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Patch Near-field Acoustical Holography in Cylindrical Geometry

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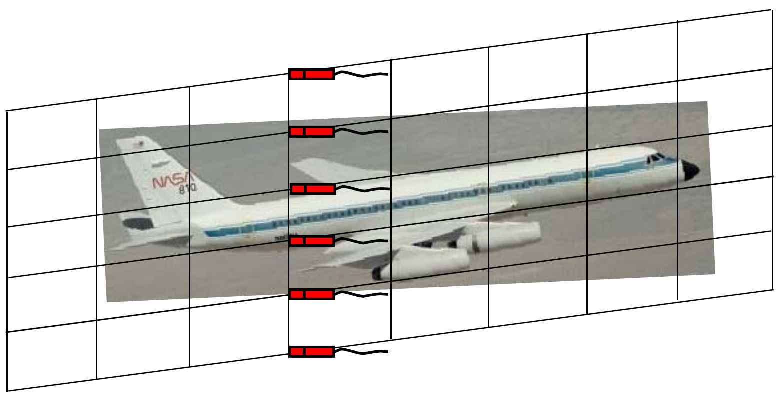
April 23 2004

What is "Patch" NAH ?

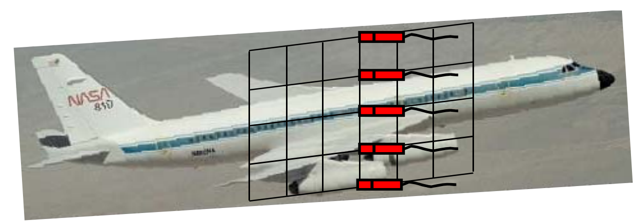


< Conventional NAH >

The measurement aperture should be extended to the region in which the sound level drops to a sufficiently low level: complete scan of the sound field

< Patch NAH >

Measurements are made over a limited region of interest: partial scan of the sound field to visualize limited region of source



Constraints of DFT-based NAH

- Large measurement aperture
- Difficult to implement NAH for large scale-structures
- Suffers from finite hologram effect

  1. Wrap-around error
     - Periodic replication of the data in the spatial domain

       : easily dealt with by zero padding
  2. Windowing effect
     - A sharp transition at the edge of measurement aperture introduces high wave number noise components

       : degrades reconstruction results when projecting towards a source due to the ill-posed nature of problem

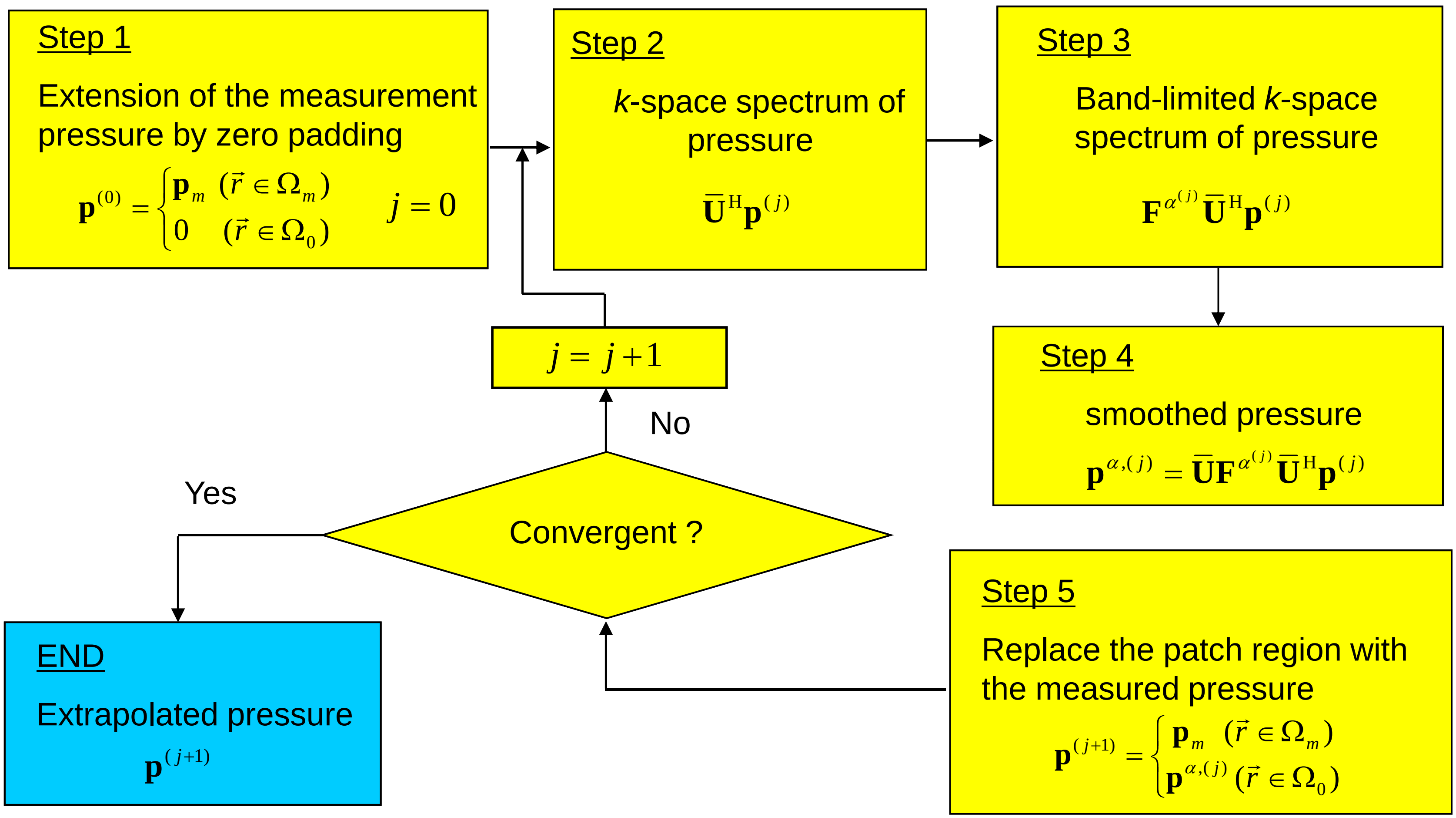
Recent Approaches to Patch NAH (1)

- Statistically optimized NAH (SONAH)
  - A plane-to-plane propagation is performed in the spatial domain by two dimensional convolution with a propagation kernel
  - *J. Hald*
- Helmholtz equation least-squares (HELS)
  - An assumed solution is expressed by an orthonormal expansion of spheroidal functions that satisfy the Helmholtz equation
  - Solve the Helmholtz equation directly and minimize errors by the least-squares method
  - *S. F. Wu*

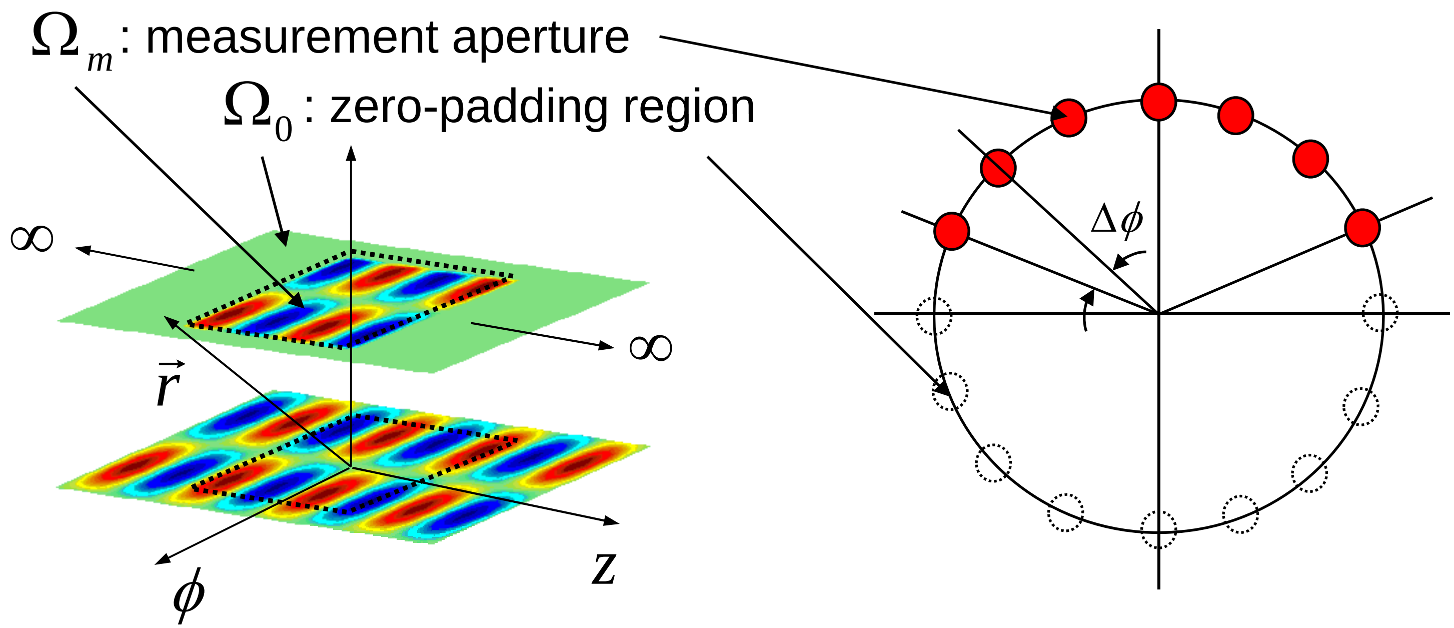
Recent Approaches to Patch NAH (2)

- Method of superposition
  - The sound field is approximated by the superposition of fields produced by a number of sources
  - Can be used to either enlarge the finite measurement aperture (extrapolation) or fill the gap in the measurement aperture (interpolation)
  - *A. Sarkissian*
- Iterative patch NAH
  - The sound field is extended into the region outside the measurement aperture by successive smoothing procedures
  - Available for both DFT and SVD-based NAH
  - *K. Saijyou and E. G. Williams*

Iterative Patch NAH Algorithm



Extension of the Measurement Aperture



Since the sound field is periodic in the circumferential direction, the number of points after extension is determined by the angular spatial sampling interval: i.e., $N_\phi = 360 / \Delta\phi$

Smoothing Procedure

- A sharp transition at the edge of the aperture can be smoothed by successive iterations based on the use of an appropriate regularization procedure
- A modified Tikhonov regularization used in conjunction with a parameter selection technique

$$\mathbf{F}^{\alpha^{(j)}} = \mathrm{diag}\left(\cdots \left|\lambda_i\right|^2 \Big/ \left(\left|\lambda_i\right|^2 + \alpha^{(j)} \left(\frac{\alpha^{(j)}}{\alpha^{(j)} + \left|\lambda_i\right|^2} \right)^2 \right) \cdots \right)$$

- Parameter selection techniques
  - Mozorov discrepancy principle
  - Generalized cross validation
  - L-curve

Convergence Criterion

- Smoothing processes repeat until the convergence condition is satisfied
- When noise variance is known (MDP)

$$\left\| \tilde{\mathbf{p}}^{(j)} - \tilde{\mathbf{p}}^{(j-1)} \right\| < \varepsilon \underbrace{\sigma^{(j)} \sqrt{M}}$$

Noise included in the hologram pressure

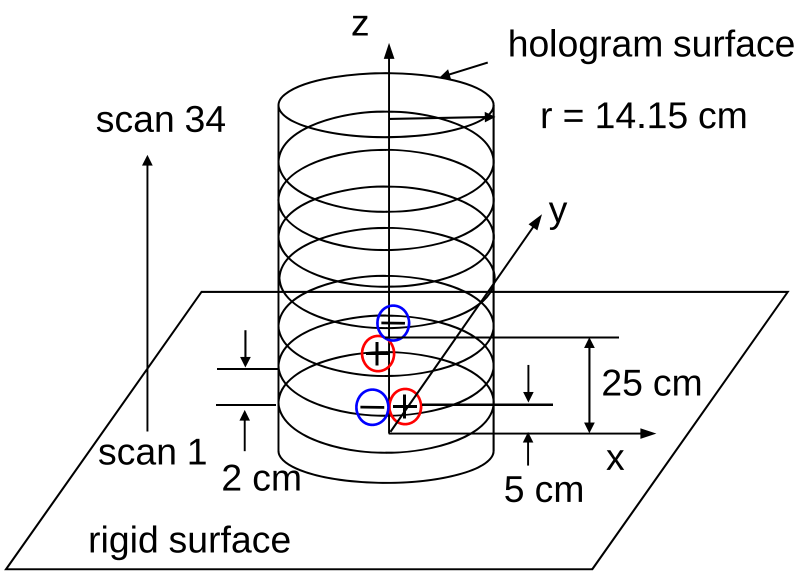
- When noise variance is not known (GCV, L-curve)

$$\left\| \alpha^{(j)} - \alpha^{(j-1)} \right\| < \varepsilon \, \alpha^{(0)}$$

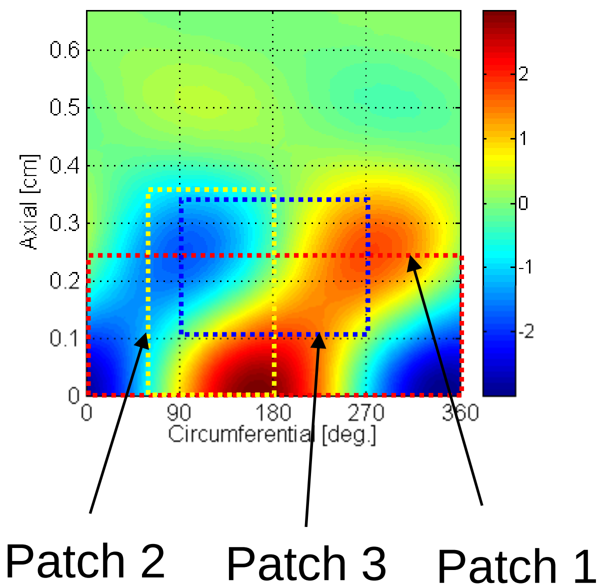
- ε is an ad-hoc factor having a “small” value

Numerical Simulation

Double dipole model

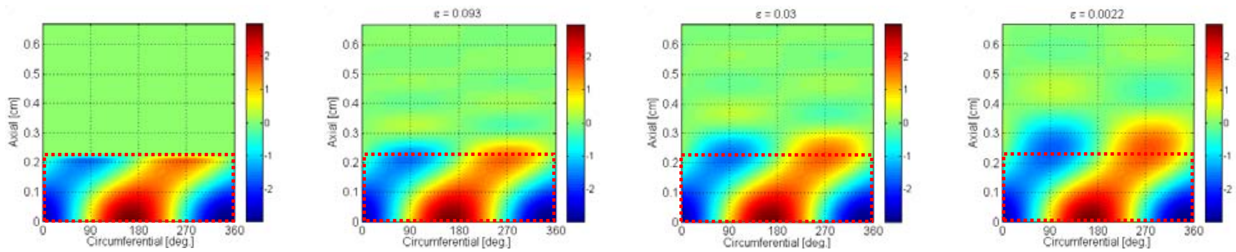


Hologram pressure and patch selection at 1kHz



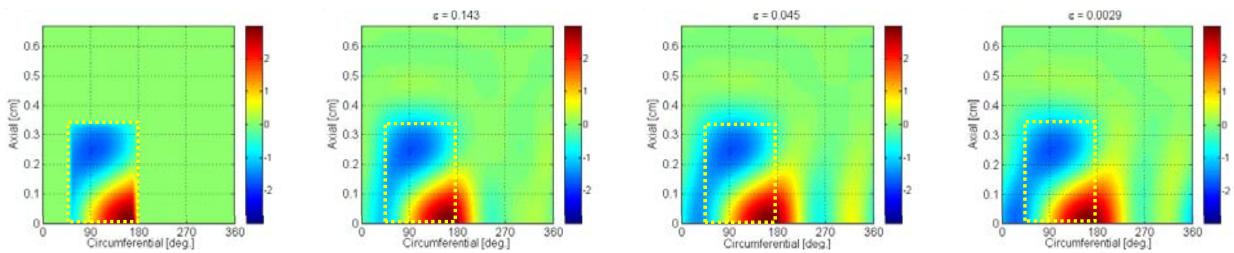
Extended Hologram Pressure at r = 14.15 cm

< Patch 1 >



0 iteration 100 iterations 800 iterations 8000 iterations

< Patch 2 >



0 iteration 100 iterations 300 iterations 8000 iterations

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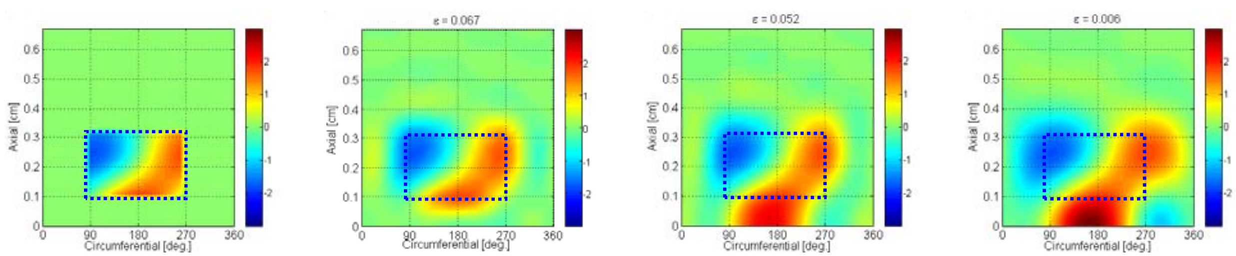
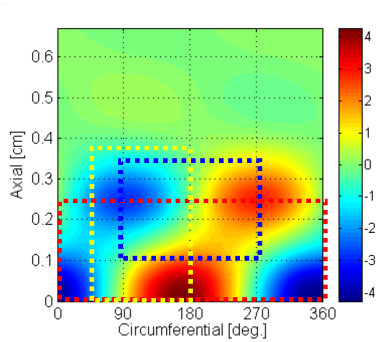


0 iteration 100 iterations 800 iterations 8000 iterations

Reconstructed Pressure at r = 9 cm



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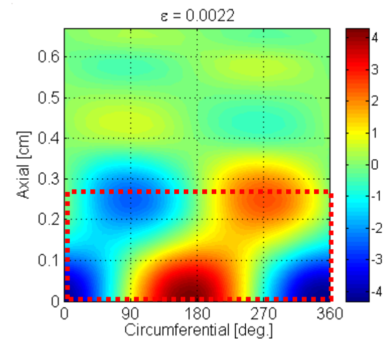

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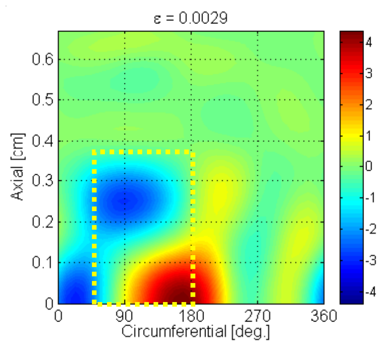

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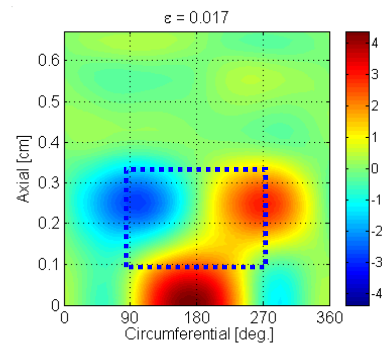


< Patch 3 >

- Reconstruction results on the surface directly under the patches show good agreement with the actual pressure

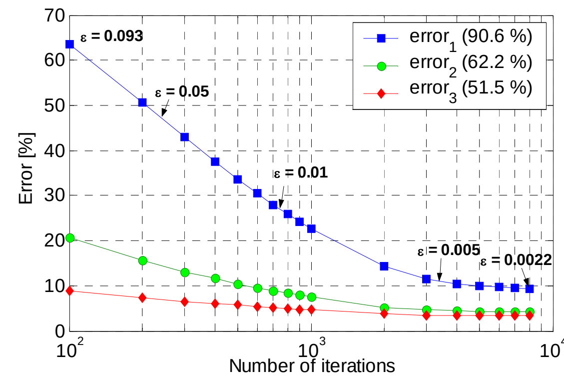
Reconstruction Error

$$error\,(\%) = \frac{\left\| \mathbf{p}_{(reconstructed)} - \mathbf{p}_{(exact)} \right\|_2}{\left\| \mathbf{p}_{(exact)} \right\|_2} \times 100$$

$error_1$: region larger than the patch

$error_2$: region directly under the patch

$error_3$: region smaller than the patch



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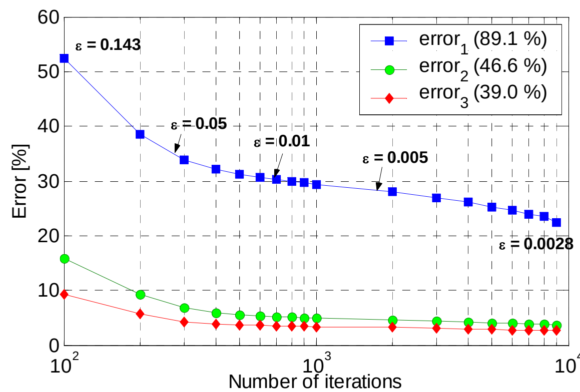

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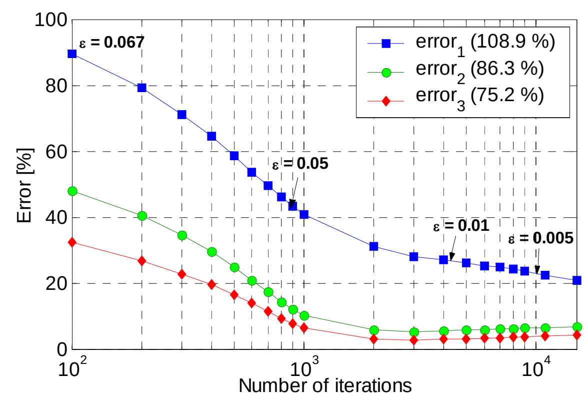
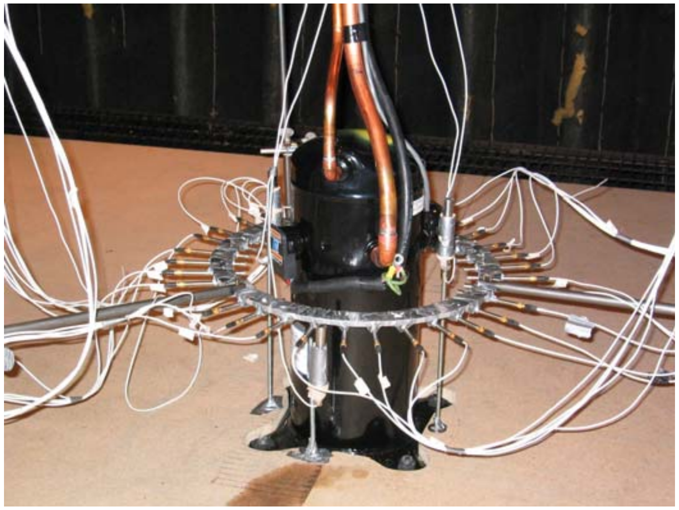


< Patch 3 >

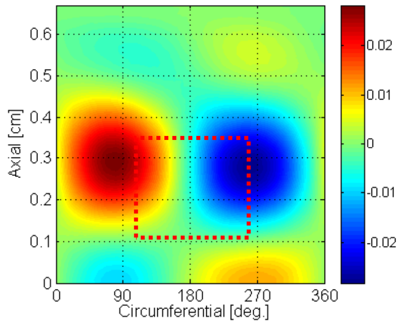
Experimental Results

Refrigeration compressor

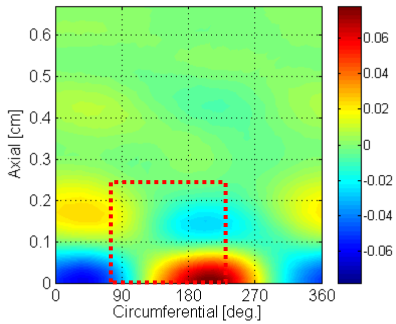


- The number of references: 4
- The number of measurement points: 32 (circumferential) by 34 (axial)

Hologram pressure and patch selection



882 Hz

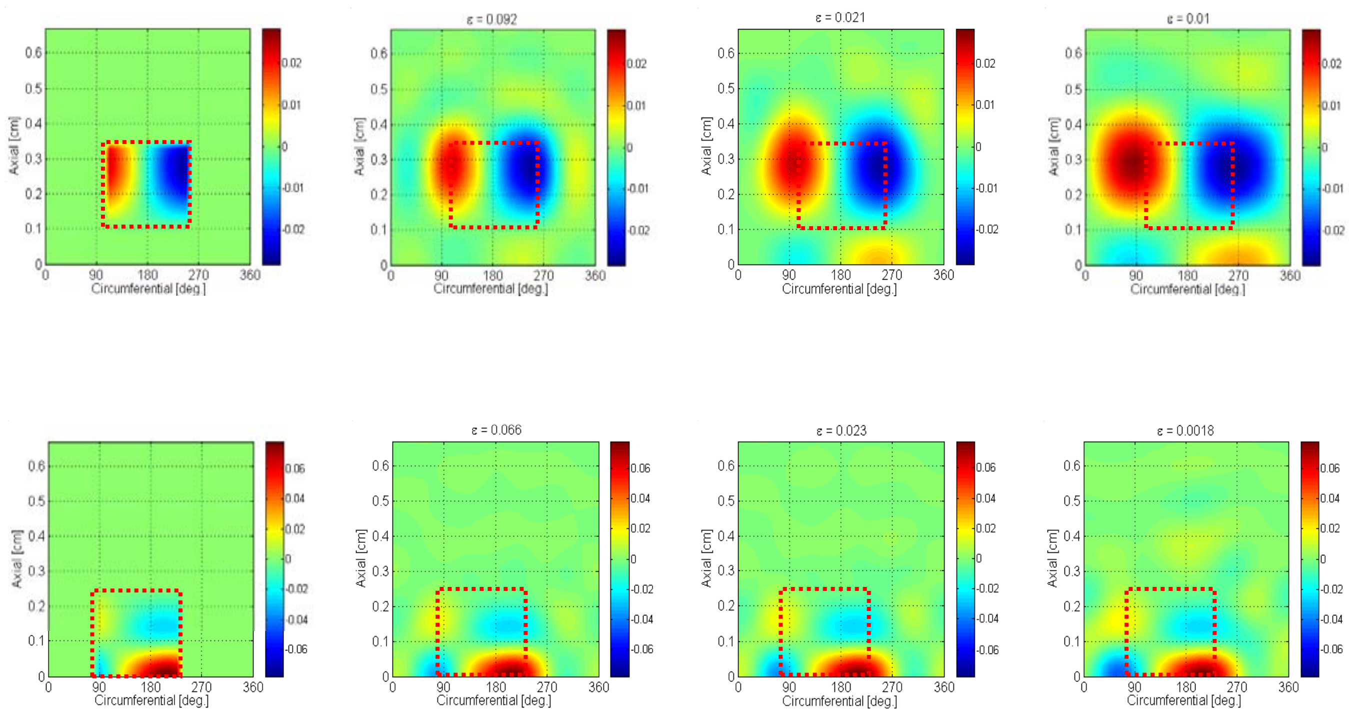


1293 Hz

Extended Hologram Pressure at r = 14.15 cm

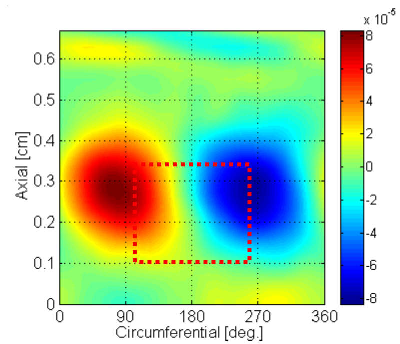
< 882 Hz >

< 1293 Hz >

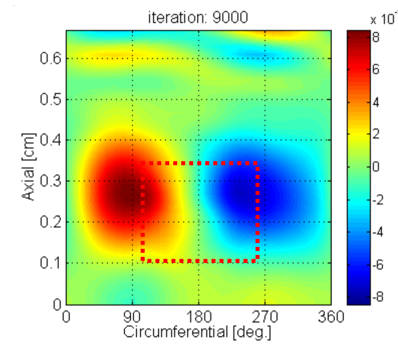


Reconstructed Surface Normal Velocity

< 882 Hz >



< Full scan >



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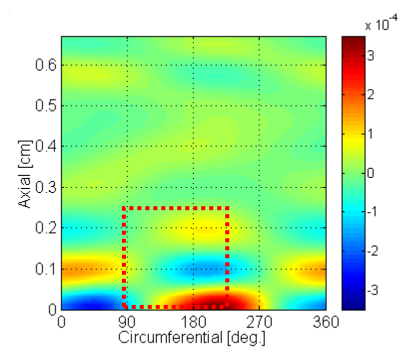

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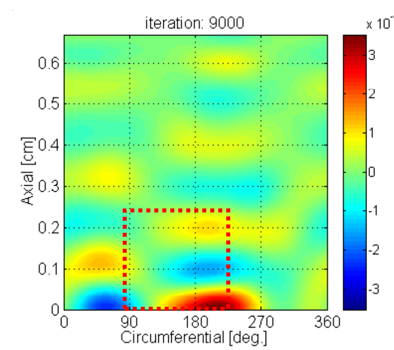
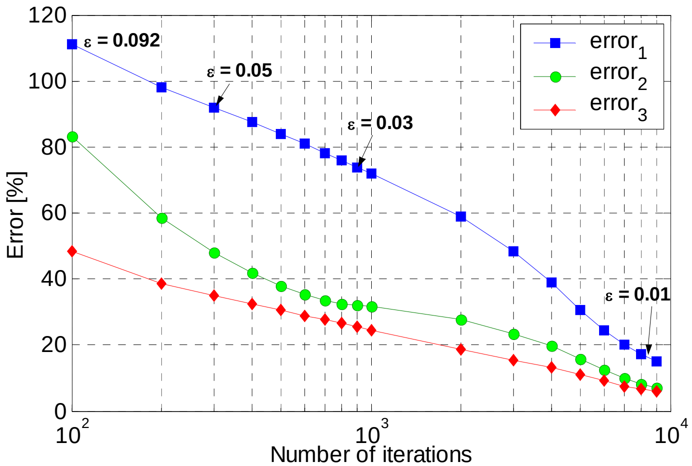


< Patch >

Reconstruction Error



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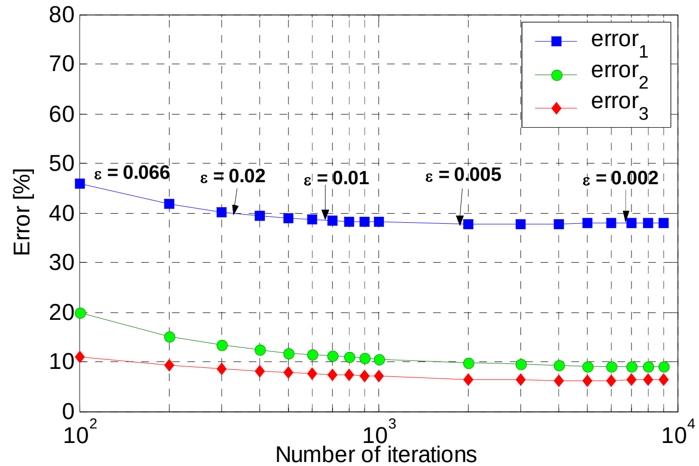

< 1293 Hz >

Summary and Conclusion

- The extrapolation procedure allows accurate reconstruction results when NAH measurement is performed in the finite region
- Useful for a large-scale source implementation since a complete scan of the sound field is not necessary
- When implemented in the cylindrical geometry, the degree of extension depends on the angular spatial sampling interval
- Practical suggestions
  - Use a larger aperture than the region of interest since the reconstruction results in the central region of the aperture are less corrupted by errors
  - Include pressure peaks in the aperture since the error drops relatively quickly to a low value