Poro-elastic material characterization methods by using standing wave tubes: history and current issues related to Biot parameter estimation

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Poro-Elastic Material Characterisation Methods by using Standing Wave Tubes: History and Current Issues Related to Biot Parameter Estimation

J. Stuart Bolton
Ray W. Herrick Laboratories, Purdue University, USA

Keynote Address: SAPEM, Bradford, UK, December 2008
Introduction

- Brief history of standing wave tube
- Four-microphone standing wave tube
- Estimation of Biot parameters based on acoustical measurements
Standing Wave Tube

- Standing wave method for measuring normal incidence absorption coefficients more than 100 years old

- Method is credited by a number of authors to J. Tuma (1902)

- Subsequent experiments conducted by Weisbach (1910) and Taylor (1913)
A DIRECT METHOD OF FINDING THE VALUE OF MATERIALS AS SOUND ABSORBERS.

By Hawley O. Taylor.

THE VALUE OF MATERIALS AS SOUND ABSORBERS.

upper limit of audibility, showing that the apparatus is applicable to any probable range of pitch.

The different parts of the apparatus used for making observations for sound absorption have already been described. The parts were assembled as shown in Fig. 14. The flue, $F$, moved along a graduated track, $T$. The one-half-inch glass tube, $IG$, from the suspended disc passed through the tone screen at $R$ and projected into the flue. The intensity of sound at the end, $G$, of this tube causes the air in the tube to vibrate and this in turn produces a deflection of the disc.

In taking observations, the flue, $F$, with the organ pipe, $P$, attached to it was moved along the track, $T$, in steps one centimeter long. Observations for intensity were made at each step, and at the region of maximum and minimum intensities for shorter steps, and thus the maximum and minimum intensities, $m$ and $n$ of the formula, were found. Applying these values in the formula, the coefficient of absorption of sound for the material, $S$, closing the end, $E$, of the flue, $F$, was calculated as already shown by an example.

Following is the coefficient of absorption of a few materials computed by using the approximate formula:

<table>
<thead>
<tr>
<th>Material</th>
<th>$m$</th>
<th>$n$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smyrna Rug.</td>
<td>6.55</td>
<td>.04</td>
<td>.26</td>
</tr>
<tr>
<td>Brussels Carpet.</td>
<td>7.30</td>
<td>.03</td>
<td>.23</td>
</tr>
<tr>
<td>Hair felt (1-inch thick)</td>
<td>24.70</td>
<td>.78</td>
<td>.51</td>
</tr>
<tr>
<td>Cellulose (1/2-inch thick)</td>
<td>57.50</td>
<td>.22</td>
<td>.25</td>
</tr>
<tr>
<td>Asbestos roll fire felt (1/2-inch thick)</td>
<td>54.50</td>
<td>.30</td>
<td>.26</td>
</tr>
<tr>
<td>Compressed cork (1/4-inch thick)</td>
<td>27.00</td>
<td>.25</td>
<td>.32</td>
</tr>
</tbody>
</table>

The last four materials were kindly furnished by the Johns-Manville Co. of New York. The values of the absorbing power given here for the first three materials check fairly well with values obtained by the reverberation method.

The absorbing power of material is increased by increasing the space
XXI.—ON THE STATIONARY-WAVE METHOD OF MEASURING SOUND-ABSORPTION AT NORMAL INCIDENCE.*

By E. T. PARIS, D.Sc., F.Inst.P.

Received January 12, 1927.

ABSTRACT.

A description is given of apparatus employed for measuring coefficients of sound-absorption by the stationary-wave method.* The apparatus differs from that used by earlier workers in the use of (1) a small tuned hot-wire microphone for determining relative pressure-amplitudes in the sound-waves; (2) the employment of a steady valve-driven source of sound with arrangements for maintaining the strength at a constant value; (3) the screening of source and experimental pipe from disturbances due to the movements of the observer. By the employment of a certain procedure the relation between the response of the microphone and the amplitude of the pressure-variation in the sound-wave is eliminated. Some examples of the employment of the apparatus for determining the coefficients of absorption at normal incidence of acoustic plasters and hair-felt are given.

---

FIG. 2.—ARRANGEMENT OF EXPERIMENTAL PIPE, SOURCE AND SPECIMEN (NOT TO SCALE).

A. Scale Read Here.
B. Sound-Chamber
C. Loud-Speaker.
D. Slider.
E. Wall of Laboratory.
F. Felt-Lined Box Enclosing Pipe.
G. Earthenware Pipe.
H. Microphone.
I. Specimen.
J. Thin Washer of Rubber.
K. Backing to Specimen (3 layers of 5-ply).
L. Packing (1-Inch Wood).
M. Cover (held by Bolts and Wing Nuts.)
N. Felt.
O. Steel Supporting Legs set in Concrete.
P. Ground-Level.
Q. Concrete.
An Acoustic Transmission Line for Impedance Measurement

WILLIAM M. HALL
Department of Electrical Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received February 17, 1939)

A method and apparatus are described for rapid and accurate measurement of acoustic impedance in terms of the characteristic acoustic impedance of a tube. The measurement consists of the simple determination of the location and relative magnitude of the maximum and minimum sound pressures along the tube. The impedance of the termination of the tube can then be read directly from a slightly modified hyperbolic tangent chart. Two methods are given for measuring the impedance of acoustic elements with cross sections different from that of the measuring tube.

**Fig. 1.** Acoustic transmission line.

**Fig. 2.** Design of acoustic transmission line.
Precision Measurement of Acoustic Impedance*

LEO L. BERANEK

Crafoft Laboratory, Harvard University, Cambridge, Massachusetts
(Received June 1, 1940)

Fig. 1. Diagram of experimental apparatus.

J. Acoust. Soc. Am. 12, 3-13, Jul 1940
<table>
<thead>
<tr>
<th>No.</th>
<th>Method</th>
<th>Authority</th>
<th>Source</th>
<th>Frequency</th>
<th>Length</th>
<th>Pressure Detector</th>
<th>Measure</th>
<th>Criticism</th>
</tr>
</thead>
</table>
| 1   | Reaction on Source          | Fay and Hall       | TelephoneReceiver | Fixed     | Variable or Fixed | None                           | Input Electrical Impedance at Telephone Receiver | a. Difficult to achieve accuracy  
    |                |                    |                 |           |                   |                                               | b. Long mathematical and graphical analysis                             |                                                                          |
| 2   | Absolute Measurement        | Not Important      | Fixed           | Fixed     | Fixed             | None                           | Pressure and Velocity at a Known Point       | Very difficult measurement                                               |
| 3   | Analysis of Standing Wave   | Taylor, Wente and Bedell | Not Important | Fixed     | Fixed             | Movable | Pressure Maximum and Minimum and Location of One | Long probe tube necessary at low frequencies with resulting noise difficulties |
| 4   | Analysis of Standing Wave   | Hall               | Fixed           | Fixed     | Movable           | Pressure Maximum and Minimum and Location of One | a. Easy to use  
    |                |                    |                 |           |                   |                                               | b. No correction for tube dissipation                                   |
| 5   | Resonance and Anti-         | Wente and Bedell   | Telephone       | Fixed     | Variable          | Fixed at Source                | Pressure Maximum and Minimum and Length Between Two Known Points | a. Tube reaction on source changes  
    | resonance       |                    | Receiver (Important) |           |                   |                                               | b. No correction for tube dissipation                                   |
| 6   | Absolute Measurement        | Wente and Bedell   | Not Important   | Fixed     | Fixed Two         | Microphones Absolute Pressure and Phase at Two Known Points | a. Easy to use  
    |                |                    |                 |           | Microphones       |                                               | b. Identical microphones necessary  
    |                |                    |                 |           |                   |                                               | c. Difficult to construct "point" pressure detector                     |
| 7   | Comparison with Standard    | Wente and Bedell   | Not Important   | Fixed     | Fixed             | Two Microphones                | Pressure for Three Terminations: Two Known   | a. Difficult to obtain known standards  
    | Standard        |                    |                 |           |                   |                                               | b. Difficult mathematical analysis                                   |
| 8   | Curve Width                 | Hunt               | Point           | Variable | Fixed             | Fixed                          | Pressure-Frequency Resonance Curve           | a. Some change of impedance with frequency  
    |                |                    |                 |           |                   |                                               | b. Difficult to measure frequency accurately                            |
| 9   | Curve Width                 | Remnek             | Point           | Fixed     | Variable          | Fixed                          | Pressure Length Resonance Curve              | a. Correction made for dissipation at all frequencies  
    |                |                    |                 |           |                   |                                               | b. Correction for source reaction                                   |
| 10  | Acoustic Impedance Bridge   | K. Schuster        | Not Important   | Fixed     | Fixed Two         | Microphones                    | Balance Bridge                                | a. Easy to use  
    |                | Robinson           |                 |           | Microphones       |                                               | b. Difficult to obtain dependable standards                             |

*Acoustic Impedance Bridge*
Some Notes on the Measurement of Acoustic Impedance

LEO L. BERANEK

Crafoord Laboratory, Harvard University, Cambridge, Massachusetts

(Received March 4, 1947)

A modified form of an earlier design of impedance tube is described here. It is capable of measuring the normal impedance of a sample by the variable length, variable frequency, or traveling microphone methods without disturbing the sample in transferring from one test method to another. The tube also may be used to measure the transmission constant with the aid of a long probe tube. Modern graphical aids to the calculation of the impedance from the quantities measured in the tube are described. Comparison of data obtained by the variable length and traveling microphone methods is made, and good agreement is found over most of the frequency range. It is shown that the clamping effects of the tube walls on the edge of a lightweight, non-rigid sample are serious enough to introduce large gyrations of the impedance curves at frequencies below 500 c.p.s.

Fig. 1. Sketch of the impedance measuring tube. The scale is calibrated in millimeters or tenths of inches. The precision screw may be removed to permit attaching another piece of Shelby tubing containing a deep sample of material. A high impedance air leak is provided at the loudspeaker end to relieve static pressures. Other details are given in the text.

A number of materials were studied for the purpose of determining the effect of the restraining action of the three-inch diameter tube on the edge of the sample. A typical set of data for a five foot deep sample of material having a volume density of 0.00754 g/cm$^3$, a specific flow resistance of 34.4 acoustic ohms per centimeter of thickness (g cm$^{-1}$ sec.$^{-1}$), a porosity of 0.996, and a volume coefficient of elasticity, $Q$, of 2800 dyne/cm$^2$, is shown in Fig. 9. Calculated curves of $R/\rho c$ and $X/\rho c$ for this sample are shown by the dashed lines. It is apparent that the calculated and measured curves differ significantly at frequencies below 500 c.p.s. The theory did not take into account the restraining action of the sidewalls of the tube on the edge of the sample. In order to take this restraining action into account, each successive layer of material must be treated as a clamped-edge diaphragm. No simple theory now exists for calculating in detail the impedance of a clamped-edge, dissipative diaphragm, but it is easy to see that it will resonate at certain frequencies and that at very low frequencies its reactance will become negative and large. The dash-dot line at the bottom left edge of Fig. 9 was calculated assuming that the material had

**Fig. 9.** Measured vs. calculated impedance curves for a very light weight sample of porous absorbing blanket. The effect of the clamping of the tube on the edges of the sample at low frequencies is clearly shown. $Q$ is the volume coefficient of elasticity of the sample in dynes/cm$^2$. 
AN APPARATUS FOR ACCURATE MEASUREMENT OF THE ACOUSTIC IMPEDANCE OF SOUND-ABSORBING MATERIALS

By R. A. SCOTT,
Manchester

ABSTRACT. This paper describes the principles and the construction of an apparatus for the precise measurement of the acoustic impedance, at normal incidence, of sound-absorbing materials by a stationary-wave method for frequencies in the range 100 to 5000 c/s. From measurements made on a sample of material 1 1/2 inches in diameter, backed by a substantially rigid wall, the magnitudes of the resistive and reactive components of the impedance may be calculated with an accuracy of about one per cent.

The theory of the method is discussed with particular reference to the influence thereon of the attenuation of sound associated with the walls of the tube in which the standing wave is formed, and it is shown that in addition to the correction which must be provided to the elementary expression for the "standing-wave ratio", the finite attenuation in the tube leads to an additional correction to the expression for the distance from the sample of the first minimum of pressure. Attention is drawn to convenient methods which facilitate calculation of the results, and typical results of measurements of the impedance of a sample of porous sound-absorbing material are shown.

Figure 3. General arrangement of measuring apparatus

Figure 4. Detail of driving unit and microphone trolley.
THE ABSORPTION OF SOUND IN A HOMOGENEOUS POROUS MEDIUM

By R. A. SCOTT,
Manchester

M.S. received 15 October 1945

Figure 1. Apparatus for measurement of propagation constant

Figure 2. Orifice of probe-tube microphone

Figure 5. Attenuation and phase velocity of sound in a typical porous medium.

Proceedings of the Physical Society of London, 58(326): 165-183 1946
Fig. 3. a) Photograph and drawings of the Standing Wave Apparatus type 4002.
b) Photograph of the six different sample holders belonging to the apparatus.
Fig. 5. Determination of the coefficients of an acoustical four-terminal network by means of a tube measurement and compensation microphones according to Wüst\textsuperscript{3}).

Fig. 6. Impedance measurements on test sample, with normal standing wave measurement. The conditions “open and short-circuited quadrupole” are fulfilled by placing the piston in the cylinder close to the sample and at a distance of $\frac{3}{4}$ wave length $\lambda$. 
Propagation of sound in highly porous open-cell elastic foams

Robert F. Lambert
Department of Electrical Engineering, Institute of Technology, University of Minnesota, Minneapolis, Minnesota 55455
(Received 7 September 1982; accepted for publication 29 December 1982)

This work presents both theoretical predictions and experimental measurements of attenuation and progressive phase constants of sound in open-cell, highly porous, elastic polyurethane foams. The foams are available commercially in graded pore sizes for which information about the static flow resistance, thermal time constant, volume porosity, dynamic structure factor, and speed of sound is known. The analysis is specialized to highly porous foams which can be efficient sound absorbers at audio frequencies. Negligible effect of internal wave coupling on attenuation and phase shift for the frequency range 16–6000 Hz was predicted and no experimentally significant effects were observed in the bulk samples studied. The agreement between predictions and measurements in bulk materials is excellent. The analysis is applicable to both the regular and compressed elastic open-cell foams.

FIG. 1. Photograph of experimental setup and propagation measurement tube.

FIG. 2. Progressive phase shift characteristics of highly porous open-cell foam materials employing mean pore size as a parameter (see Table 1). (a) Characteristics for $a_p = 0.019, 0.026,$ and $0.079$ cm. (b) Characteristics for $a_p = 0.023$ and $0.048$ cm.

FIG. 3. Attenuation characteristics of highly porous open-cell foam materials employing mean pore size as a parameter (see Table 1). (a) Characteristics for $a_p = 0.019, 0.026,$ and $0.079$ cm. (b) Characteristics for $a_p = 0.023$ and $0.048$ cm.
Standing wave apparatus for measuring fundamental properties of acoustic materials in air

Jason D. McIntosh, Michael T. Zuroski, and Robert F. Lambert
Department of Electrical Engineering, Institute of Technology, University of Minnesota, Minneapolis, Minnesota 55455

(Received 29 January 1990; accepted for publication 26 June 1990)

A standing wave apparatus employing multiple microphones for measuring fundamental properties of acoustic materials is analyzed. The apparatus is basically a tube with a compression driver at one end with a rigid plug at the other and a finite length sample of acoustic material strategically located in between. A microphone array is then used in obtaining the pressure and velocity boundary conditions of the sample from which basic acoustic properties can be calculated. Such properties as internal propagation constant, characteristic admittance (inverse impedance), and bulk modulus are measured and data for a 100-pore-per-inch (ppi) open-cell acoustic foam is presented. A relatively new parameter called the complex flow impedance is measured under low- and high-intensity levels and is shown to exhibit finite amplitude properties. Of particular concern is how finite sample lengths effect the accuracy of the measurements. It is shown that in general a long sample length (on the order of a wavelength or more) is required for the accurate measurement of propagation constant, characteristic admittance, and bulk modulus measurements while a short length (much less than a wavelength) is required for good finite amplitude flow impedance measurements. Previous studies of flow impedance have been carried out using a similar but not identical apparatus [W. E. Zorunski and T. L. Parrott, "Nonlinear Acoustic Theory for Rigid Porous Materials," U.S. NASA TN-6196, 1971; K. U. Ingard and T. A. Dear, "Measurements of the Acoustic Flow Resistance," J. Sound Vib. 103, 567-572 (1985)]. However, the previous studies have lacked an analysis of the possible errors in the measurement methods that can be very significant for these types of flow impedance measurements.

FIG. 3. Block diagram of the measurement system.
Measurement of the characteristic impedance and propagation constant of materials having high flow resistivity

Yvan Champoux and Michael A. Stinson
Institute for Microstructural Sciences, National Research Council, Ottawa, Ontario K1A 0R6, Canada

(Received 20 November 1990; accepted for publication 8 May 1991)

A technique for the measurement of characteristic acoustical impedance and propagation constant of porous materials is presented. Samples are mounted in an impedance tube and both the surface impedance and a transfer function along the sample length are determined; characteristic impedance and propagation constant may be calculated using these quantities. For this implementation, a classical standing wave procedure is used for the determination of surface impedance. Measurements are obtained for frequencies between 100 and 4000 Hz. A special procedure that makes use of extension tubes, between sample and the fixed part of the impedance tube, is used to achieve the low-frequency results. The new technique yields results that are comparable in accuracy to the "two-cavity" technique described by Utsuno et al. [J. Acoust. Soc. Am. 86, 637-643 (1989)] for samples of low flow resistivity (17.2 and 43.6 cgs-rayl cm$^{-1}$), but substantially better for a sample of higher flow resistivity (380 cgs-rayl cm$^{-1}$).

FIG. 1. Sketch of the sample end of the measurement tube, showing the principle of operation of the measurement technique. In panel (a), measurement of the impedance $Z_s$ at the surface of the sample ($r = 0$) and the transfer function between microphones at $x_s$ and $x_a$ allows the material to be characterized acoustically. The microphones are calibrated against each other by removing the sample, as in (b).
FIG. 2. Sketch of the measurement system. The surface impedance is determined using a classical standing wave technique, with probe microphone moving over a 35 cm range. To extend the technique to lower frequencies, extension tubes may be inserted between sample and the fixed part of the standing wave tube (one such extension is shown in this figure). The signals from the reference and cavity microphones are accepted by the lock-in amplifier, and the transfer function between them determined.
### Impedance Measurements

- **Test No.**
- **Material:**
- **Layers:** 1, 2
- **Room Temp:**
- **Ambient Press.:**
- **BFO Output:**

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Top</th>
<th>Backing</th>
<th>$V_{inc}$ (Volts)</th>
<th>$V_{in}$ (Volts)</th>
<th>$N$</th>
<th>$X_1$ (ω)</th>
<th>$X_2$ (ω)</th>
<th>$X_4$ (ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1.96 mV</td>
<td>.340 mV</td>
<td>2.46</td>
<td>1.83</td>
<td>6.18</td>
<td>.210</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1.42 mV</td>
<td>.398 mV</td>
<td>2.07</td>
<td>1.84</td>
<td>6.19</td>
<td>.211</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>4</td>
<td>1.48 mV</td>
<td>.340 mV</td>
<td>3.35</td>
<td>1.82</td>
<td>6.19</td>
<td>.208</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>4</td>
<td>1.54 mV</td>
<td>.353 mV</td>
<td>4.54</td>
<td>1.80</td>
<td>6.18</td>
<td>.206</td>
</tr>
<tr>
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<td>2</td>
<td>3</td>
<td>1.64 mV</td>
<td>.346 mV</td>
<td>4.74</td>
<td>1.82</td>
<td>6.18</td>
<td>.209</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>1.66 mV</td>
<td>.362 mV</td>
<td>4.56</td>
<td>1.81</td>
<td>6.18</td>
<td>.207</td>
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<td>3</td>
<td>1</td>
<td>4</td>
<td>1.66 mV</td>
<td>.385 mV</td>
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<td>6.18</td>
<td>.210</td>
</tr>
<tr>
<td>3</td>
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<td>1.70 mV</td>
<td>.393 mV</td>
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<td>1.81</td>
<td>6.17</td>
<td>.208</td>
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<td>.390 mV</td>
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<td>6.19</td>
<td>.207</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
<td>1.69 mV</td>
<td>.382 mV</td>
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<td>1.89</td>
<td>6.20</td>
<td>.216</td>
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<tr>
<td>4</td>
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<td>3</td>
<td>1.74 mV</td>
<td>.365 mV</td>
<td>4.77</td>
<td>1.84</td>
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<td>.211</td>
</tr>
<tr>
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<td>4</td>
<td>1.75 mV</td>
<td>.380 mV</td>
<td>4.61</td>
<td>1.84</td>
<td>6.19</td>
<td>.211</td>
</tr>
</tbody>
</table>

\[
\bar{N} = \frac{4.49}{100} = 0.0449,
\bar{r} = \frac{\bar{N} - 2}{N + 2} = -0.635
\]

\[
\left( \frac{X_1}{2 (X_4 - X_3)} \right) = \frac{2.090}{100} = 0.0209
\]

\[
\Delta = \pi \left[ 1 - 4 \left( \frac{X_1}{2 (X_4 - X_3)} \right) \right] = 0.523
\]

\[
R = \frac{4 - r}{\rho Q_0} = 2.01
\]

\[
\frac{X}{\rho v_c} = \frac{2 r \sin \theta \Delta}{1 + r^2 + 2 \rho v_c \Delta} = 2.09
\]
Experimental determination of acoustic properties using a two-microphone random-excitation technique

A. F. Seybert

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D. F. Ross

Arvin Industries, Incorporated, Lafayette, Indiana 47906

(Received 17 December 1976)

An experimental technique is presented for the determination of normal acoustic properties in a tube, including the effect of mean flow. An acoustic source is driven by Gaussian white noise to produce a randomly fluctuating sound field in a tube terminated by the system under investigation. Two stationary, wall-mounted microphones measure the sound pressure at arbitrary but known positions in the tube. Theory is developed, including the effect of mean flow, showing that the incident- and reflected-wave spectra, and the phase angle between the incident and reflected waves, can be determined from measurement of the auto- and cross-spectra of the two microphone signals. Expressions for the normal specific acoustic impedance and the reflection coefficient of the tube termination are developed for a random sound field in the tube. Three no-flow test cases are evaluated using the two-microphone random-excitation technique: a closed tube of specified length, an open, baffled tube of specified length, and a prototype automotive muffler. Comparison is made between results using the present method and approximate theory and results from the traditional standing-wave method. In all cases agreement between the two-microphone random-excitation method and comparison data is excellent. The two-microphone random-excitation technique can be used to evaluate acoustic properties very rapidly since no traversing is necessary and since random excitation is used (in each of three test cases only 7 sec of continuous data was needed). In addition, the bandwidth may be made arbitrarily small, within limits, so that the computed properties will have a high degree of frequency resolution.

FIG. 3. Experimental set-up used in determination of impedance and power reflection coefficient using the two-microphone random-excitation method.
Four Microphone Method

- Transfer matrix and scattering matrix methods
- Estimation of complex density and sound speed
- Random incidence TL prediction
  - Closer to real application
  - No simple relationship with normal incidence TL
- Grazing incidence measurements
  - Performance of acoustic materials in channel lining applications
History

- Two-microphone tube (E1050)
  - Chung and Blaser
  - Seybert and Ross

- Four-microphone tube for silencer testing
  - Munjal (Duct Acoustics)
  - Two-load method
  - Two-source method

- Four-microphone tube for material testing
  - Suggested by Joseph Pope
  - Yun and Bolton (1997 SAE)
  - Song and Bolton (2000 JASA) introduce transfer matrix approach
  - Many articles since then
Transfer Matrix Method

\[ P_1 = (A e^{-jkx_1} + B e^{jkx_1}) e^{j\omega t} \]
\[ P_2 = (A e^{-jkx_2} + B e^{jkx_2}) e^{j\omega t} \]

\[ A = \frac{j(P_1 e^{jkx_1} - P_2 e^{jkx_1})}{2 \sin k(x_1 - x_2)} \]
\[ B = \frac{j(P_1 e^{jkx_1} - P_2 e^{jkx_1})}{2 \sin k(x_1 - x_2)} \]

- Sound pressure and velocity relationship
- Symmetric sample
- Transmission loss
- Transfer matrix

\[ T_{11} = T_{22}, \quad T_{11} T_{22} - T_{12} T_{21} = 1 \]
\[ T_{11} = \frac{P_{x=d} V_{x=d} + P_{x=0} V_{x=0}}{P_{x=d} V_{x=d} + P_{x=0} V_{x=0}} \]
\[ T_{21} = \frac{V_{x=0} - V_{x=d}}{P_{x=d} V_{x=d} + P_{x=0} V_{x=0}} \]

\[ T_L = 20 \log_{10}(1/|T|) \]

\[ k_p = \frac{1}{d} \cos^{-1} T_{11} \]
\[ \rho_p = \frac{1}{c_p} \sqrt{\frac{T_{12}}{T_{21}}} \]
Two Load Method

- Measurement 1:
  - Open termination
  - Measurement 1:
    - Solve for $T_{11}$, $T_{12}$, $T_{21}$, $T_{22}$
  - Advantage: no requirement that sample be symmetric
  - Disadvantage: twice as many measurements

- Measurement 2:
  - Hard termination
  - Measurement 2:
    - Solve for $T_{11}$, $T_{12}$, $T_{21}$, $T_{22}$
Plane wave model – Scattering Matrices
(Jorgen Hald – INTER-NOISE 2006)

\[ P = (Ae^{-jkx} + Be^{jkx}) \quad P = (Ce^{-jkx} + De^{jkx}) \]

- Seen from sample:
  A and D incident
  B and C outgoing

- Rewrite to left/right:

\[
\begin{bmatrix}
  A \\
  B
\end{bmatrix} = \begin{bmatrix}
  a_{11} & a_{12} \\
  a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
  C \\
  D
\end{bmatrix} = \begin{bmatrix}
  1/t_{12} & -r_2/t_{12} \\
  r_1/t_{12} & t_{21} - r_1r_2/t_{12}
\end{bmatrix}
\]

- Transmission Loss:

\[ TL_n = 10\log\left(\frac{A^2}{C^2|_{D=0}}\right) = 10\log\left(a_{11}^2\right) = 10\log\left(\frac{1}{t_{12}}^2\right) \]
## Materials

- **THL3**
  - polyester staple fibers
  - Lower density than TC3303
  - Lower TL and absorption coefficient
  - Thinner than TC3303

- **TC3303**
  - blown micro fibers with mix of polypropylene and polyester staple fibers

<table>
<thead>
<tr>
<th></th>
<th>THL3</th>
<th>TC3303</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness [cm]</td>
<td>3.95</td>
<td>4.98</td>
</tr>
<tr>
<td>Mass per unit area [g/m²]</td>
<td>156</td>
<td>376</td>
</tr>
</tbody>
</table>
Two-load and transfer matrix methods show good agreement both on magnitude and phase.

Two-load method needs two different terminations: Total of two measurements are required.
Typical Results

For anechoically terminated sample

\[ \alpha = 1 - |R_a|^2 \]

\[ \alpha_d = 1 - |R_a|^2 - |T_a|^2 \]

\[ T = \frac{2e^{jkd}}{T_{11} + (T_{12} / \rho_0 c) + \rho_0 cT_{21} + T_{22}} \]

\[ R_a = \frac{T_{11} + (T_{12} / \rho_0 c) - \rho_0 cT_{21} - T_{22}}{T_{11} + (T_{12} / \rho_0 c) + \rho_0 cT_{21} + T_{22}} \]

\[ TL = 10 \log_{10} \left( \frac{1}{|T_a|^2} \right) \]

- \( \alpha_d \) represents the fraction of the incident energy dissipated within the sample
Complex Density and Complex Wave Number

- Normalized complex densities show that TC3303 has higher density
- Phase speed: \( \omega/\text{Re}\{k\rho\} \)
- Attenuation per m: \( \text{Im}\{k\rho\} \)
- These values can be used in SEA, FE predictions and plane wave predictions
Normal Incidence TL

- 10 sets of two samples

- Because of leakage problem, two layers were used for normal incidence TL test.
- Edge-constraint effects appear at low frequencies in the measurements
- Higher duct modes appear in sample around 3 kHz and cause increased standard deviation
Anechoic Transmission Loss

Aviation grade glass fiber (density=9.6 kg/m³, flow resistivity= 31000 Rayls/m)

![Graph showing transmission loss vs frequency](image)

**Increase in TL due to edge constraint**

- Above shearing resonance – finite size sample represents infinite sample
- Below shearing resonance – all properties affected by edge-constraint
Laser Measurement Setup
(Large Tube, 1” Sample A)
The 1\textsuperscript{st} and 2\textsuperscript{nd} Mode Shapes of the Edge-constrained Sample (1′′)

<table>
<thead>
<tr>
<th>FEM</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="1st Mode at 100 Hz" /></td>
<td><img src="image2" alt="1st Mode at 100 Hz" /></td>
</tr>
<tr>
<td><img src="image3" alt="2nd Mode at 350 Hz" /></td>
<td><img src="image4" alt="2nd Mode at 350 Hz" /></td>
</tr>
</tbody>
</table>
Higher Order Modes in Samples

- Since wave speed in porous materials is subsonic, higher order modes may “cut-on” in the sample at lower frequencies than in the tube
  - TC3303 – accurate at frequencies < 2700 Hz
  - THL3 – accurate at frequencies < 3000 Hz
- This effect limits high frequency accuracy of the measurements
Random Incidence Transmission Loss
(for rigid or limp porous materials)

**Region I**
\[
\begin{align*}
P_1 &= e^{-jk_x x - jk_y y} + Re^{jk_x x - jk_y y} \\
U_{1x} &= -\frac{1}{j\omega p_0} \frac{\partial P_1}{\partial x} = -\frac{1}{j\omega p_0} (-jk_x e^{-jk_x x - jk_y y} + jk_x Re^{jk_x x - jk_y y})
\end{align*}
\]

**Region II**
\[
\begin{align*}
P_2 &= Ae^{-jk_{px} x - jk_{py} y} + Be^{jk_{px} x - jk_{py} y} \\
U_{2x} &= -\frac{1}{j\omega p_p} \frac{\partial P_2}{\partial x} = -\frac{1}{j\omega p_p} (-jk_{px} Ae^{-jk_{px} x - jk_{py} y} + jk_{px} Be^{jk_{px} x - jk_{py} y})
\end{align*}
\]

**Region III**
\[
\begin{align*}
P_3 &= Te^{-jk_x x - jk_y y} \\
U_{3x} &= -\frac{1}{j\omega p_0} \frac{\partial P_3}{\partial x} = -\frac{1}{j\omega p_0} (-jk_x Te^{-jk_x x - jk_y y})
\end{align*}
\]

\[
T = \frac{2 \rho_p k_x}{\rho_0 k_{px}} e^{-jk_x x} \left\{ \begin{array}{c} j \sin k_{px} \left( \frac{\rho_p k_x}{\rho_0 k_{px}} \right)^2 + 1 \right\} + 4 \frac{\rho_p k_x}{\rho_0 k_{px}} \cos k_{px}
\]

\[
TL = 10 \log_{10} \left( \frac{1}{\overline{T}} \right)
\]

where \( \overline{T} = \int_0^{\pi/2} |T|^2 \sin 2\theta d\theta \)

**\( k_p \) and \( \rho_p \) can be acquired from normal incidence TL test**

- Transmission coefficient
- Transmission loss
Random Incidence Transmission Loss

- Test was performed in reverberation room with intensity probe.
- TL was calculated by averaging TL at 25 points over sample
- Two layers of each material were used
- Predictions based on complex density and wave number and direct measurements show good agreement
Introduction

Grazing Incidence Applications

- Porous materials are often used to line “channels”
- Previous work shows that the properties that control grazing direction attenuation are somewhat different from those that control normal absorption.

Here introduce anisotropic theory to account for different flow resistivity of the lining material in normal and grazing directions.
Glass fiber (yellow and green) properties

<table>
<thead>
<tr>
<th></th>
<th>Glass fiber (yellow)</th>
<th>Glass fiber (green)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bulk Young's modulus [N/m²]</strong></td>
<td>3360</td>
<td>8120</td>
</tr>
<tr>
<td><strong>Possion ratio</strong></td>
<td>0.4</td>
<td>0.45</td>
</tr>
<tr>
<td><strong>Loss factor</strong></td>
<td>0.35</td>
<td>0.275</td>
</tr>
<tr>
<td><strong>Flow resistivity [MKS Rayls/m]</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>7500</td>
<td>16000</td>
</tr>
<tr>
<td>Y</td>
<td>15000</td>
<td>26000</td>
</tr>
<tr>
<td><strong>Tortuosity</strong></td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td><strong>Porosity</strong></td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td><strong>Bulk density [kg/m³]</strong></td>
<td>6.7</td>
<td>9.6</td>
</tr>
</tbody>
</table>
Square duct system and glass fiber layer

- 4-microphone transfer matrix method used to measure attenuation at grazing incidence
Effect of Material Anisotropy

Measurement of TL in two configurations for glass fiber (green)

Case 1: 9 pieces layered in vertical direction

Case 2: 9 pieces layered in horizontal direction

- Acoustic properties depend on material orientation – flow resistivity larger in normal direction
Anisotropic Duct Lining

- Prediction and measurement for Yellow glass fiber by using anisotropic poro-elastic model
- Different flow resistivities [MKS Rayls/m] in x- and y-directions

\[
s_\sigma^x = 7500 \quad s_\sigma^y = 15000
\]

\(\text{Attenuation ratio in dB/0.5m}\)

![Graph showing attenuation ratio vs. frequency](image)

Frequency in Hz
Anisotropic Duct Lining

• Prediction and measurement for Green glass fiber by using anisotropic poro-elastic model

• Different flow resistivity [MKS Rayls/m] in x- and y-directions

\[ \sigma_x = 16000 \quad \sigma_y = 26000 \]

\[
\begin{align*}
\text{Air} \\
45 \text{mm} \\
25 \text{mm}
\end{align*}
\]

\[
\begin{align*}
\text{x} & \quad \text{y}
\end{align*}
\]
Estimation of Biot Parameters

- Software available to estimate Biot parameters by performing optimal fit to measure acoustical data
  - ESI-FOAM-X (rigid, limp)
  - COMET/Trim (rigid, limp, elastic)

- Original software based on transversely infinite layered representation: i.e., edge constraint effects are not included
Infinite Panel Model: COMET/TRIM
Infinite Panel Model: Limitation

Note that this model does not simulate the low frequency transmission loss fluctuation caused by shearing resonance of the sample.
Finite Element Models: COMET/SAFE

- The software COMET/SAFE is used to model and compute the absorption and transmission loss having a finite depth and finite size layer of porous material.
- A finite element based program that allows for the analysis of sound traveling through various media including fluids, solids and foam-like substances.
- Finite element implementation is based on $u-U$ and $p-U$ versions of Biot theory.
- All models used in this work involved axisymmetric elements.
- **The new version of TRIM supports automated inverse characterization capability based on SAFE.**
Finite Element Model

Note that finite model can simulate the low frequency transmission loss fluctuation caused by shearing resonance of the sample.
Variation of Shear Modulus

- As shear modulus increases, the minimum location moves to higher frequencies.
Flow Resistivity

- Flow resistivity controls TL in **low and high frequency limit**

```
Flow resistivity=20000 MKS Rayls/m
Flow resistivity=40000 MKS Rayls/m
Flow resistivity=60000 MKS Rayls/m
```

Loss Factor

- Loss factor controls depth of TL minimum

```
loss factor = 0.1
loss factor = 0.3
loss factor = 0.5
```
Finite Element Model
4 mic standing wave tube: 100 Hz
Finite Element Model
2 mic standing wave tube: 100 Hz
Inverse Characterization

- Questions: Is it possible to determine the Biot parameters from acoustical measurements? Do parameters act independently? How many parameters can be estimated?
- To help answer these questions, introduce a procedure based on Singular Value Decomposition
- *Singular Value Decomposition* is widely used linear algebraic method to identify the principal component in the field of image processing and signal processing.
Sensitivity Matrix Analysis

Procedures

1. Linearize absorption and transmission coefficient close to a certain parameter set

2. Use absorption and/or transmission coefficient values for certain number of frequencies to construct a sensitivity matrix

3. Perform singular value decomposition on the sensitivity matrix and extract singular values to determine effective rank (number of independent parameters)

4. Calculate condition number (the smaller the better)
Sensitivity Matrix Analysis

- Linearize the expression for the absorption and transmission coefficient in the vicinity of a certain parameter set.
  For 1 frequency

\[
\alpha_{f_i}(x) = \alpha_{f_i}(x_0) + \sum_{i=1}^{9} \left( \frac{\partial \alpha_{x_i}}{\partial x_i} \right) dx_i
\]

Real Solution

Approximate Solution

Calculate by using “central difference scheme” ±1 % difference of material properties
Sensitivity Matrix Analysis

- For \( n \) frequencies, the equation can be combined as a matrix

\[
\begin{bmatrix}
\alpha_{f_1}(x)
& \ldots \\
\alpha_{f_2}(x)
& \ldots \\
\alpha_{f_n}(x)
& \ldots
\end{bmatrix}
= \begin{bmatrix}
\alpha_{f_1}(x_0)
& \ldots \\
\alpha_{f_2}(x_0)
& \ldots \\
\alpha_{f_n}(x_0)
& \ldots
\end{bmatrix}
+ \begin{bmatrix}
\frac{\partial \alpha_{f_1}}{\partial x_1} & \frac{\partial \alpha_{f_1}}{\partial x_2} & \ldots & \frac{\partial \alpha_{f_1}}{\partial x_9} \\
\frac{\partial \alpha_{f_2}}{\partial x_1} & \frac{\partial \alpha_{f_2}}{\partial x_2} & \ldots & \frac{\partial \alpha_{f_2}}{\partial x_9} \\
\frac{\partial \alpha_{f_n}}{\partial x_1} & \frac{\partial \alpha_{f_n}}{\partial x_2} & \ldots & \frac{\partial \alpha_{f_n}}{\partial x_9}
\end{bmatrix}
\begin{bmatrix}
dx_1 \\
\ldots \\
dx_9
\end{bmatrix}
\]

Sensitivity Matrix
Perform singular value decomposition:
\( M = U \Sigma V^* \)

The rank of the matrix \( M \) equals the number of non-zero singular values which is the same as the number of non-zero elements in the matrix \( \Sigma \).
Rigid Foam
Sensitivity Matrix Analysis

- Use COMET/TRIM rigid foam type material that has 5 material properties. E.g., Porosity, flow resistivity, tortuosity, viscous and thermal characteristic length.
- The nominal values of the material properties are

<table>
<thead>
<tr>
<th>Porosity</th>
<th>Flow Resistivity</th>
<th>Tortuosity</th>
<th>VCL</th>
<th>TCL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.98</td>
<td>50,000</td>
<td>2.0</td>
<td>3.0*10^-5</td>
<td>9.0*10^-5</td>
</tr>
</tbody>
</table>
Rigid Foam Sensitivity Matrix Analysis

- Effect of adding frequency data for absorption coefficient

Adding additional frequency data reduces the condition number, but the condition number is too big to consider that the sensitivity matrix is well-posed.
Adding additional frequency data reduces the condition number, but the condition number is too big to consider that the sensitivity matrix is well-posed.
Rigid Foam
Sensitivity Matrix Analysis

- Sensitivity matrix

Sensitivity of porosity and flow resistivity is quite close to each other for both absorption and transmission coefficients.
Rigid Foam
Sensitivity Matrix Analysis

- Fixed porosity case result for absorption coefficient

Fixing porosity reduces the condition number significantly and makes the sensitivity matrix well-posed.
Rigid Foam
Sensitivity Matrix Analysis

- Combine both absorption and transmission coefficient sensitivity matrix

<table>
<thead>
<tr>
<th>Singular Value</th>
<th>Condition Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1197</td>
<td>11</td>
</tr>
<tr>
<td>0.87038</td>
<td></td>
</tr>
<tr>
<td>0.39976</td>
<td></td>
</tr>
<tr>
<td>0.18173</td>
<td></td>
</tr>
</tbody>
</table>

Adding other acoustical measurements reduces the condition number further
Rigid Foam Sensitivity Matrix Analysis

To verify the effect of low and high condition number during the automatic inverse characterization in COMET/TRIM, two different cases were studied.

<table>
<thead>
<tr>
<th></th>
<th>Solution</th>
<th>Initial value</th>
<th>Found value 1</th>
<th>Found value 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity</td>
<td>0.98</td>
<td>0.5</td>
<td>0.54</td>
<td><strong>0.98</strong></td>
</tr>
<tr>
<td>Flow resistivity</td>
<td>50,000</td>
<td>125,000</td>
<td>165,000</td>
<td>51,050</td>
</tr>
<tr>
<td>Tortuosity</td>
<td>2.0</td>
<td>6.0</td>
<td>1.47</td>
<td>2.073</td>
</tr>
<tr>
<td>Viscous C.L</td>
<td>3.0*10^{-5}</td>
<td>9.0*10^{-5}</td>
<td>1.77*10^{-5}</td>
<td>3.08*10^{-5}</td>
</tr>
<tr>
<td>Thermal C.L</td>
<td>9.0*10^{-5}</td>
<td>2.7*10^{-4}</td>
<td>7.85*10^{-4}</td>
<td>9.06*10^{-5}</td>
</tr>
</tbody>
</table>
Elastic Foam
Sensitivity Matrix Analysis

- Use COMET/TRIM elastic foam type material that has 9 material properties.
- The nominal values of the material properties are

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity</td>
<td>0.98</td>
</tr>
<tr>
<td>Flow resistivity (MKS Rayls/m)</td>
<td>50,000</td>
</tr>
<tr>
<td>Tortuosity</td>
<td>2.0</td>
</tr>
<tr>
<td>Viscous Char. Length (m)</td>
<td>3.0*10^{-5}</td>
</tr>
<tr>
<td>Thermal Char. Length (m)</td>
<td>9.0*10^{-5}</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>9.0</td>
</tr>
<tr>
<td>Young’s molulus (N/m²)</td>
<td>50,000</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.4</td>
</tr>
<tr>
<td>Loss factor</td>
<td>0.3</td>
</tr>
</tbody>
</table>
The condition number is too big to consider that the sensitivity matrix is well-posed.
The condition number is substantially lower than infinite panel model case (2336).
Finite Element Model
7 Material Property Search

![Graphs showing material property search results](image-url)
Conclusions
Acknowledgments

- P. E. Doak
- Joe Pope
- L&L Products
- United Technologies Research Center
- 3M Corporation (Jon Alexander)
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- NASA (Richard Silcox)
- Richard Yun, Heuk Jin (Bryan) Song, Jinho Song, Jeong-woo Kim, Taewook Yoo, Kwanwoo Hong, Kang Hou
- Tanya Wulf
Plane wave model – Scattering Matrices

\[ P = (A e^{-j k x} + B e^{j k x}) \quad P = (C e^{-j k x} + D e^{j k x}) \]

- Seen from sample:
  A and D incident
  B and C outgoing

- Rewrite to left/right:

- No sample:
Plane wave model – Based on Scattering Matrix

- One measurement: 2 equations, 4 unknowns:

\[
\begin{bmatrix}
A \\
B
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
C \\
D
\end{bmatrix}
\]

- Two independent measurements:

\[
\begin{bmatrix}
A^{(a)} & A^{(b)} \\
B^{(a)} & B^{(b)}
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
C^{(a)} & C^{(b)} \\
D^{(a)} & D^{(b)}
\end{bmatrix}
\]

- Transmission Loss:

\[
TL_n = 10 \log \left( |a_{11}|^2 \right)
\]

\[
a_{11} = \frac{A^{(a)}D^{(b)} - A^{(b)}D^{(a)}}{C^{(a)}D^{(b)} - C^{(b)}D^{(a)}} = \frac{R^{(b)} \cdot A^{(a)}/C^{(a)} - R^{(a)} \cdot A^{(b)}/C^{(b)}}{R^{(b)} - R^{(a)}}, R^{(s)} \equiv \frac{D^{(s)}}{C^{(s)}}
\]
Transmission Loss

- **Shear modulus** controls minimum location in TL curve

Absorption Coefficient

- \[ G = \frac{E}{2(1+v)} = 2845 \text{ Pa} \]
Solid Phase of Constrained Sample (SDX)

- **200 Hz**
- **1100 Hz**
- **500 Hz**
- **1800 Hz**
Fluid Phase of Constrained Sample (ADX)

- **200 Hz**
- **1100 Hz**
- **500 Hz**
- **1800 Hz**
Rigid Foam
Sensitivity Matrix Analysis

- Fixed porosity case result for transmission coefficient

Effect of adding frequency

Fixing porosity reduces the condition number significantly and makes the sensitivity matrix well-posed.
Elastic Foam
Sensitivity Matrix Analysis

The condition number is too big to consider that the sensitivity matrix is well-posed.
Elastic Foam
Sensitivity Matrix Analysis

- Combine both absorption and transmission coefficient sensitivity matrix

<table>
<thead>
<tr>
<th>Singular Value</th>
<th>Condition Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1245</td>
<td>1650</td>
</tr>
<tr>
<td>1.8882</td>
<td></td>
</tr>
<tr>
<td>1.442</td>
<td></td>
</tr>
<tr>
<td>1.0938</td>
<td></td>
</tr>
<tr>
<td>0.5967</td>
<td></td>
</tr>
<tr>
<td>0.2557</td>
<td></td>
</tr>
<tr>
<td>0.1469</td>
<td></td>
</tr>
<tr>
<td>0.1185</td>
<td></td>
</tr>
<tr>
<td>0.0019</td>
<td></td>
</tr>
</tbody>
</table>
Elastic Foam Sensitivity Matrix Analysis

- Fixed Young’s modulus case result for absorption coefficient

Fixing Young’s modulus or Poisson’s ratio reduces condition number substantially
Elastic Foam
Sensitivity Matrix Analysis

- Fixed Young’s modulus case result for transmission coefficient

Fixing Young’s modulus or Poisson’s ratio reduces condition number substantially
Elastic Foam
Sensitivity Matrix Analysis

- Combine both absorption and transmission coefficient sensitivity matrix

<table>
<thead>
<tr>
<th>Singular Value</th>
<th>Condition Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0884</td>
<td>26</td>
</tr>
<tr>
<td>1.8401</td>
<td></td>
</tr>
<tr>
<td>1.4242</td>
<td></td>
</tr>
<tr>
<td>1.0931</td>
<td></td>
</tr>
<tr>
<td>0.5967</td>
<td></td>
</tr>
<tr>
<td>0.2555</td>
<td></td>
</tr>
<tr>
<td>0.1469</td>
<td></td>
</tr>
<tr>
<td>0.1185</td>
<td></td>
</tr>
</tbody>
</table>
9 Material Property Search
Condition number = 1650

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Solution</th>
<th>Starting Value</th>
<th>Found Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity</td>
<td>0.98</td>
<td>0.9</td>
<td>0.69</td>
</tr>
<tr>
<td>Flow resistivity</td>
<td>50,000</td>
<td>45,000</td>
<td>57,528</td>
</tr>
<tr>
<td>Tortuosity</td>
<td>2.0</td>
<td>1.8</td>
<td>2.17</td>
</tr>
<tr>
<td>Viscous Char. Length</td>
<td>3.0*10^{-5}</td>
<td>2.7*10^{-5}</td>
<td>1.69*10^{-4}</td>
</tr>
<tr>
<td>Thermal Char. Length</td>
<td>9.0*10^{-5}</td>
<td>8.1*10^{-5}</td>
<td>1.69*10^{-4}</td>
</tr>
<tr>
<td>Density</td>
<td>9.0</td>
<td>8.1</td>
<td>8.3</td>
</tr>
<tr>
<td>Young’s molulus</td>
<td>50,000</td>
<td>45,000</td>
<td>8,101,700</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.4</td>
<td>0.36</td>
<td>0.16</td>
</tr>
<tr>
<td>Loss factor</td>
<td>0.3</td>
<td>0.27</td>
<td>0.99</td>
</tr>
</tbody>
</table>

The inverse characterization process diverges and the found value is not close to the solution.
8 Material Property Search
Condition number = 26

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Solution</th>
<th>Starting Value</th>
<th>Found Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity</td>
<td>0.98</td>
<td>0.9</td>
<td>0.99</td>
</tr>
<tr>
<td>Flow resistivity</td>
<td>50,000</td>
<td>45,000</td>
<td>43,676</td>
</tr>
<tr>
<td>Tortuosity</td>
<td>2.0</td>
<td>1.8</td>
<td>1.9</td>
</tr>
<tr>
<td>Viscous Char. Length</td>
<td>3.0*10^{-5}</td>
<td>2.7*10^{-5}</td>
<td>2.7*10^{-5}</td>
</tr>
<tr>
<td>Thermal Char. Length</td>
<td>9.0*10^{-5}</td>
<td>8.1*10^{-5}</td>
<td>1.0*10^{-4}</td>
</tr>
<tr>
<td>Density</td>
<td>9.0</td>
<td>8.1</td>
<td>9.4</td>
</tr>
<tr>
<td>Young’s molulus</td>
<td>50,000</td>
<td>50,000</td>
<td>50,000</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.4</td>
<td>0.36</td>
<td>0.396</td>
</tr>
<tr>
<td>Loss factor</td>
<td>0.3</td>
<td>0.27</td>
<td>0.35</td>
</tr>
</tbody>
</table>

The inverse characterization process found much closer material properties to the solution.
Elastic Foam: Finite Element Model
Sensitivity Matrix Analysis

Transmission coefficient

The condition number is substantially lower than infinite panel model case (1309).
Elastic Foam: Finite Element Model
Sensitivity Matrix Analysis

- Combine both absorption and transmission coefficient sensitivity matrix

<table>
<thead>
<tr>
<th>Singular Value</th>
<th>Condition Number</th>
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<tbody>
<tr>
<td>1.8276</td>
<td>35</td>
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<tr>
<td>1.2253</td>
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<tr>
<td>0.7164</td>
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<tr>
<td>0.5647</td>
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<tr>
<td>0.4859</td>
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<tr>
<td>0.2339</td>
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<tr>
<td>0.1046</td>
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<tr>
<td>0.0686</td>
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<tr>
<td>0.0515</td>
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</table>

The condition number is substantially lower than infinite panel model case (1650).
### Finite Element Model
#### 7 Material Property Search

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Solution</th>
<th>Starting Value</th>
<th>Found Value</th>
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</thead>
<tbody>
<tr>
<td>Porosity</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
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<tr>
<td>Flow resistivity</td>
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<td>30,000</td>
<td>51,011</td>
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<tr>
<td>Tortuosity</td>
<td>2.0</td>
<td>1.5</td>
<td>3.21</td>
</tr>
<tr>
<td>Viscous Char. Length</td>
<td>3.0*10^{-5}</td>
<td>2.0*10^{-5}</td>
<td>4.82*10^{-5}</td>
</tr>
<tr>
<td>Thermal Char. Length</td>
<td>9.0*10^{-5}</td>
<td>6.0*10^{-5}</td>
<td>6.65*10^{-5}</td>
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<tr>
<td>Density</td>
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<td>9.0</td>
<td>9.0</td>
</tr>
<tr>
<td>Young’s modulus</td>
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<td>30,000</td>
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<td>Poisson’s ratio</td>
<td>0.4</td>
<td>0.3</td>
<td>0.391</td>
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<tr>
<td>Loss factor</td>
<td>0.3</td>
<td>0.2</td>
<td>0.351</td>
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