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Measurement of Transmission Loss of Materials Using a Standing Wave Tube

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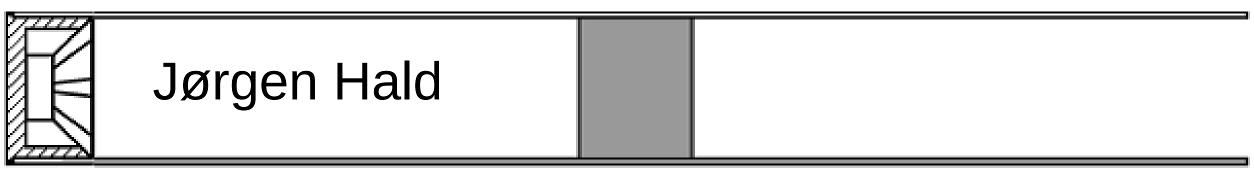


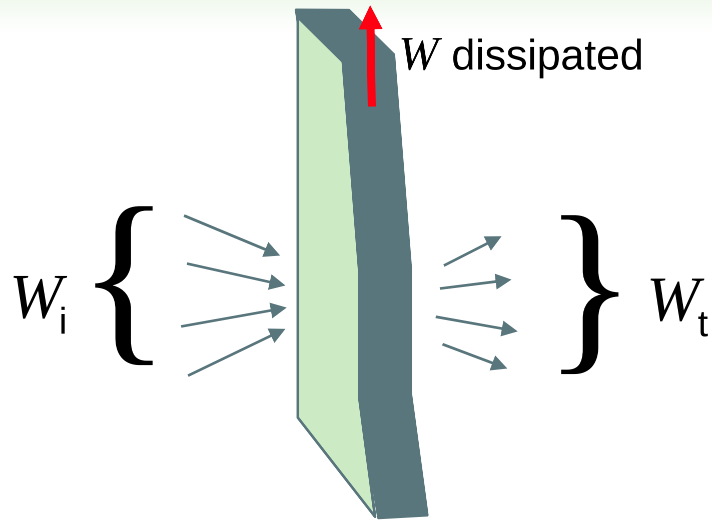
Program

- Definitions of Transmission Loss
- Plane wave model for tube measurements
- Scattering Matrix model of sample
  – Two-Load method
  – One-Load method
- Transfer Matrix model of sample
  – Two-load method
  – One-load method
  – Determination of material properties
- Experiments
- Summary

Transmission Loss

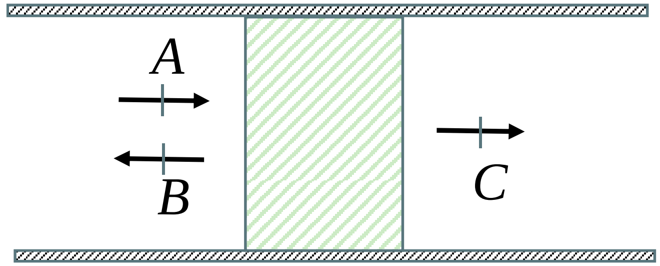
- In the case of a partition between two rooms

$$TL(\omega) \equiv 10\log_{10}\left(\frac{W_i}{W_t}\right)$$



- In the case of a plane-wave tube with a perfectly anechoic termination:

$$TL_n(\omega) = 10\log_{10}\left(\left|\frac{A^{(anechoic)}}{C^{(anechoic)}}\right|^2\right)$$



Plane wave model – Determination of coeff. *A*, *B*, *C* and *D*

$$P = (Ae^{-jkx} + Be^{jkx}) \qquad P = (Ce^{-jkx} + De^{jkx})$$

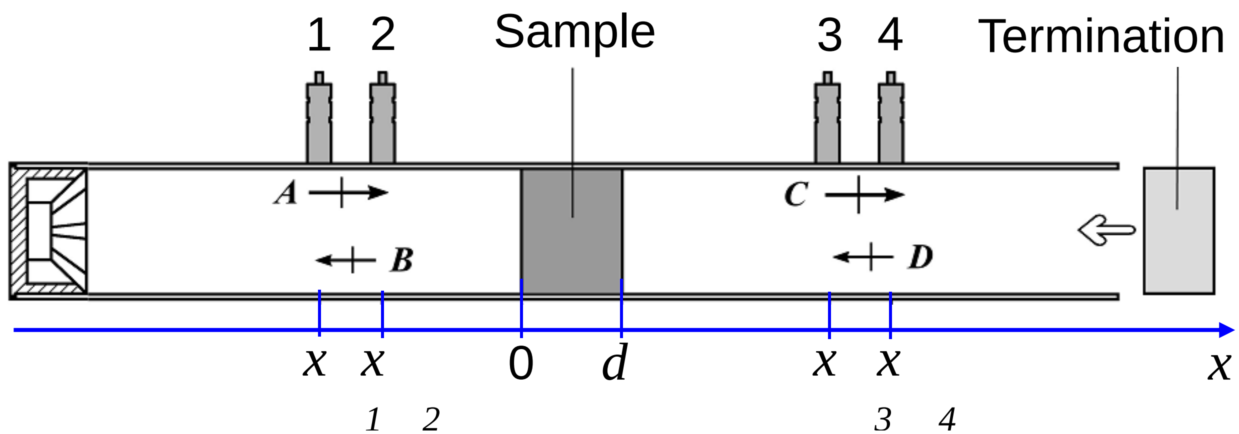
<u>Microphone pressures:</u>

$$P_1 = (Ae^{-jkx_1} + Be^{jkx_1})$$

$$P_2 = (Ae^{-jkx_2} + Be^{jkx_2})$$

$$P_3 = (Ce^{-jkx_3} + De^{jkx_3})$$

$$P_4 = (Ce^{-jkx_4} + De^{jkx_4})$$



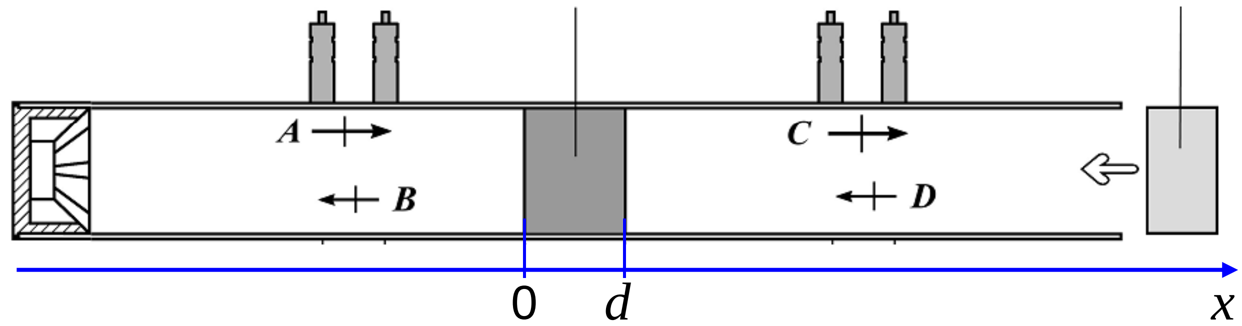
<u>Plane wave amplitudes in terms of microphone pressures</u>

$$A = \frac{j(P_1 e^{jkx_2} - P_2 e^{jkx_1})}{2\sin k(x_1 - x_2)}, \qquad C = \frac{j(P_3 e^{jkx_4} - P_4 e^{jkx_3})}{2\sin k(x_3 - x_4)},$$

$$B = \frac{j(P_2 e^{-jkx_1} - P_1 e^{-jkx_2})}{2\sin k(x_1 - x_2)}, \qquad D = \frac{j(P_4 e^{-jkx_3} - P_3 e^{-jkx_4})}{2\sin k(x_3 - x_4)}$$

Plane wave model – Scattering Matrices

$$P = (Ae^{-jkx} + Be^{jkx}) \qquad P = (Ce^{-jkx} + De^{jkx})$$



Seen from sample:

A and D incident

B and C outgoing

$$B = r_1 A + t_{21} D \qquad C = t_{12} A + r_2 D$$

$$\begin{bmatrix} B \\ C \end{bmatrix} = \begin{bmatrix} r_1 & t_{21} \\ t_{12} & r_2 \end{bmatrix} \begin{bmatrix} A \\ D \end{bmatrix}$$

Rewrite to left/right:

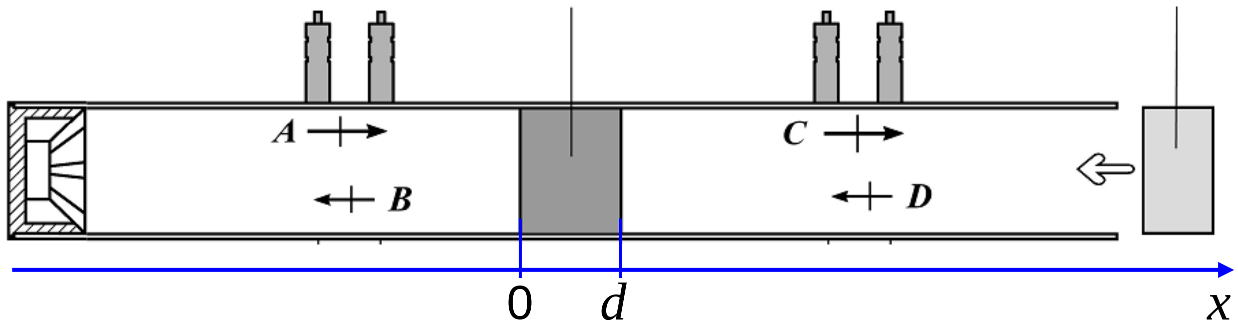
$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 1/t_{12} & -r_2/t_{12} \\ r_1/t_{12} & t_{21} - r_1 r_2/t_{12} \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix}$$

No sample:

$$\begin{bmatrix} r_1 & t_{21} \\ t_{12} & r_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Plane wave model – Scattering Matrices

$$P = (Ae^{-jkx} + Be^{jkx}) \qquad P = (Ce^{-jkx} + De^{jkx})$$



<u>Seen from sample:</u>

A and D incident

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$$B = r_1 A + t_{21} D \qquad C = t_{12} A + r_2 D$$

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<u>Transmission Loss:</u>

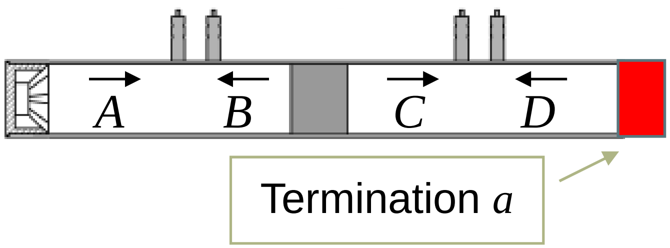
$$TL_n \equiv 10\log\left(\left|\frac{A}{C}\right|^2_{D=0}\right) = 10\log\left(|a_{11}|^2\right) = 10\log\left(\left|\frac{1}{t_{12}}\right|^2\right)$$

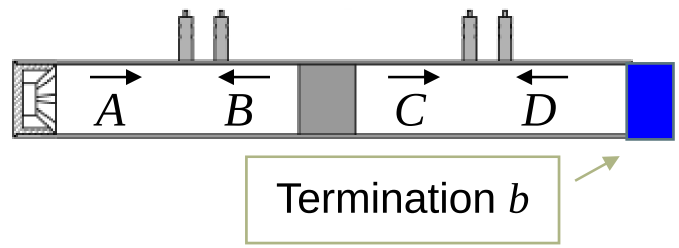
Two-load method – Based on Scattering Matrix

- One measurement: 2 equations, 4 unknowns:

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix}$$

- Two independent measurements:





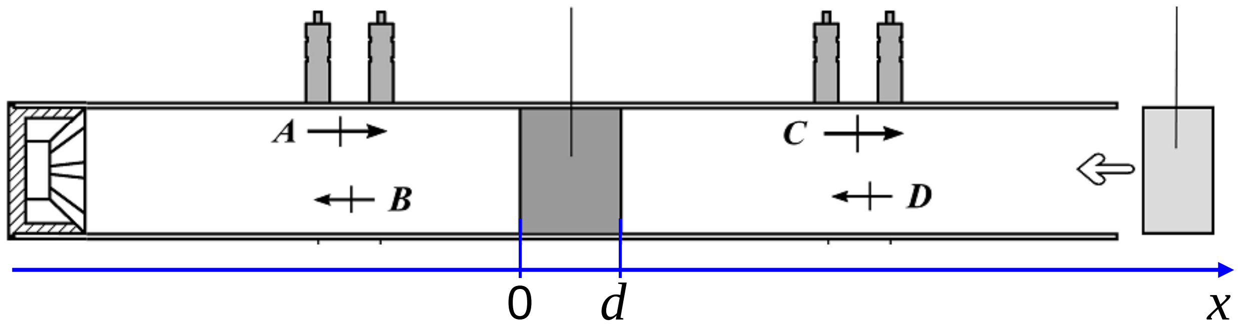
$$\begin{bmatrix} A^{(a)} & A^{(b)} \\ B^{(a)} & B^{(b)} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} C^{(a)} & C^{(b)} \\ D^{(a)} & D^{(b)} \end{bmatrix}$$

- Transmission Loss: $TL_n = 10\log\left(\left|a_{11}\right|^2\right)$

$$a_{11} = \frac{A^{(a)}D^{(b)} - A^{(b)}D^{(a)}}{C^{(a)}D^{(b)} - C^{(b)}D^{(a)}} = \frac{R^{(b)} \cdot A^{(a)}/C^{(a)} - R^{(a)} \cdot A^{(b)}/C^{(b)}}{R^{(b)} - R^{(a)}}, \quad R^{(s)} \equiv \frac{D^{(s)}}{C^{(s)}}$$

One-load method – Based on Scattering Matrix

$$P = (Ae^{-jkx} + Be^{jkx}) \qquad P = (Ce^{-jkx} + De^{jkx})$$



Symmetric reflection and transmission:

$$t_{21} = t_{12} \qquad r_2 = r_1$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1/t_{12} & -r_2/t_{12} \\ r_1/t_{12} & t_{21} - r_1 r_2/t_{12} \end{bmatrix}$$

Therefore:

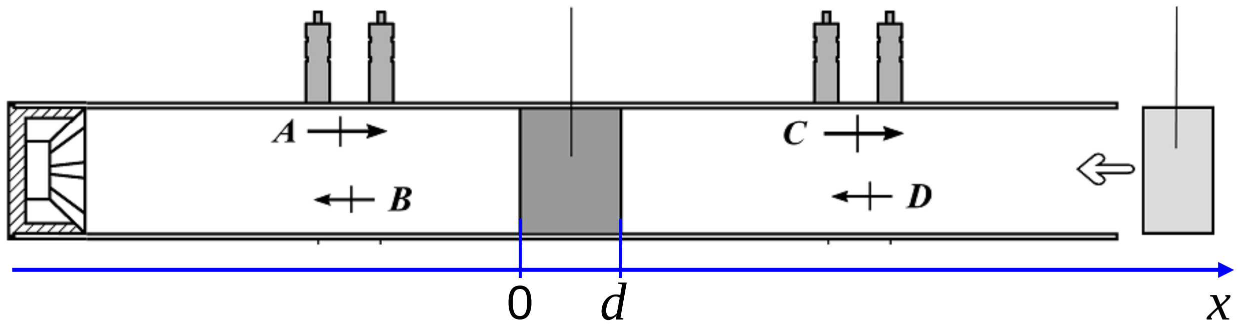
$$a_{21} = -a_{12} \qquad a_{11}a_{22} - a_{21}a_{12} = t_{21}/t_{12} = 1$$

One termination condition sufficient:

$$TL_n = 10\log\left(\left|a_{11}\right|^2\right) = 10\log\left(\left|\frac{A^2 - D^2}{AC - BD}\right|^2\right)$$

One-load method – Based on Scattering Matrix

$$P = (Ae^{-jkx} + Be^{jkx}) \qquad P = (Ce^{-jkx} + De^{jkx})$$



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Therefore:

$$a_{21} = -a_{12} \qquad a_{11}a_{22} - a_{21}a_{12} = t_{21}/t_{12} = 1$$

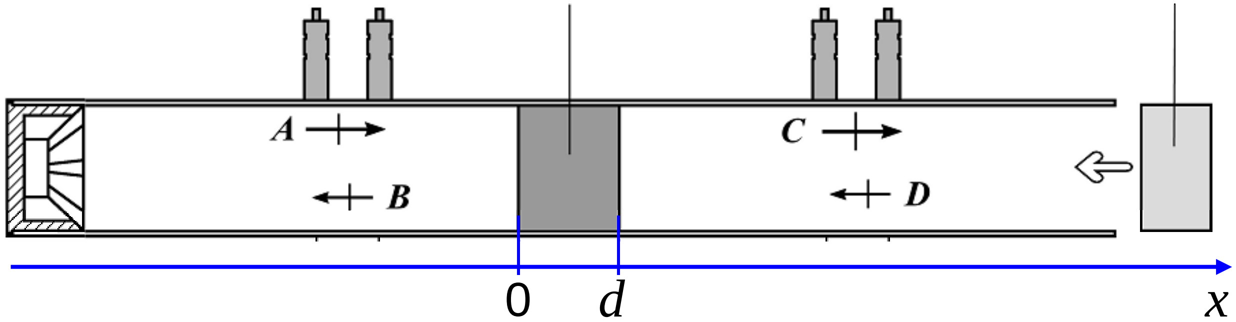
Anechoic reflection coefficient:

$$R_a = r_1 = \frac{a_{21}}{a_{11}} = \frac{AB - CD}{A^2 - D^2}$$

Transfer Matrix – Boundary values on sample

$$P = (Ae^{-jkx} + Be^{jkx})$$
$$V = (Ae^{-jkx} - Be^{jkx}) / \rho_0 c$$

$$P = (Ce^{-jkx} + De^{jkx})$$
$$V = (Ce^{-jkx} - De^{jkx}) / \rho_0 c$$



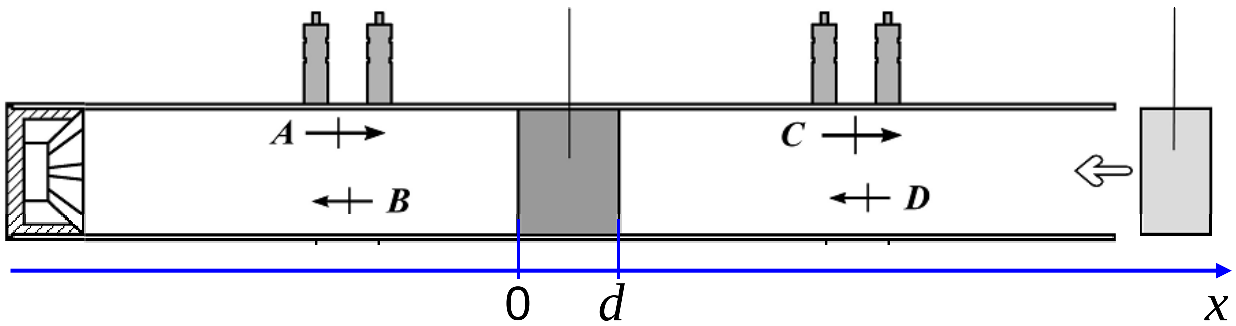
$$P\big|_{x=0} = (A + B)$$
$$V\big|_{x=0} = (A - B) / \rho_0 c$$

$$P\big|_{x=d} = (Ce^{-jkd} + De^{jkd})$$
$$V\big|_{x=d} = (Ce^{-jkd} - De^{jkd}) / \rho_0 c$$

Transfer Matrix

$$P = (Ae^{-jkx} + Be^{jkx}) \qquad P = (Ce^{-jkx} + De^{jkx})$$

$$V = (Ae^{-jkx} - Be^{jkx}) \big/ \rho_0 c \qquad V = (Ce^{-jkx} - De^{jkx}) \big/ \rho_0 c$$



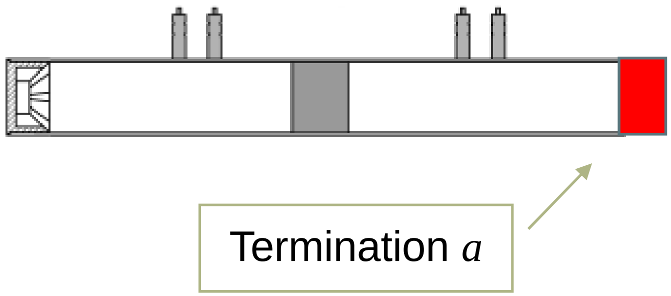
Transfer Matrix:

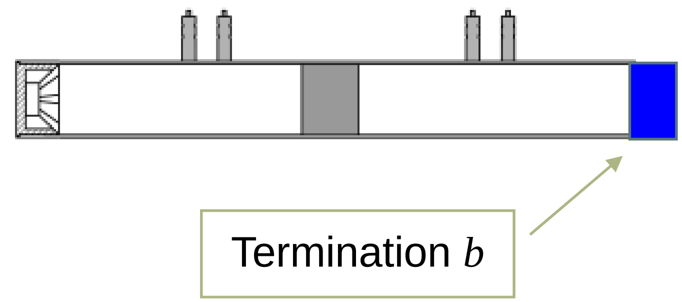
$$\begin{bmatrix} P \\ V \end{bmatrix}_{x=0} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} P \\ V \end{bmatrix}_{x=d}$$

- Elements characterize only sample (independent of excitation and termination)
- Provides complete two-port model of sample – allows TL_n to be obtained
- One measurement provides two equations for determination of four matrix elements

Two-load method – Determination of Transfer Matrix

- Two additional, independent equations can be generated by measuring a second termination condition





$$\begin{bmatrix} P^{(a)} & P^{(b)} \\ V^{(a)} & V^{(b)} \end{bmatrix}_{x=0} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} P^{(a)} & P^{(b)} \\ V^{(a)} & V^{(b)} \end{bmatrix}_{x=d}$$

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \frac{1}{P^{(a)}\big|_{x=d} V^{(b)}\big|_{x=d} - P^{(b)}\big|_{x=d} V^{(a)}\big|_{x=d}} \begin{bmatrix} P^{(a)}\big|_{x=0} V^{(b)}\big|_{x=d} - P^{(b)}\big|_{x=0} V^{(a)}\big|_{x=d} & -P^{(a)}\big|_{x=0} P^{(b)}\big|_{x=d} + P^{(b)}\big|_{x=0} P^{(a)}\big|_{x=d} \\ V^{(a)}\big|_{x=0} V^{(b)}\big|_{x=d} - V^{(b)}\big|_{x=0} V^{(a)}\big|_{x=d} & -P^{(b)}\big|_{x=d} V^{(a)}\big|_{x=0} + P^{(a)}\big|_{x=d} V^{(b)}\big|_{x=0} \end{bmatrix}.$$

One-load method – Determination of Transfer Matrix

- Symmetric reflection and transmission provides two equations:

$$T_{11} = T_{22}$$

$$T_{11}T_{22} - T_{12}T_{21} = 1$$

- One termination condition is then sufficient

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \frac{1}{P\big|_{x=0} V\big|_{x=d} + P\big|_{x=d} V\big|_{x=0}} \begin{bmatrix} P\big|_{x=d} V\big|_{x=d} + P\big|_{x=0} V\big|_{x=0} & P\big|_{x=0}^{2} - P\big|_{x=d}^{2} \\ V\big|_{x=0}^{2} - V\big|_{x=d}^{2} & P\big|_{x=d} V\big|_{x=d} + P\big|_{x=0} V\big|_{x=0} \end{bmatrix}.$$

Transmission Loss based on Transfer Matrix

- Relation between Transfer Matrix and Plane Wave Scattering Matrix:

$$\begin{bmatrix} A^{(s)} \\ B^{(s)} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} C^{(s)} \\ D^{(s)} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\left(T_{11} + \frac{T_{12}}{\rho_0 c} + \rho_0 c T_{21} + T_{22}\right)e^{-jkd} & \frac{1}{2}\left(T_{11} - \frac{T_{12}}{\rho_0 c} + \rho_0 c T_{21} - T_{22}\right)e^{+jkd} \\ \frac{1}{2}\left(T_{11} + \frac{T_{12}}{\rho_0 c} - \rho_0 c T_{21} - T_{22}\right)e^{-jkd} & \frac{1}{2}\left(T_{11} - \frac{T_{12}}{\rho_0 c} - \rho_0 c T_{21} + T_{22}\right)e^{+jkd} \end{bmatrix}$$

- Transmission Loss:

$$TL_n = 10\log_{10}\left(\left|a_{11}\right|^2\right) = 10\log_{10}\left(\frac{1}{4}\left|T_{11} + \frac{T_{12}}{\rho_0 c} + \rho_0 c T_{21} + T_{22}\right|^2\right)$$

Characteristic impedance and wave number

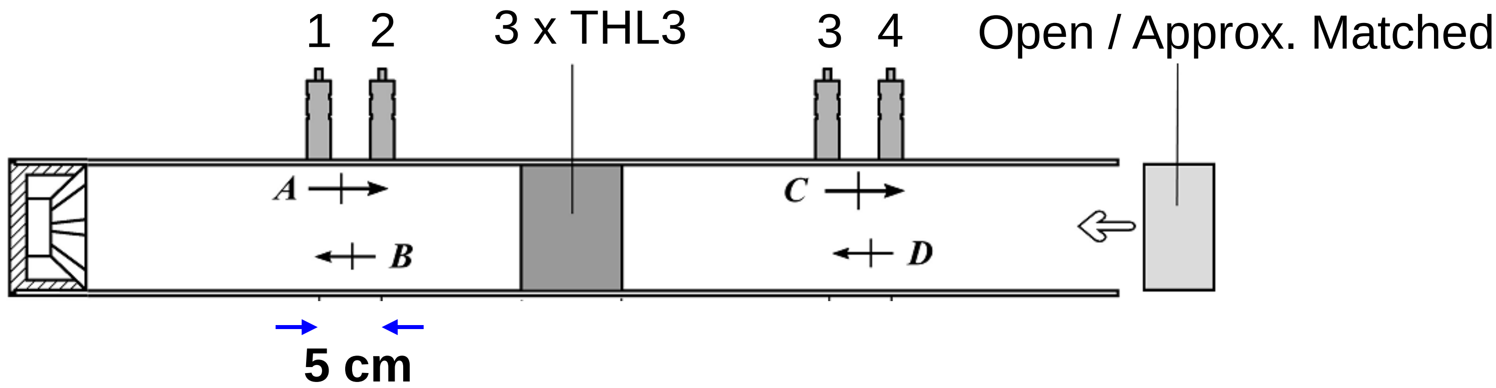
- A homogeneous, isotropic porous material that is (approximately) limp or rigid can be modeled as an equivalent fluid. The complex wave number k_p and the complex characteristic impedance $\rho_p c_p$ can be calculated from:

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \cos k_p d & j\rho_p c_p \sin k_p d \\ j \sin k_p d / \rho_p c_p & \cos k_p d \end{bmatrix}$$

$$k_p = \frac{1}{d} cos^{-1} T_{11}$$

$$\rho_p c_p = \sqrt{\frac{T_{12}}{T_{21}}}$$

Experiments

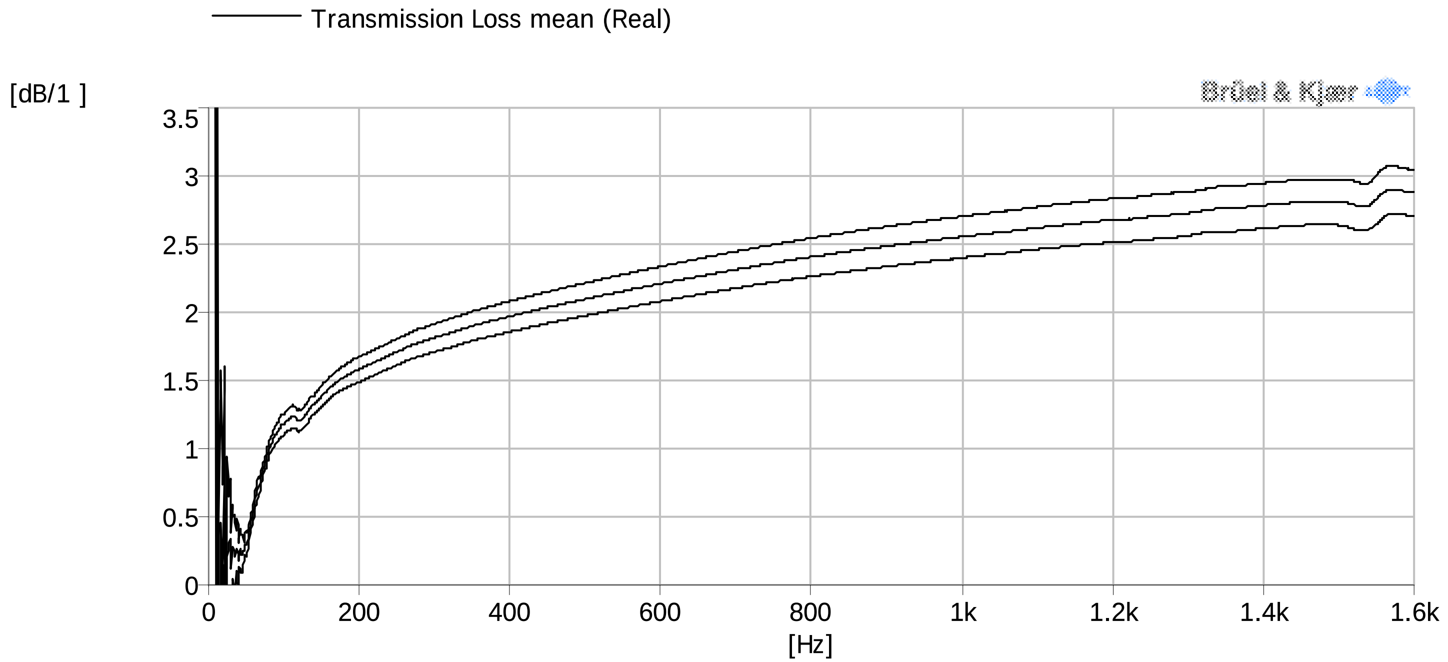


- **10 cm inner tube diameter**
- **50 Hz to 1600 Hz**

Transmission Loss: **Two-load method**

Load conditions measured:

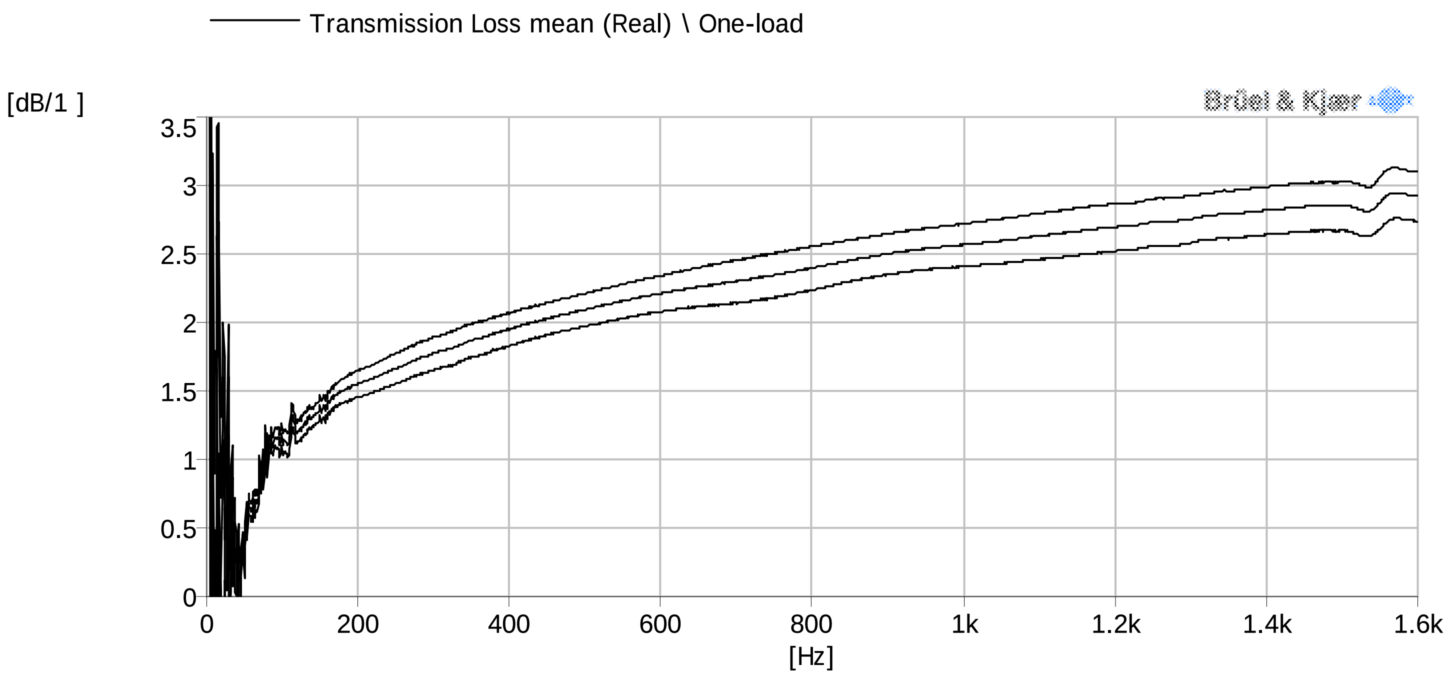
1. Open termination
2. Approximate matched termination



Transmission Loss: One-load method

Load condition measured:

Approximate matched termination



Summary

- The Scattering Matrix formulation for measurement of the normal incidence transmission loss in a plane wave tube has been presented. Neither the two-load nor the one-load implementations require knowledge of the properties of the termination.
- The Scattering Matrix formulation is very intuitive and simple for description of transmission and reflection.
- The Transfer Matrix formulation for measurement of the same quantity has been presented, including the two-load and one-load cases.
- The Transfer Matrix formulation has been implemented, because it is better suited for extraction of material properties, such as complex wave number and complex characteristic impedance for limp or rigid materials.
- Measurement results have been presented.