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Development of Axisymmetric Finite Elements for Poroelastic Materials

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DEVELOPMENT OF AXISYMMETRIC FINITE ELEMENTS FOR POROELASTIC MATERIALS

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• Introduction

• Axisymmetrical Foam Finite Elements

• Sound Transmission through Cylindrical & Conical Foam Plug
  - validation with 3-D Cartesian solution
  - comparison with experimental results
  - effect of finite termination impedance

• Sound Attenuation in Foam-Lined Duct
  - comparison with Morse’s solution
  - comparison with experiment results
  - effect of circumferential boundary condition
INTRODUCTION

• Cartesian Finite Elements of Poroelastic Materials
  - Normal incidence absorption coefficient
  - Normal incidence sound transmission loss
    (J. P. Coyette and H. Wynendaele, Inter-Noise 95)
    (N. Attala and R. Panneton, Inter-Noise 95)
  - Sound transmission through poroelastic wedges

• Sound Propagation along Lined Ducts
  - Axisymmetric circular ducts, Locally reacting
  - Rectangular ducts, Extended & Locally reacting
Elastic Porous Material Theory based on Biot

3D \((r, \theta, z)\) \rightarrow 2D \((r, z)\)

Dynamic Relations \rightarrow Stress-Strain Relations

Two Wave Equations

Weak Forms

Galerkin’s Approximations

Foam Finite Elements
• Uncoupled System Equations

\[
\begin{bmatrix}
K_a \\
K_f
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix} p \\ u_r \\ u_z \\ U_r \\ U_z \end{bmatrix}
\end{bmatrix}
= 
\begin{bmatrix}
- j\omega \rho_0 2\pi \int r \Phi_i (n_r v_r + n_z v_z) \, d\Gamma \\
\int_{r} r \Phi_i (n_r \sigma_r + n_z \tau_r) \, d\Gamma \\
\int_{r} r \Phi_i (n_r \tau_r + n_z \sigma_z) \, d\Gamma \\
\int_{z} r \Phi_i n_z s \, d\Gamma \\
\int_{z} r \Phi_i n_z s \, d\Gamma
\end{bmatrix}
\]

need to be coupled using appropriate boundary conditions at the interface of two systems
• Boundary Conditions

- Velocity continuity :
  \[ v_a = j\omega (1-h)u + j\omega hU \]

- Force equilibrium (fluid part) :
  \[ hp_n_a = sn_f \]

- Force equilibrium (frame part) :
  \[ (1-h)p_n_a = r(\sigma_r n_{fr} + \tau_{zr} n_{fz})i + r(\tau_{zr} n_{fr} + \sigma_z n_{fz})k \]
• Coupled Acoustic-Foam System Equations

\[
\begin{bmatrix}
[K_a] & [K_{af}] \\
[K_{af}'] & [K_f]
\end{bmatrix}
\begin{bmatrix}
p \\
u_r \\
u_z \\
U_r \\
U_z
\end{bmatrix}
= \begin{bmatrix}
Q \\
F^1 \\
F^2 \\
F^3 \\
F^4
\end{bmatrix}
\]
SOUND TRANSMISSION THROUGH CYLINDRICAL FOAM PLUG

- Axisymmetric vs. 3-D Cartesian

<table>
<thead>
<tr>
<th></th>
<th>Air I</th>
<th>Foam</th>
<th>Air II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>3.3 cm</td>
<td>2.8 cm</td>
<td>3.3 cm</td>
</tr>
<tr>
<td>FOAM</td>
<td>49 elements</td>
<td></td>
<td>1456 elements</td>
</tr>
<tr>
<td>AIR I</td>
<td>98 elements</td>
<td></td>
<td>2912 elements</td>
</tr>
<tr>
<td>NODES</td>
<td>192</td>
<td></td>
<td>5082</td>
</tr>
</tbody>
</table>

* R = 1.45 cm

* It takes 5500 times longer solution time at each frequency!
SOUND TRANSMISSION THROUGH CYLINDRICAL FOAM PLUG

- Validation with 3-D Cartesian Solution

![Graph showing comparison between Axisymmetric Model and 3-D Model]

- Frequency (Hz)
- Transmission Loss (dB)

Axisymmetric Model
3-D Model
**Experimental Setup**

- Dual Channel Signal Analyzer
  - B & K Type 2032

- Computer

- Signal Amplifier

- CH A
- CH B

- Signal Generator

- Microphones

- Anechoic Termination

- New Sample Holder

- Two-Microphone Impedance Measurement Tube
  - B & K Type 4206

**Anechoic Termination**

- Measured impedance data was phase-corrected when it was applied to the model.

*Note:* Measured impedance data was phase-corrected when it was applied to the model.
SOUND TRANSMISSION THROUGH CYLINDRICAL FOAM PLUG

- Effect of Circumferential Boundary Conditions

measured
FEM

Free edge

Glued edge

Transmission Loss (dB)

Frequency (Hz)

Frequency (Hz)
SOUND TRANSMISSION THROUGH CONICAL FOAM PLUG

free edge

Transmission Loss (dB)

Frequency (Hz)

measured
FEM

AXISYMMETRIC POROELASTIC FINITE ELEMENTS

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• Effect Finite Termination Impedance

![Graph showing transmission loss through cylindrical and conical foam plugs.](image_url)
SOUND ATTENUATION IN FOAM-LINED CIRCULAR DUCT

- Macroscopic Physical Properties of Foams Obtained by Measurement and Optimization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Foam Type</th>
<th>Foam A (polyester)</th>
<th>Foam B (polyether)</th>
<th>Foam C (polyester)</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow resistivity (mks Rayls/m)</td>
<td>Foam Type</td>
<td>Foam A (polyester)</td>
<td>Foam B (polyether)</td>
<td>Foam C (polyester)</td>
<td>measured</td>
</tr>
<tr>
<td>Tortuosity (Structure factor)</td>
<td>Foam Type</td>
<td>Foam A (polyester)</td>
<td>Foam B (polyether)</td>
<td>Foam C (polyester)</td>
<td>optimized</td>
</tr>
<tr>
<td>Porosity</td>
<td>Foam Type</td>
<td>Foam A (polyester)</td>
<td>Foam B (polyether)</td>
<td>Foam C (polyester)</td>
<td>assumed</td>
</tr>
<tr>
<td>Bulk density (kg/m^3)</td>
<td>Foam Type</td>
<td>Foam A (polyester)</td>
<td>Foam B (polyether)</td>
<td>Foam C (polyester)</td>
<td>measured</td>
</tr>
<tr>
<td>Bulk Young’s Modulus (Pa)</td>
<td>Foam Type</td>
<td>Foam A (polyester)</td>
<td>Foam B (polyether)</td>
<td>Foam C (polyester)</td>
<td>optimized</td>
</tr>
<tr>
<td>Loss factor</td>
<td>Foam Type</td>
<td>Foam A (polyester)</td>
<td>Foam B (polyether)</td>
<td>Foam C (polyester)</td>
<td>assumed</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>Foam Type</td>
<td>Foam A (polyester)</td>
<td>Foam B (polyether)</td>
<td>Foam C (polyester)</td>
<td>assumed</td>
</tr>
</tbody>
</table>
SOUND ATTENUATION IN FOAM-LINED CIRCULAR DUCT

- Measured and Predicted Absorption Coefficient

Foam A

Foam C

**AXISYMMETRIC POROELASTIC FINITE ELEMENTS**

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SOUND ATTENUATION IN FOAM-LINED CIRCULAR DUCT

• Experimental Setup

![Diagram of experimental setup]

- Impedance Tube: B&K 4206
- Microphone
- Foam lining
- Anechoic Termination 1 m 0.6 m
- Amplifier
- Frequency Analyzer: HP35670A
- Conditioning Amplifier: B&K type 2690

• Open Area Fraction (radius of airway / radius of tube)

![Illustration of open area fractions]

0.5 0.6 0.7 0.8
• Performance of Anechoic Termination
SOUND ATTENUATION IN FOAM-LINED CIRCULAR DUCT

- Comparison with Experimental Results (Foam A)

![Graph showing sound attenuation vs. frequency for different open area fractions with FEM and measured data.]

OPEN AREA FRACTION

- 0.5
- 0.6
- 0.7
- 0.8
Comparison with Experimental Results (Foam B)

- Open area fraction 0.5:
- Open area fraction 0.6:
- Open area fraction 0.7:
- Open area fraction 0.8:

![Graph showing sound attenuation vs. frequency for different open area fractions](graph.png)

**Open Area Fraction**
- 0.5
- 0.6
- 0.7
- 0.8
• Comparison with Experimental Results (Foam C)
SOUND ATTENUATION IN FOAM-LINED CIRCULAR DUCT

- Bulk Reacting Vs. Locally Reacting Liner

![Graphs showing sound attenuation in foam-lined circular duct for different foams and frequency ranges.](attachment:graph.png)

<table>
<thead>
<tr>
<th>Foam</th>
<th>FEM:</th>
<th>Morse's sol:</th>
<th>Measured:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foam A</td>
<td>0.5, 0.8</td>
<td>0.5, 0.8</td>
<td>0.5, 0.8</td>
</tr>
<tr>
<td>Foam C</td>
<td>0.5, 0.8</td>
<td>0.5, 0.8</td>
<td>0.5, 0.8</td>
</tr>
</tbody>
</table>

AXISYMMETRIC POROELASTIC FINITE ELEMENTS

Seoul National University
SOUND ATTENUATION IN FOAM-LINED CIRCULAR DUCT

- Effect of Boundary Condition

Foam A

FEM (constrained): 0.5, 0.8
FEM (lubricated): 0.5, 0.8
Measured: 0.5, 0.8

Foam C

FEM (constrained): 0.5, 0.8
FEM (lubricated): 0.5, 0.8
Measured: 0.5, 0.8

Frequency (Hz)

Attenuation (dB/m)
CONCLUSION

• The AXISYMMETRICAL FOAM FINITE ELEMENTS has been formulated and validated for its accuracy and efficiency.

• It has many applications such as sound transmission and attenuation in axisymmetric configurations.

• Constraining the circumference of the foam plugs decreased the transmission loss at high frequencies.

• Finite termination impedance had rippling effect on sound transmission loss at low frequencies.

• Thicker liner does not always guarantee high sound attenuation.

• Locally reacting assumption is valid for some limited cases.