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## Model-Based Performance Analysis of a Single Borehole in Ground Heat Exchanger

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### ABSTRACT

Ground Source Heat Pumps (GSHP) are high efficient HVAC equipment that provide heating and cooling for buildings. The high initial cost of drilling boreholes is one of the main reasons that hinder the widespread application of GSHP. It is very important to develop proper design methods to predict and evaluate the performance of the Ground Heat Exchanger (GHE) throughout its entire life span. In this paper, heat transfer inside the borehole is simplified as a one-dimensional (vertical) quasi-steady state model. The equivalent circuit method is used to solve the constant heat flux at the borehole wall that becomes the boundary condition for heat transfer outside the borehole. A two-dimensional (vertical and radial) transient model was developed to analyze heat transfer outside the borehole. These highly coupled equations are solved by the bisection method. This numerical model is used for a building located in Chicago as a case study. The results show that the ground temperature rises about 2.0 °C after 10 years operation at a location 5.0 meters away from the borehole. A hybrid GHE system that integrates cooling towers may be considered in this case to balance the net energy input to the GHE system.

### 1. INTRODUCTION

Ground-source heat pump (GSHP) systems use stored energy in the ground at a relatively constant temperature as both a heat source and a heat sink so that they offer higher energy efficiency than air-source systems. There are two kinds of closed-loop ground heat exchangers (GHE): horizontal and vertical. The advantages of the vertical GHE are: 1) to efficiently utilize available land for installing GHE; 2) to reduce the influence of the ground temperature fluctuations on the performance of the GHE. However, drilling of deep boreholes causes high initial cost, which hinders the application of the GSHP system. In order to use the GSHP, it is necessary to accurately predict the heat extraction and injection rates of the GHE during its entire life span in the design phase.

Many models have been developed to predict the performance of the GHE and to optimize the operation schedule of a GSHP system. The theoretical models are line source model by Ingersoll et al. (1954) and cylindrical source model by Carslaw and Jaeger (1959). Both models assume that a constant heat source occurs in an infinite homogeneous medium. The temperature distribution of the medium at different times is given under these one-dimensional transient models. Hart and Couvillion (1986) and Eskilson (1987) introduced the far-field radius, which depends on the operating time and on the thermal conductivity of the ground. Beyond the far-field radius, the ground temperature is assumed to be undistributed and constant. IGSHA (1991) developed an approximate solution from line source model to calculate the seasonal performance and energy consumption using the monthly bin method. An equivalent pipe diameter was suggested by Bose (1984) to consider the effect of the U-tube configuration inside the borehole. On the other hand, numerical models were also developed to account for the geometric complexities of a borehole and the variable properties of the ground around the borehole. Eskilson (1987) proposed a dimensionless temperature response factor, called g-functions, to describe the response of a single borehole to a unit step heat pulse. Then the energy flow rate profile is decomposed as the sum of a set of unit step heat pulses. The response of a single borehole to each unit step heat pulse is superimposed to determine the overall response to the energy flow rate profile. Hellstrom (1991) developed a duct storage model for a densely packed GHE, which was used for seasonal thermal storage. The ground is subdivided into two different parts: a local region and a global region. The local region is treated as quasi-steady state with constant heat flux and the global region is regarded as a transient process.

Rottmayer et al (1997) investigated a two-dimensional thermal resistance network for a single borehole. A pie-sector shaped pipe with the same perimeter as the circular U-tube is used to approximate heat transfer inside the borehole. A geometry factor is proposed to modify the variation effects from the shape change. Muraya (1995) used a transient two-dimensional finite element model to study the thermal interference between the U-tube legs. Li and Zheng (2009) developed a three-dimensional unstructured finite volume model for a single borehole to retain the geometric structure in the borehole. Delaunay triangulation method is used to mesh the cross-section domain of the borehole. Good accuracy is shown by a comparison between the model predictions and experimental data.

This paper aims to investigate the performance of a single borehole throughout its entire life span. The models mentioned above treat the ground as homogeneous medium. To accurately simulate heat transfer in the vertical direction, the ground is divided into several layers with different thermal conductivities in the vertical direction. The schematic of a single borehole in the ground is shown in Figure 1. In order to shorten the computing time, heat transfer inside the borehole is treated as one-dimensional (vertical) quasi-steady state within a relative long time step. Heat transfer outside the borehole is regarded as a two-dimensional (radial and vertical) transient process with a constant heat flux boundary condition at the borehole wall. To further improve the accuracy, the ground is divided into several layers in the vertical direction based on typical thermal properties of soil. Based on the numerical calculation, the design of a GHE for a medium sized office building in Chicago is evaluated over its life span in this paper.

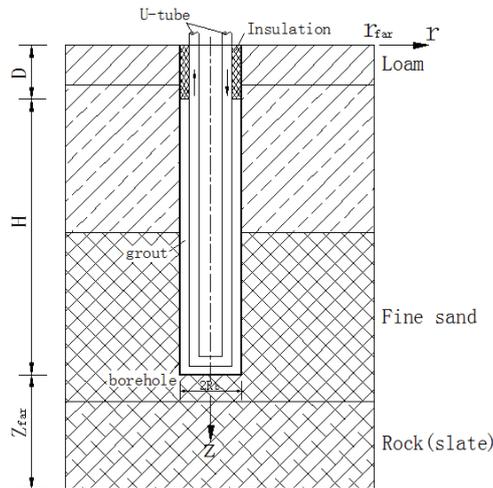


Figure 1: The schematic of a single borehole in ground

## 2. NUMERICAL MODEL

Discreted heat transfer equations are developed to describe the heat transfer phenomena both inside and outside the borehole. In order to develop the numerical model, the following assumptions are made:

### 2.1 Assumptions

- 1) The layers of the ground have homogeneous properties;
- 2) The properties of all the materials remain constant;
- 3) No moisture transport in the ground;
- 4) The effect of groundwater is ignored;
- 5) No contact resistance between U-tubes and grout;
- 6) Thermal capacity of the grout is ignored;
- 7) Quasi-steady state is maintained inside the borehole for a time step;
- 8) Boundary condition is adiabatic at the bottom of the borehole;
- 9) Fluid flow rate is uniform in the U-tube;
- 10) Fluid flow in the U-tube is turbulent and fully developed.

**2.2 Heat transfer inside the borehole**

In this model, On-Off control is used based on the requirement of a medium sized office building located in Chicago. When the circulation pump of the GHE is on, heat transfer inside the borehole is a forced flow process. When the circulation pump is off, there is no flow assumed in the U-tube. Heat transfer equations for these two processes are developed.

2.2.1 Forced flow process: In the borehole, the wall temperature is highly coupled with the fluid temperature. The fluid temperature at every grid point is solved simultaneously. According to Claesson (1986), if the length of each time step is greater than  $5r_b^2/a_g$ , heat transfer inside the borehole can be regarded as quasi-steady state within this time step. For the typical engineering application, this timescale is about 4 hours.

Figure 2 shows the grid scheme and equivalent thermal circuit inside the borehole. The fluid, pipe and grout between two grids are chosen as a control volume. The average borehole wall temperature in a time step is noted as  $\bar{T}_b$ . The equivalent thermal circuits are expressed for downstream and upstream flow in Equation (1) and (2). Therefore, it is a one-dimensional (vertical) quasi-steady state heat transfer inside the borehole.

Downstream flow:

$$\dot{m}c(T_{f,i} - T_{f,i-1}) = \frac{\Delta z_{i-1}}{2} \left( \frac{\bar{T}_{b,i-1} - T_{f,i-1}}{R_d} + \frac{\bar{T}_{b,i-1} - T_{f,i}}{R_d} + \frac{T_{f,2n_f-i+2} - T_{f,i-1}}{R_{ud}} + \frac{T_{f,2n_f-i+2} - T_{f,i}}{R_{ud}} \right) \quad (1)$$

$(2 \leq i \leq n_f)$

Upstream flow:

$$-\dot{m}c(T_{f,i} - T_{f,i-1}) = \frac{\Delta z_{i-1}}{2} \left( \frac{\bar{T}_{b,i-1} - T_{f,i-1}}{R_u} + \frac{\bar{T}_{b,i-1} - T_{f,i}}{R_u} + \frac{T_{f,i-1} - T_{f,2n_f-i+2}}{R_{ud}} + \frac{T_{f,i} - T_{f,2n_f-i+2}}{R_{ud}} \right) \quad (2)$$

$(n_f + 2 \leq i \leq 2n_f)$

The thermal resistances are quoted from Lee (2007). The line heat flux of a borehole is the sum of heat flux from borehole wall to the two legs of the U-tube:

$$q'_{i-1} = \frac{\bar{T}_{b,i-1} - \frac{T_{f,i-1} + T_{f,i-2}}{2}}{R_d} + \frac{\bar{T}_{b,2n_f-i+2} - \frac{T_{f,2n_f-i+2} + T_{f,2n_f-i+1}}{2}}{R_u} \quad (2 \leq i \leq n_f) \quad (3)$$

where  $\bar{T}_{b,i-1} = \bar{T}_{b,2n_f-i+2} \quad (4)$

Because of assumption 8), the fluid temperature at the bottom of the U-tube is:

$$T_{f,n_f+1} = T_{f,n_f} \quad (5)$$

The inlet fluid temperature is known as:

$$T_{f,1} = T_{f,in} \quad (6)$$

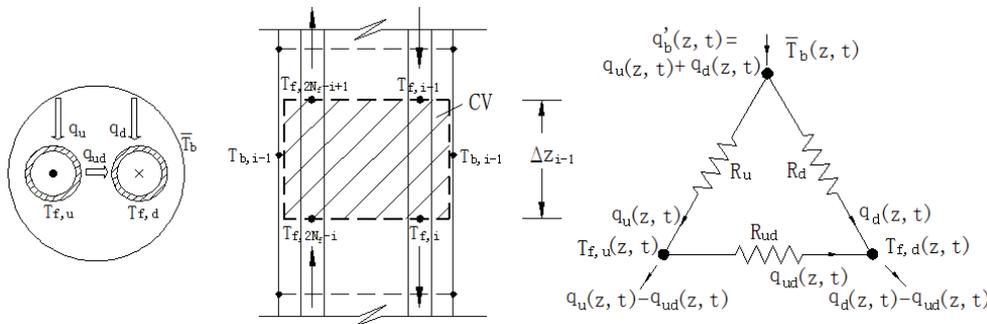


Figure 2: The grid scheme and equivalent thermal circuit inside borehole

2.2. 2 No flow process: If there is no flow going through the U-tube, the boundary condition is assumed as adiabatic at the borehole wall because the thermal capacities of the grout, the U-tube and fluid inside the U-tube are negligible to that of the ground around the borehole. Therefore, when the heat pump is off, the line heat flux is:

$$q'_{i-1} = 0 \quad (2 \leq i \leq n_f) \quad (7)$$

### 2.3 Heat transfer outside borehole

The ground is meshed in a cylinder coordinate system to describe the two-dimensional (radial and vertical) transient heat transfer phenomena. A control volume, as shown in Figure 3, in the ground is studied. The energy balance is implemented in this control volume and its implicit difference equations are:

$$\rho_g c_g V \frac{T_{g,i,j}^{k+1} - T_{g,i,j}^k}{\Delta t} = q_{r,j-1}'' A_{r,j-1} - q_{r,j}'' A_{r,j} + q_{z,i-1}'' A_{z,i-1} - q_{z,i}'' A_{z,i} \quad (2 \leq i \leq n_z, 2 \leq j \leq n_r) \quad (8)$$

where

$$q_{r,j-1}'' = \frac{T_{g,i,j-1}^{k+1} - T_{g,i,j}^{k+1}}{r_{j-1} \ln\left(\frac{r_j}{r_{j-1}}\right) / k_g} \quad (8.a)$$

$$q_{r,j}'' = \frac{T_{g,i,j}^{k+1} - T_{g,i,j+1}^{k+1}}{r_j \ln\left(\frac{r_{j+1}}{r_j}\right) / k_g} \quad (8.b)$$

$$q_{z,i-1}'' = \frac{T_{g,i-1,j}^{k+1} - T_{g,i,j}^{k+1}}{\Delta z_{i-1} / k_g} \quad (8.c)$$

$$q_{z,i}'' = \frac{T_{g,i,j}^{k+1} - T_{g,i+1,j}^{k+1}}{\Delta z_i / k_g} \quad (8.d)$$

For the boundary condition, according to Eskilson (1987), the far-field radius is:

$$r_{far} = 3\sqrt{a_g t_{max}} \quad (9)$$

Similarly, the far-field depth is also pre-defined as:

$$z_{far} = \sqrt{a_g t_{max}} \quad (10)$$

The far-field boundary is assumed adiabatic. If the ground temperature change at the far-field boundary is not negligible during the life span, the far-field boundary should be enlarged to cover the domain of the ground that is impacted by the borehole. At the borehole wall, the boundary condition is constant heat flux when the circulation pump runs or adiabatic when the circulation pump is off, as shown in Equations (3) and (7). Above and below the borehole wall, the boundary condition of the ground is adiabatic. That is,

$$q_{r, far}'' = q_{z, far}'' = q_{r, b, z < D}'' = q_{r, b, z > H}'' = 0 \quad (11)$$

At the top surface of the ground, convective heat transfer is assumed to occur between the soil and sol-air temperature (ASHRAE Handbook, 2005) with a constant annual average convective heat transfer coefficient of ,

$$q_{z=0}'' = hA(T_{sol-air}^{k+1} - T_{g,1,j}^{k+1}) \quad (12)$$

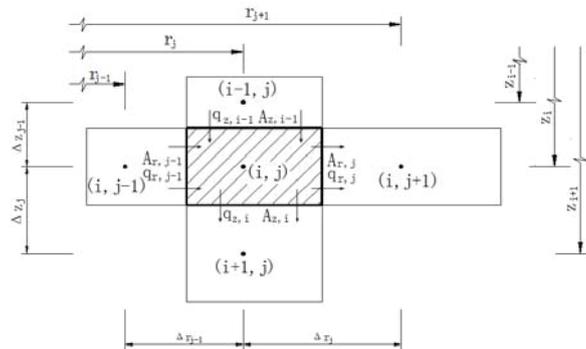


Figure 3: The schematic of energy balance in control volume in ground

### 3. NUMERICAL SCHEME

The size of the grid is set to get an accurate solution without extensive computation time. The principles of sizing the grid are to set a fine grid where the temperature changes more dramatically and a coarse grid where the temperature gradient is relatively small. Obviously, in the radial direction, the temperature gradient near to the borehole is much larger than that away from the borehole. In the vertical direction, the temperature also changes a lot near the top and bottom of the borehole. In this paper, the first node along the radial direction is at the borehole wall and the first grid spacing is equal to  $r_b$ . Incremental grid spacing is used in the radial direction. A fine-coarse-fine-coarse grid is meshed from top to bottom to catch the trend of temperature gradient along the vertical direction.

Different minimum grid spacing and ratio in the radial and vertical directions are used to check the accuracy and computation time. The simulation period is one month for heating. The result is shown in Table 1.

Table 1: The building information in the case study

Case No.	Min radial grid (m), Radial ratio	Min vertical grid (m), Vertical ratio	Cell numbers	Computing time (s)	Relative error at borehole wall (%)
1	0.05, 1.1	1.0, 1.2	2,352	1,920	-1.1
2	0.05, 1.2	1.0, 1.2	1,260	360	+2.3
3	0.05, 1.3	1.0, 1.2	1,176	180	+2.7
4	0.05, 1.05	0.9, 1.1	5,040	16,800	--

Considering the computation time and the accuracy, the grid of Case 1 is chosen to mesh the domain of the borehole and the ground with an acceptable tradeoff between the computation time and the accuracy.

According to Claesson (1986), the appropriate time step is determined to regard the heat transfer inside borehole as quasi-steady state. The bisection method is used to solve the temperature of fluid and ground within one time step. The numerical scheme is as below:

- 1) set the low and high limit of borehole wall temperature;
- 2) the borehole wall average temperature is assumed as  $(T_{low} + T_{high})/2$ , the fluid temperature and line heat flux are calculated based on Equation (1), (2) and (3);
- 3) the ground temperature is calculated from the governing Equation (4) at the end of this time step;
- 4) if the assumed borehole wall temperature is higher than that calculated by step 3 beyond the tolerance,  $T_{new,high} = (T_{low} + T_{high})/2$ , vice versa;
- 5) repeat step 2) until the assumed and calculated borehole wall temperature converges, then go to the next time step.

### 4. CASE STUDY

In this case study, the numerical model is used to evaluate the design of the GHE for a GSHP system which provides heating and cooling for a medium sized office building in Chicago. The information of this building is listed in Table 2. The operation parameters of the GHE and the heat pump system are shown in Table 3. The COP of heating and cooling for the GSHP are assumed to be 4.0 and 5.5 respectively based on typical operating conditions. Properties of the U-tube, grout, and the ground are listed in Table 4. The distribution range of the ground is also shown in Table 4.

Table 2: The building information in the case study

Item	Value	Item	Value
Location	Chicago, IL	floor	0.294
Building area (m <sup>2</sup> )	1660.73*3	R_value	1.949
Conditioned Building area (m <sup>2</sup> )	4982.19	(m·K/W)	2.611
Window-Wall ratio (%)	33.0	roof	0.315
Heating occupied set point (°C)	21	Heating unoccupied set point (°C)	15.6
Cooling occupied set point (°C)	24	Cooling unoccupied set point (°C)	30
Heating energy (GJ)	800	Cooling energy (GJ)	1336

Table 3: The parameters of GHE and the operation of GSHP

Item	Value	Item	Value
$T_{g,initial}$ ( $^{\circ}\text{C}$ )	17	$d_i$ (m)	0.024
$T_{f,initial}$ ( $^{\circ}\text{C}$ )	7	$d_o$ (m)	0.032
$\rho_f$ ( $\text{kg}/\text{m}^3$ )	998.3	$d_{leg}$ (m)	0.06
$c_f$ ( $\text{J}/\text{kg}\cdot\text{K}$ )	4183	$d_b$ (m)	0.10
$k_f$ ( $\text{W}/\text{m}\cdot\text{K}$ )	0.586	$H$ (m)	100
$\dot{m}$ ( $\text{kg}/\text{s}$ )	0.5	$D$ (m)	5
$COP_{heating}$	4.0	$COP_{cooling}$	5.5

Table 4: The properties of U-tube, grout and the formation of ground

Name	Density ( $\text{kg}/\text{m}^3$ )	Specific heat ( $\text{J}/\text{kg}\cdot\text{K}$ )	Conductivity ( $\text{W}/\text{m}\cdot\text{K}$ )	Distribution range (m)
U-tube	950	2250	1.2	0-105
Grout	1400	1200	1.3	0-105
Loam	1490	1300	0.8	0-4
Loam and clay (1:1)	1580	1320	1.1	4-45
Fine sand	1800	1200	2.0	45-120
Rock (hard)	2050	1150	2.3	120+

The modeling horizon of the GHE is assumed to be 10 years. The initial fluid temperature is  $10\text{ }^{\circ}\text{C}$  and the initial ground temperature is  $17\text{ }^{\circ}\text{C}$ . The time step is 4 hours in this case study. The fluid inlet temperature is updated at the beginning of every time step based on the heating/cooling load of the building which is calculated using commercially available software. There are 30 boreholes for this GSHP system. The thermal interference between the boreholes is assumed to be negligible in this case. A prediction of the GHE performance on December 31<sup>st</sup> (heating mode) in different years is plotted in Figure 5-8. The y axial shows the vertical location below the top of the borehole. The value 0 means the top level of the borehole and the value 1 means the bottom level of the borehole. Figure 5 shows the borehole wall temperature change from the initial value after different years. The borehole is extracting heat from the ground, so the borehole wall temperature change is negative along the borehole. Because the cooling load is higher than heating load in this building, heat is built up in the soil around the borehole, as proved by the gradual increase of borehole wall temperature with the operating years. The borehole load profile is shown in Figure 6. Because the thermal conductivity rises from  $1.1\text{ W}/\text{m}\cdot\text{K}$  to  $2.0\text{ W}/\text{m}\cdot\text{K}$  at the interface of loam/clay and fine sand, the load of borehole jumps from  $-9\text{ W}/\text{m}$  to  $-15\text{ W}/\text{m}$  at the interface. It shows the load profile is very sensitive to the thermal conductivity of ground. It also shows a relatively constant load profile at the lower part of borehole. At the higher part of borehole, some variation of load profile is demonstrated, especially running after a few years.

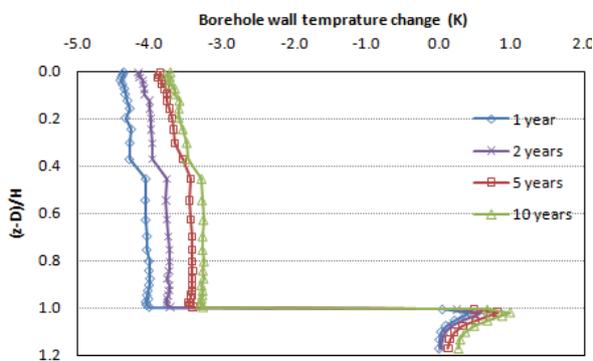


Figure 5: Borehole wall temperature profiles

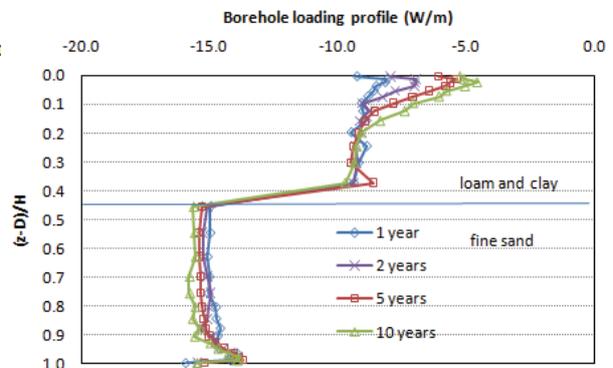


Figure 6: Borehole loading profiles

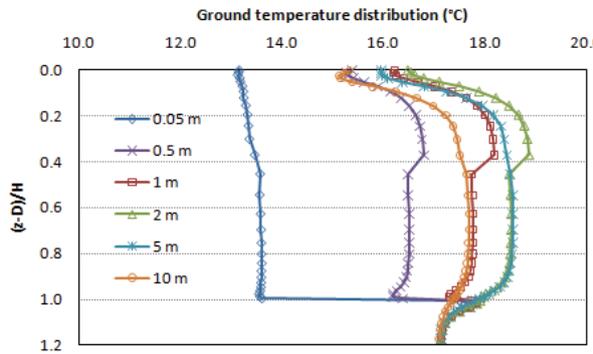


Figure 7: Ground temperature profiles after 5 years

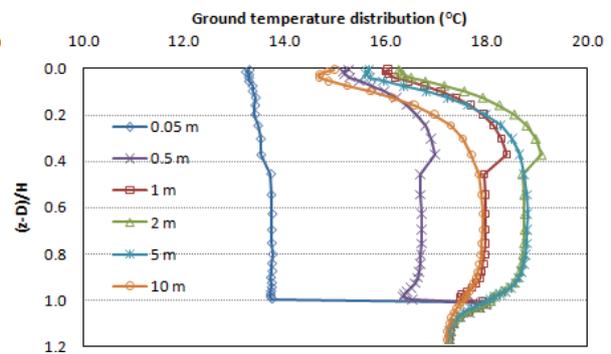


Figure 8: Ground temperature profiles after 10 years

Figure 7 and Figure 8 show the ground temperature profiles at different distances from the borehole after 5 and 10 years. On December 31<sup>st</sup> (the middle of heating season), the ground within 0.5 meter from the borehole is responding to the short term heat flux from the ground to the fluid. The ground away from the borehole is dominated by the long term net energy input to the GHE system.

In this case study, the thermal interference between boreholes is not included. Because the imbalance of cooling and heating load in this building, the ground temperature 5 meters away from the borehole rises about 2.0 °C after 10 years. A GHE system integrating cooling towers may be considered to balance heat transferred from the ground to guarantee long term performance of the GSHP system.

## 5. CONCLUSION

In this paper, heat transfer inside the borehole is simplified as a one-dimensional (vertical) quasi-steady state model with a time step of 4 hours. The equivalent circuit method is used to determine the constant heat flux at the borehole wall, which acts as the boundary condition for heat transfer outside the borehole. A two-dimensional (vertical and radial) transient model is developed to analyze heat transfer outside the borehole. These highly coupled equations are solved by the bisection method. This numerical model is used for a building located in Chicago as a case study. The simulation results show that the load profile is very sensitive to the thermal conductivity of the ground. The net energy input to the GHE is built up after a few years operation because of the imbalance of cooling and heating load in this building. A GHE system that integrates cooling towers will be further studied in the future to deal with the imbalance of building load.

## NOMENCLATURE

$A$	area	(m <sup>2</sup> )	<b>Subscripts</b>	
$a$	thermal diffusivity	(m <sup>2</sup> /s)	$b$	borehole wall
$c$	specific heat	(J/kg·K)	$d$	downstream of fluid
$D$	depth of insulation above borehole	(m)	$f$	fluid
$d$	diameter	(m)	$far$	far-field
$H$	length of borehole	(m)	$g$	ground
$h$	convective transfer rate	(W/m <sup>2</sup> ·K)	$i$	index of vertical direction
$k$	thermal conductivity	(W/m·K)	$in$	inlet
$\dot{m}$	fluid flow rate	(kg/s)	$j$	index of radial direction
$q''$	heat flux	(W/m <sup>2</sup> )	$max$	life years
$q'$	line heat flux	(W/m)	$o$	outside
$R$	thermal resistance	(m·K/W)	$r$	radial direction
$r$	radial distance from borehole	(m)	$u$	upstream of fluid
$T$	temperature	(°C)	$z$	vertical direction
$t$	time	(s)	<b>Superscripts</b>	

$V$	volume	(m <sup>3</sup> )	$k$	k-th time step
$Z$	depth from the bottom of borehole to far-field depth	(m)	—	average value in time step
$z$	depth from the top surface of ground	(m)		
$\rho$	density	(kg/m <sup>3</sup> )		

## REFERENCES

- Lee, C.K., Lam, H.N., 2008, Computer simulation of borehole ground heat exchanger for geothermal heat pump systems, *Renewable Energy*, vol. 33, no. 6: p. 1286-1296.
- Li, Z., Zheng, M., 2009, Development of a numerical model for the simulation of vertical U-tube ground heat exchangers, *Applied Thermal Engineering*, vol. 29, no. 5, : p. 920-924.
- Rottmayer, S.P., Beckman, W.A., Mitchell, J.W., 1997, Simulation of a single vertical U-tube ground heat exchanger in an infinite medium, *ASHRAE Transactions*, vol. 103, no. 2: p. 651-659.
- ASHRAE Handbook: Fundamentals, 2005, American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc, Atlanta.
- Bose, J.E., 1984, *Closed loop ground coupled heat pump design manual*, *Engineering Technology Extension*, Oklahoma State University.
- Carslaw, H.S., Jaeger, J.C., 1959, *Conduction of heat in solids*, Clarendon Press, Oxford, ?p.
- Claesson, J., Eskilson, P., 1986, *Conductive heat extraction to a deep borehole: Thermal analyses and dimensioning rules*, Dept. of Mathematical Physics and Building Technology, University of Lund, Sweden.
- Eskilson P., 1987, *Thermal analysis of heat extraction boreholes*, Doctoral Thesis, Dept. of Mathematical Physics and Building Technology, University of Lund, Sweden.
- Hart, D.P., Couvillion, R., 1986, *Earth coupled heat transfer*, National Water Well Association.
- Hellstrom, G., 1991, *Ground heat storage. Thermal analysis of duct storage systems: part I. Theory*, Doctoral Thesis, Dept. of Mathematical Physics, University of Lund, Sweden.
- IGSHPA, 1991, *Design and installation standards, Stillwater*, International Ground Source Heat Pump Association, Oklahoma.
- Ingersoll, L.R., Zobel, O.J., Ingersoll, A.C., 1954, *Heat conduction: with engineering, geological and other applications*, Madison University of Wisconsin Press, Madison, 325p.
- Muraya, N.K., 1995, *Numerical modeling of the transient thermal interference of vertical U-tube heat exchanger*, Doctoral Thesis, Texas A&M University, College Station.