The Use of a Four-Microphone Standing Wave Tube to Estimate the Anisotropic Properties of Fibrous Noise Control Materials

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The use of a four-microphone standing wave tube to estimate the anisotropic properties of fibrous noise control materials

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Overall Approach

1. Measuring TL and absorption coefficient in square tube

2. Estimating material properties (COMET/Trim)
   - Flow resistivity, Characteristic lengths, tortuosity, porosity, bulk density, Young’s modulus, Poisson’s ratio, loss factor

3. Predicting performance using FEM (COMET/SAFE)
   - Partially filled cases
Introduction & Objectives

• Fibrous material
  – Blown in certain direction: Thus are anisotropic
  – Level of anisotropy depends on density of fiber, fiber’s material and physical dimensions

• Anisotropy
  – Performance differs with respect to wave direction with respect to fiber orientation
  – When is the performance better?
  – Estimation of material properties in different fiber orientations
  – Prediction of performance with the estimated properties
Material Properties I

- Nine properties represent a material

1. Flow resistivity
   - Pressure drop from low velocity flow in material
   - One of the most sensitive properties to performance

2. Viscous characteristic length
   - Depth of viscous boundary layer
   - Sensitive properties
   - Range: $1 \times 10^{-5}$~$9.99 \times 10^{-4}$ m

3. Thermal characteristic length
   - Depth of thermal boundary layer
   - Sensitive properties
   - Range: $1 \times 10^{-5}$~$9.99 \times 10^{-4}$ m
Material properties II

4. Tortuosity
   – Complexity in structure (usually 1.1 for fibrous material)

5. Porosity
   – Ratio of open to closed volume in bulk material
   – Usually 0.99 for fibrous material

6. Bulk density
   – Weight per unit volume

7. Young’s modulus
   – Axial stiffness of bulk material (usually less than 20 kPa for fibrous media)

8. Poisson’s ratio
   – Ratio of change in dimension in two different directions

9. Loss factor
   – Energy dissipation in solid material
# Materials

<table>
<thead>
<tr>
<th></th>
<th>THL</th>
<th>TC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness [cm]</td>
<td>3.7</td>
<td>4</td>
</tr>
<tr>
<td>Mass per unit area [g/m²]</td>
<td>156</td>
<td>376</td>
</tr>
<tr>
<td>Material</td>
<td>Polyester</td>
<td>Polypropylene + Polyester</td>
</tr>
<tr>
<td>Fiber size</td>
<td>Normal</td>
<td>Fine</td>
</tr>
</tbody>
</table>
Measurements of TL – square or round

1. Measuring sound pressure:

\[ P_1 = (A e^{-jkx_1} + B e^{jkx_1}) e^{j\omega t} \]
\[ P_2 = (A e^{-jkx_2} + B e^{jkx_2}) e^{j\omega t} \]

2. Calculate complex amplitude of waves:

\[ A = \frac{j(P_1 e^{jkx_1} - P_2 e^{jkx_1})}{2 \sin k(x_1 - x_2)} \]
\[ B = \frac{j(P_2 e^{-jkx_1} - P_1 e^{-jkx_1})}{2 \sin k(x_1 - x_2)} \]
\[ C = \frac{j(P_1 e^{jkx_3} - P_2 e^{jkx_3})}{2 \sin k(x_3 - x_4)} \]
\[ D = \frac{j(P_2 e^{-jkx_3} - P_1 e^{-jkx_3})}{2 \sin k(x_3 - x_4)} \]

3. Estimate transfer matrix elements:

\[ T_{11} = \frac{P_{|x=d}^{P} |V|_{x=d} + P_{|x=0}^{P} |V|_{x=0}}{P_{|x=d}^{P} |V|_{x=d} + P_{|x=0}^{P} |V|_{x=0}} \]
\[ T_{12} = \frac{P_{|x=d}^{P} |V|_{x=d} - P_{|x=0}^{P} |V|_{x=0}}{P_{|x=d}^{P} |V|_{x=d} + P_{|x=0}^{P} |V|_{x=0}} \]
\[ T_{21} = \frac{V_{|x=d}^{P} |V|_{x=d} - V_{|x=0}^{P} |V|_{x=0}}{P_{|x=d}^{P} |V|_{x=d} + P_{|x=0}^{P} |V|_{x=0}} \]
\[ T_{22} = \frac{P_{|x=d}^{P} |V|_{x=d} + P_{|x=0}^{P} |V|_{x=0}}{P_{|x=d}^{P} |V|_{x=d} + P_{|x=0}^{P} |V|_{x=0}} \]

4. Obtain Transmission loss:

\[ TL = 20 \log_{10} \left( \frac{1}{|T|} \right) \]

Where,

\[ T = \frac{2 e^{jkd}}{T_{11} + (T_{12} / \rho_0 c) + \rho_0 c T_{21} + T_{22}} \]

* Measurements are averaged over 10 sets of samples
TL’s in Two Orientations

• Averaged over ten measurements
  • Normal: using 4 layers
  • Parallel: samples were cut to have same weight as normal cases

THL: Not much difference
TC: Large difference
Measurement of Absorption coefficient

1. Sound pressures

\[ P_1 = \left( A e^{-jkx_1} + B e^{jkx_1} \right) e^{j\omega t} \]
\[ P_2 = \left( A e^{-jkx_2} + B e^{jkx_2} \right) e^{j\omega t} \]

2. Measuring transfer function

\[ H_{21} = \frac{A e^{-jkx_2} + B e^{jkx_2}}{A e^{-jkx_1} + B e^{jkx_1}} \]
\[ H_{21} = \frac{e^{-jkx_2} + \text{Re} e^{jkx_2}}{e^{-jkx_1} + \text{Re} e^{jkx_1}} \]

3. Solve for R

4. Absorption coefficient

\[ \alpha = 1 - |R|^2 \]

* Measurements are averaged over 10 sets of samples
Absorptions in Two Orientations

- Averaged over ten measurements
- Normal: using 4 layers
- Parallel: samples were cut to have same weight as normal cases

\[ \alpha_n \text{ of 4 layers of THL in square tube} \]

\[ \alpha_n \text{ of 4 layers of TC in square tube} \]
Estimation of TC Properties

Using inverse characterization in COMET/Trim

<table>
<thead>
<tr>
<th>Material</th>
<th>TC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber direction</td>
<td>Normal</td>
</tr>
<tr>
<td>Thickness [mm]</td>
<td>40</td>
</tr>
<tr>
<td>Porosity</td>
<td>0.99</td>
</tr>
<tr>
<td>Flow resistivity [Rayls/m]</td>
<td>5500</td>
</tr>
<tr>
<td>Tortuosity</td>
<td>1.1</td>
</tr>
<tr>
<td>Viscous characteristic length [m]</td>
<td>7.90E-05</td>
</tr>
<tr>
<td>Thermal characteristic length [m]</td>
<td>1.52E-04</td>
</tr>
<tr>
<td>Bulk density [kg/m³]</td>
<td>9.4</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Loss factor</td>
<td>0.3</td>
</tr>
</tbody>
</table>
Estimation of THL Properties

Using inverse characterization in COMET/Trim

<table>
<thead>
<tr>
<th>Material</th>
<th>THL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber direction</td>
<td>Normal</td>
</tr>
<tr>
<td>Thickness [mm]</td>
<td>37</td>
</tr>
<tr>
<td>Porosity</td>
<td>0.99</td>
</tr>
<tr>
<td>Flow resistivity [Rayls/m]</td>
<td>2300</td>
</tr>
<tr>
<td>Tortuosity</td>
<td>1.1</td>
</tr>
<tr>
<td>Viscous characteristic length [m]</td>
<td>3.00E-04</td>
</tr>
<tr>
<td>Thermal characteristic length [m]</td>
<td>8.29E-04</td>
</tr>
<tr>
<td>Bulk density [kg/m3]</td>
<td>4.16</td>
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<tr>
<td>Young's modulus [Pa]</td>
<td>3000</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Loss factor</td>
<td>0.3</td>
</tr>
</tbody>
</table>

![Graph 1](image1.png)

![Graph 2](image2.png)
Finite Element Method

- Dimensions are same as experiment setup

\[ \frac{\lambda}{4} = \frac{c}{4 \cdot f} = \frac{340}{4 \cdot 2700} = 0.0315 \quad > 0.01 \]
Partially Filled Cases

- Using single layer of TC and THL

1. Traveling in empty space
2. Traveling inside the material
   - Wave moves in $z$ and $y$ directions
   - Anisotropy affects the results
Test and FEM results

Good agreement between test and prediction with both materials
- THL shows a small difference with different fiber orientations
- TC shows large difference with different fiber orientations
Variation in Lining Thickness

As the sample fills the duct, TL increases.

The prediction and test show good agreement.
Conclusions

• There were large differences in performance and in estimated material properties
  – When the fiber orientation was normal to the wave direction, higher flow resistivity. Thus higher transmission loss and absorption were found
  – Depending on fiber orientation, TC material has much larger difference in performance than THL because of finer fiber size and scrims

• Homogeneous model predicts anisotropic materials when proper material properties are used
  – Material properties were successfully estimated
  – Performances in partially filled tube were accurately predicted using finite element model