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Development of Axisymmetric Finite Elements for Poroelastic Materials

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DEVELOPMENT OF AXISYMMETRIC FINITE ELEMENTS FOR POROELASTIC MATERIALS

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OUTLINE

• Introduction

• Axisymmetrical Foam Finite Elements

• Sound Transmission through Cylindrical & Conical Foam Plug
  - validation with 3-D Cartesian solution
  - comparison with experimental results
  - effect of finite termination impedance

• Sound Attenuation in Foam-Lined Duct
  - comparison with Morse’s solution
  - comparison with experiment results
  - effect of circumferential boundary condition
INTRODUCTION

• Cartesian Finite Elements of Poroelastic Materials
  
  - Normal incidence absorption coefficient
  
  - Normal incidence sound transmission loss
    ( J. P. Coyette and H. Wynendaele, Inter-Noise 95 )
    ( N. Attala and R. Panneton, Inter-Noise 95 )
  
  - Sound transmission through poroelastic wedges

• Sound Propagation along Lined Ducts
  
  - Axisymmetric circular ducts, Locally reacting
  
  - Rectangular ducts, Extended & Locally reacting
Elastic Porous Material Theory based on Biot

3D \( (r, \theta, z) \) \rightarrow 2D \( (r, z) \)

Dynamic Relations

Stress-Strain Relations

Two Wave Equations

Weak Forms

Galerkin’s Approximations

Foam Finite Elements
• Uncoupled System Equations

\[
\begin{bmatrix}
[K_a] \\
K_f
\end{bmatrix}
\begin{bmatrix}
p \\
u_r \\
u_z \\
U_r \\
U_z
\end{bmatrix}
= \begin{cases}
- j\omega \rho_0 2\pi \int_{\Gamma} r \phi_i (n_{ar} v_r + n_{az} v_z) d\Gamma \\
\int_{\Gamma_r} r \phi_i (n_r \sigma_r + n_z \tau_{rz}) d\Gamma \\
\int_{\Gamma_r} r \phi_i (n_r \tau_{rz} + n_z \sigma_z) d\Gamma \\
\int_{\Gamma_r} r \phi_i n_r s d\Gamma \\
\int_{\Gamma_r} r \phi_i n_z s d\Gamma
\end{cases}
\]

need to be coupled using appropriate boundary conditions at the interface of two systems
• Boundary Conditions

- Velocity continuity: \[ v_a = j \omega (1 - h)u + j \omega h U \]

- Force equilibrium (fluid part): \[ h p n_a = s n_f \]

- Force equilibrium (frame part): \[ (1 - h) p n_a = r (\sigma_r n_{fr} + \tau_{zr} n_{fz}) i + r (\tau_{fr} n_{fr} + \sigma_z n_f z) k \]
- Coupled Acoustic-Foam System Equations

\[
\begin{bmatrix}
K_a \\
K_{af}' \\
K_{af}
\end{bmatrix}
\begin{bmatrix}
K_{af} & K_f \\
K_{af} & K_f
\end{bmatrix}
\begin{bmatrix}
p \\
u_r \\
u_z \\
U_r \\
U_z
\end{bmatrix}
= 
\begin{bmatrix}
Q \\
F^1 \\
F^2 \\
F^3 \\
F^4
\end{bmatrix}
\]
SOUND TRANSMISSION THROUGH CYLINDRICAL FOAM PLUG

- Axisymmetric vs. 3-D Cartesian

![Diagram showing axisymmetric and 3-D Cartesian models]

- Axisymmetric model:
  - 49 foam elements
  - 98 air elements
  - 192 nodes
  - Radius, $R = 1.45$ cm

- 3-D Cartesian model:
  - 1456 foam elements
  - 2912 air elements
  - 5082 nodes

* It takes 5500 times longer solution time at each frequency!
• Validation with 3-D Cartesian Solution

![Graph showing sound transmission loss through a cylindrical foam plug comparing an axisymmetric model to a 3-D model. The graph plots frequency in Hz on the x-axis and transmission loss in dB on the y-axis. The axisymmetric model shows a smooth increase in transmission loss, while the 3-D model has a peak at a specific frequency.]
SOUND TRANSMISSION THROUGH FOAM PLUG

• Experimental Setup

Dual Channel Signal Analyzer
B & K Type 2032

Computer

Signal Generator

Signal Amplifier

Microphones

Anechoic Termination

New Sample Holder

Two-Microphone Impedance Measurement Tube
B & K Type 4206

• Anechoic Termination

Normalized Impedance

Normalized Impedance

* Note:

Measured impedance data was phase-corrected when it was applied to the model.
SOUND TRANSMISSION THROUGH CYLINDRICAL FOAM PLUG

- Effect of Circumferential Boundary Conditions

free edge

![Diagram of free edge]

glued edge

![Diagram of glued edge]

- Loss (dB)

- Transmission

- Frequency (Hz)

<table>
<thead>
<tr>
<th>measured</th>
<th>FEM</th>
</tr>
</thead>
</table>

AXISYMMETRIC POROELASTIC FINITE ELEMENTS

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SOUND TRANSMISSION THROUGH CONICAL FOAM PLUG

Free edge

Transmission Loss (dB)

Frequency (Hz)

measured
FEM

AXISYMMETRIC POROELASTIC FINITE ELEMENTS

Seoul National University
SOUND TRANSMISSION THROUGH CYLINDRICAL & CONICAL FOAM PLUG

- Effect Finite Termination Impedance

![Graph showing transmission loss vs frequency for measured and ideal terminations with different terminations: cone, glue, free.](graph.png)
### SOUND ATTENUATION IN FOAM-LINED CIRCULAR DUCT

- Macroscopic Physical Properties of Foams Obtained by Measurement and Optimization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Foam A (polyester)</th>
<th>Foam B (polyether)</th>
<th>Foam C (polyester)</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow resistivity (mks Rayls/m)</td>
<td>13666</td>
<td>30814</td>
<td>46417</td>
<td>measured</td>
</tr>
<tr>
<td>Tortuosity (Structure factor)</td>
<td>3.58</td>
<td>4.28</td>
<td>6.13</td>
<td>optimized</td>
</tr>
<tr>
<td>Porosity</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>assumed</td>
</tr>
<tr>
<td>Bulk density (kg/m$^3$)</td>
<td>32</td>
<td>29</td>
<td>32</td>
<td>measured</td>
</tr>
<tr>
<td>Bulk Young's Modulus (Pa)</td>
<td>30400</td>
<td>25200</td>
<td>85800</td>
<td>optimized</td>
</tr>
<tr>
<td>Loss factor</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>assumed</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>assumed</td>
</tr>
</tbody>
</table>
SOUND ATTENUATION IN FOAM-LINED CIRCULAR DUCT

- Measured and Predicted Absorption Coefficient

Foam A

Foam C

Measured
- Optimized by analytical approach

Optimized by FEM
SOUND ATTENUATION IN FOAM-LINED CIRCULAR DUCT

• Experimental Setup

![Diagram of experimental setup](image)

- Impedance Tube B&K 4206
- Microphone
- Foam lining
- Anechoic Termination
- Amplifier B&K type 2690
- Frequency Analyzer HP35670A

• Open Area Fraction (radius of airway / radius of tube)

<table>
<thead>
<tr>
<th>Open Area Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>0.6</td>
</tr>
<tr>
<td>0.7</td>
</tr>
<tr>
<td>0.8</td>
</tr>
</tbody>
</table>
• Performance of Anechoic Termination

![Graph showing impedance real and imaginary parts for anechoic termination and normalized impedance.](image)

**Axes:**
- X-axis: Frequency (Hz)
- Y-axis: Normalized Impedance

**Lines:**
- Solid line: Anechoic termination
- Dashed line: Impedance real part
- Dashed-dotted line: Impedance imaginary part
Comparison with Experimental Results (Foam A)

- Open area fraction 0.5: FEM, measured
- Open area fraction 0.6: FEM, measured
- Open area fraction 0.7: FEM, measured
- Open area fraction 0.8: FEM, measured

SOUND ATTENUATION IN FOAM-LINED CIRCULAR DUCT

AXISYMMETRIC POROELASTIC FINITE ELEMENTS

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• Comparison with Experimental Results (Foam B)

Open area fraction 0.5: Open area fraction 0.6: Open area fraction 0.7: Open area fraction 0.8:

OPEN AREA FRACTION

0.5

0.6

0.7

0.8
• Comparison with Experimental Results (Foam C)
SOUND ATTENUATION IN FOAM-LINED CIRCULAR DUCT

- Bulk Reacting Vs. Locally Reacting Liner

Foam A

<table>
<thead>
<tr>
<th>Method</th>
<th>0.5, 0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM:</td>
<td></td>
</tr>
<tr>
<td>Morse’s sol:</td>
<td>0.5, 0.8</td>
</tr>
<tr>
<td>Measured:</td>
<td>0.5, 0.8</td>
</tr>
</tbody>
</table>

Foam C

<table>
<thead>
<tr>
<th>Method</th>
<th>0.5, 0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM:</td>
<td></td>
</tr>
<tr>
<td>Morse’s sol:</td>
<td>0.5, 0.8</td>
</tr>
<tr>
<td>Measured:</td>
<td>0.5, 0.8</td>
</tr>
</tbody>
</table>

AXISYMMETRIC POROELASTIC FINITE ELEMENTS

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SOUND ATTENUATION IN FOAM-LINED CIRCULAR DUCT

- Effect of Boundary Condition

Foam A

- FEM (constrained): 0.5, 0.8
- FEM (lubricated): 0.5, 0.8
- Measured: 0.5, 0.8

Foam C

- FEM (constrained): 0.5, 0.8
- FEM (lubricated): 0.5, 0.8
- Measured: 0.5, 0.8

Axisymmetric Poroelastic Finite Elements

Seoul National University
CONCLUSION

- The AXISYMMETRICAL FOAM FINITE ELEMENTS has been formulated and validated for its accuracy and efficiency.

- It has many applications such as sound transmission and attenuation in axisymmetric configurations.

- Constraining the circumference of the foam plugs decreased the transmission loss at high frequencies.

- Finite termination impedance had rippling effect on sound transmission loss at low frequencies.

- Thicker liner does not always guarantee high sound attenuation.

- Locally reacting assumption is valid for some limited cases.