Patch Near-field Acoustical Holography in Cylindrical Geometry

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Patch Near-field Acoustical Holography In Cylindrical Geometry

Moohyung Lee and J. Stuart Bolton

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What is “Patch” NAH?

< Conventional NAH >
The measurement aperture should be extended to the region in which the sound level drops to a sufficiently low level: complete scan of the sound field.

< Patch NAH >
Measurements are made over a limited region of interest: partial scan of the sound field to visualize limited region of source.

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Constraints of DFT-based NAH

- Large measurement aperture
- Difficult to implement NAH for large scale-structures
- Suffers from finite hologram effect
  1. Wrap-around error
     - Periodic replication of the data in the spatial domain
       : easily dealt with by zero padding
  2. Windowing effect
     - A sharp transition at the edge of measurement aperture introduces high wave number noise components
       : degrades reconstruction results when projecting towards a source due to the ill-posed nature of problem
Recent Approaches to Patch NAH (1)

- Statistically optimized NAH (SONAH)
  - A plane-to-plane propagation is performed in the spatial domain by two dimensional convolution with a propagation kernel
  - *J. Hald*

- Helmholtz equation least-squares (HELS)
  - An assumed solution is expressed by an orthonormal expansion of spheroidal functions that satisfy the Helmholtz equation
  - Solve the Helmholtz equation directly and minimize errors by the least-squares method
  - *S. F. Wu*
Recent Approaches to Patch NAH (2)

- Method of superposition
  - The sound field is approximated by the superposition of fields produced by a number of sources
  - Can be used to either enlarge the finite measurement aperture (extrapolation) or fill the gap in the measurement aperture (interpolation)
  - A. Sarkissian

- Iterative patch NAH
  - The sound field is extended into the region outside the measurement aperture by successive smoothing procedures
  - Available for both DFT and SVD-based NAH
  - K. Saijyou and E. G. Williams
Iterative Patch NAH Algorithm

**Step 1**
Extension of the measurement pressure by zero padding
\[ p^{(0)} = \begin{cases} p_m & (\tilde{r} \in \Omega_m) \\ 0 & (\tilde{r} \in \Omega_0) \end{cases} j = 0 \]

**Step 2**
\[ k \text{-space spectrum of pressure} \]
\[ \bar{U}^H p^{(j)} \]

**Step 3**
Band-limited \( k \)-space spectrum of pressure
\[ F^{(j)} \bar{U}^H p^{(j)} \]

**Step 4**
Smoothed pressure
\[ p^{(j+1)} = \bar{U} F^{(j)} \bar{U}^H p^{(j)} \]

**Step 5**
Replace the patch region with the measured pressure
\[ p^{(j+1)} = \begin{cases} p_m & (\tilde{r} \in \Omega_m) \\ p^{(j)} & (\tilde{r} \in \Omega_0) \end{cases} \]

Convergent? Yes/No

**END**
Extrapolated pressure
\[ p^{(j+1)} \]
Extension of the Measurement Aperture

Ωₘ: measurement aperture
Ω₀: zero-padding region

Since the sound field is periodic in the circumferential direction, the number of points after extension is determined by the angular spatial sampling interval: i.e., \( N_φ = 360 / \Delta φ \)
Smoothing Procedure

- A sharp transition at the edge of the aperture can be smoothed by successive iterations based on the use of an appropriate regularization procedure.

- A modified Tikhonov regularization used in conjunction with a parameter selection technique:

  \[ F^{\alpha^{(j)}} = \text{diag} \left( \frac{\lambda_i^2}{\lambda_i^2 + \alpha^{(j)} \left( \frac{\alpha^{(j)}}{\lambda_i^2 + \lambda_i^2} \right)^2} \right) \]

- Parameter selection techniques:
  - Mozorov discrepancy principle
  - Generalized cross validation
  - L-curve
Convergence Criterion

- Smoothing processes repeat until the convergence condition is satisfied
- When noise variance is known (MDP)
  \[ \left\| \tilde{p}^{(j)} - \tilde{p}^{(j-1)} \right\| < \epsilon \sigma^{(j)} \sqrt{M} \]
  Noise included in the hologram pressure
- When noise variance is not known (GCV, L-curve)
  \[ \left\| \alpha^{(j)} - \alpha^{(j-1)} \right\| < \epsilon \alpha^{(0)} \]
- \( \epsilon \) is an ad-hoc factor having a “small” value
Numerical Simulation

**Double dipole model**

- hologram surface
- \( r = 14.15 \text{ cm} \)
- scan 34
- scan 1
- rigid surface
- 2 cm
- 5 cm
- 25 cm
- Patch 1, Patch 2, Patch 3

**Hologram pressure and patch selection at 1kHz**

- Axial [cm]
- Circumferential [deg]
- Patch 1, Patch 2, Patch 3

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Extended Hologram Pressure at $r = 14.15$ cm

- **< Patch 1 >**
  - 0 iteration
  - 100 iterations
  - 800 iterations
  - 8000 iterations

- **< Patch 2 >**
  - 0 iteration
  - 100 iterations
  - 300 iterations
  - 8000 iterations

- **< Patch 3 >**
  - 0 iteration
  - 100 iterations
  - 800 iterations
  - 8000 iterations

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Reconstructed Pressure at \( r = 9 \text{ cm} \)

- Reconstruction results on the surface directly under the patches show good agreement with the actual pressure.

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Reconstruction Error

\[ \text{error} (\%) = \frac{\| \mathbf{p}_{\text{reconstructed}} - \mathbf{p}_{\text{exact}} \|_2}{\| \mathbf{p}_{\text{exact}} \|_2} \times 100 \]

- \( \text{error}_1 \) : region larger than the patch
- \( \text{error}_2 \) : region directly under the patch
- \( \text{error}_3 \) : region smaller than the patch

---

**< Patch 1 >**

**< Patch 2 >**

**< Patch 3 >**
Experimental Results

Refrigeration compressor

- The number of references: 4
- The number of measurement points: 32 (circumferential) by 34 (axial)

Hologram pressure and patch selection

- 882 Hz
- 1293 Hz
Extended Hologram Pressure at r = 14.15 cm

< 882 Hz >

< 1293 Hz >
Reconstructed Surface Normal Velocity

< 882 Hz >

< Full scan >

< Patch >

< 1293 Hz >

< Full scan >

< Patch >
Reconstruction Error

\[ \varepsilon = 0.092 \]
\[ \varepsilon = 0.05 \]
\[ \varepsilon = 0.03 \]
\[ \varepsilon = 0.01 \]
\[ \varepsilon = 0.005 \]
\[ \varepsilon = 0.002 \]

\[< 882 \text{ Hz}>\]

\[< 1293 \text{ Hz}>\]
Summary and Conclusion

• The extrapolation procedure allows accurate reconstruction results when NAH measurement is performed in the finite region
• Useful for a large-scale source implementation since a complete scan of the sound field is not necessary
• When implemented in the cylindrical geometry, the degree of extension depends on the angular spatial sampling interval
• Practical suggestions
  ➢ Use a larger aperture than the region of interest since the reconstruction results in the central region of the aperture are less corrupted by errors
  ➢ Include pressure peaks in the aperture since the error drops relatively quickly to a low value