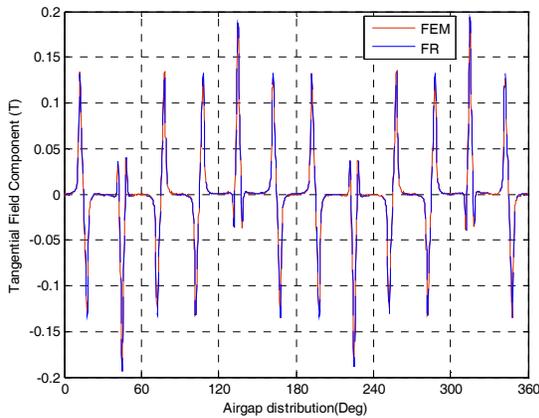


Fig 7: Tangential field component comparison FEM vs. FR

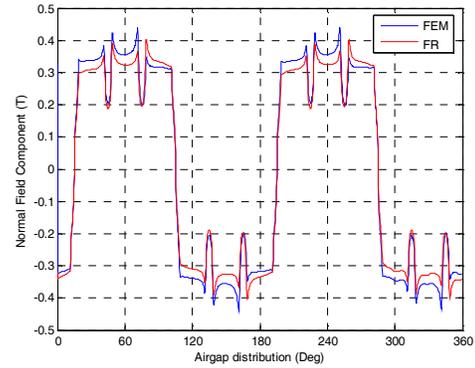


Fig 10: Normal field component comparison FEM vs. FR

The second set of results is shown in Figs. 8-11. In this case the voltage shown in Fig. 8 is applied to the stator windings. This waveform is generated in a way that there would not be any saturation in the magnetic materials because the saturation causes the magnetic characteristics to be nonlinear and as mentioned the field reconstruction is not suitable for this case and has considerable errors. The rotor speed is 2500 rpms.

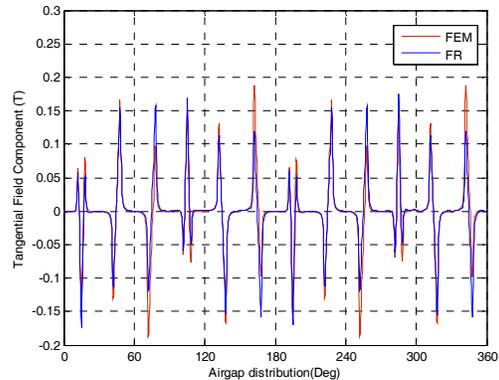


Fig 11: Tangential field component comparison FEM vs. FR

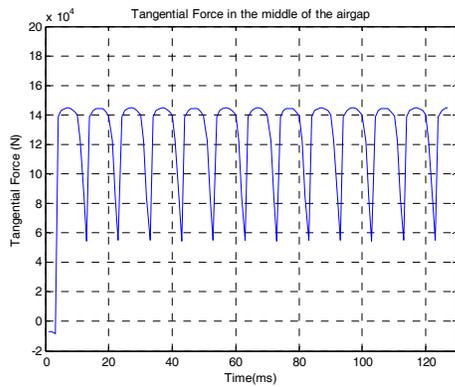


Fig 12: Tangential Force In the middle of the airgap

Fig. 9 depicts the flux linkages of phase A of the PM machine achieved from FEM method compared to the field reconstruction results. It is clear that the two profiles are in good agreement. The magnetic field normal and tangential components are shown in Figs 10 and 11. It can be seen that the field reconstruction method has an acceptable accuracy in computing the field components. Fig. 12 depicts the tangential force in the middle of the airgap.

## VI. CONCLUSION

In this paper a field reconstruction method for the voltage fed permanent magnet machine has been developed. The advantage of this method is that it reduces the computation time and cost significantly while the accuracy of the results are acceptable. Two cases have been simulated in this paper and in both cases the magnetic field computed using field reconstruction matches properly to the one obtained using MAGNET. This method can be used to applications like torque ripple minimization that requires the online computation of the field components in the airgap of the machine.

## References

- [1] T. M. Jahns and W. L. Soong, "Pulsating torque minimization techniques for permanent magnet ac motor drives-a review," *IEEE Trans. on Ind. Electron.*, vol. 43, no. 2, pp. 321-330, Apr. 1996.
- [2] D. C. Hanselman, "Minimum torque ripple, maximum efficiency excitation of brushless permanent magnet motors," *IEEE Trans. on Ind. Electron.*, vol. 41, no. 3, pp. 292-300, Jun. 1994.
- [3] D. C. Hanselman, "Effect of skew, pole count and slot count on brushless motor radial force, cogging torque and back EMF," *Inst. Elect. Eng. Proc.—Elect. Power Appl.*, vol. 144, no. 5, pp. 325-330, Sep. 1997.
- [4] R. Tirnovan, A. N'diaye, A. Miraoui, and R. Munteanu, "Analysis of feed currents influence on the electromagnetic forces in ac brushless motor with outer rotor," in *Proc. IEEE Int. Elect. Mach. Drives Conf.*, vol. 3, Jun. 1-4, 2003, pp. 1585-1589.
- [5] Q. Weizhe, S. K. Panda, X. Jian-Xin, "Torque ripple minimization in PM synchronous motors using iterative learning control," *IEEE Trans. on Ind. Electron.*, vol. 19, no.2, pp. 272-279, Mar 2004.
- [6] L. Parsa, K. Taehyung, "Reducing Torque Pulsation of Multi-Phase Interior Permanent Magnet Machines", *IEEE Conf. on Ind. Appl.*, vol. 4, pp. 1978-1983, Oct. 2006.
- [7] L. Parsa, H. A. Toliyat, "Five-phase interior permanent magnet motor with low torque pulsation", *IEEE Conf. on Ind. Appl.*, vol. 3, pp. 1770-1775, Oct. 2005.

- [8] A. Kioumars, M. Moallem, B. Fahimi, "Mitigation of Torque Ripple in Interior Permanent Magnet Motors by Optimal Shape Design", *IEEE Trans. on Magnetics*, vol. 42, no.11, pp. 3706-3711, Nov 2006.
- [9] W. Zhu, S. Pekarek, B. Fahimi, B. J. Deken, "Investigation of Force Generation in a Permanent Magnet Synchronous Machine", *IEEE Trans. on Energy Conversion*, vol. 22, no.3, pp. 557-565, Sept 2007.
- [10] W. Zhu, B. Fahimi, S. Pekarek, "A field reconstruction method for optimal excitation of permanent magnet synchronous machines", *IEEE Trans. on Energy Conversion*, vol. 21, no.2, pp. 305-313, June 2006.
- [11] B. Fahimi, "Qualitative approach to electromechanical energy conversion: Reinventing the art of design in adjustable speed drives", *ICEMS Int. Conf. on Electrical machines and Systems*, pp. 432-439, Oct 2007.
- [12] W. Jiang, M. Moallem, B. Fahimi, S. Pekarek, "Qualitative Investigation of Force Density Components in Electromechanical Energy Conversion Process", *IEEE Conf. on Ind. Electron.*, pp.1113-1118, Nov. 2006.
- [13] W. Zhu, B. Fahimi, S. Pekarek, "Optimal excitation of permanent magnet synchronous machines via direct computation of electromagnetic force components", *IEEE Int. Conf. on Electrical machines and Drives*, pp. 918-925, May 2005.
- [14] W. Zhu, S. Pekarek, B. Fahimi, "On the effect of stator excitation on radial and tangential flux and force densities in a permanent magnet synchronous machine", *IEEE Int. Conf. on Electrical machines and Drives*, pp. 346-353, May 2005.
- [15] A. Belahcen, "Overview of the calculation methods for forces in magnetized iron cores of electrical machines," *presented at the Seminar on Modeling and Simulation of Multi-Technological Machine Systems*, vol. 29, pp. 41-47, Nov. 1999.
- [16] P. C. Krause, *Analysis of Electric Machinery*, McGraw- Hill, 1986, New York.