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# Acoustical Modeling of Tensioned, Permeable Membranes

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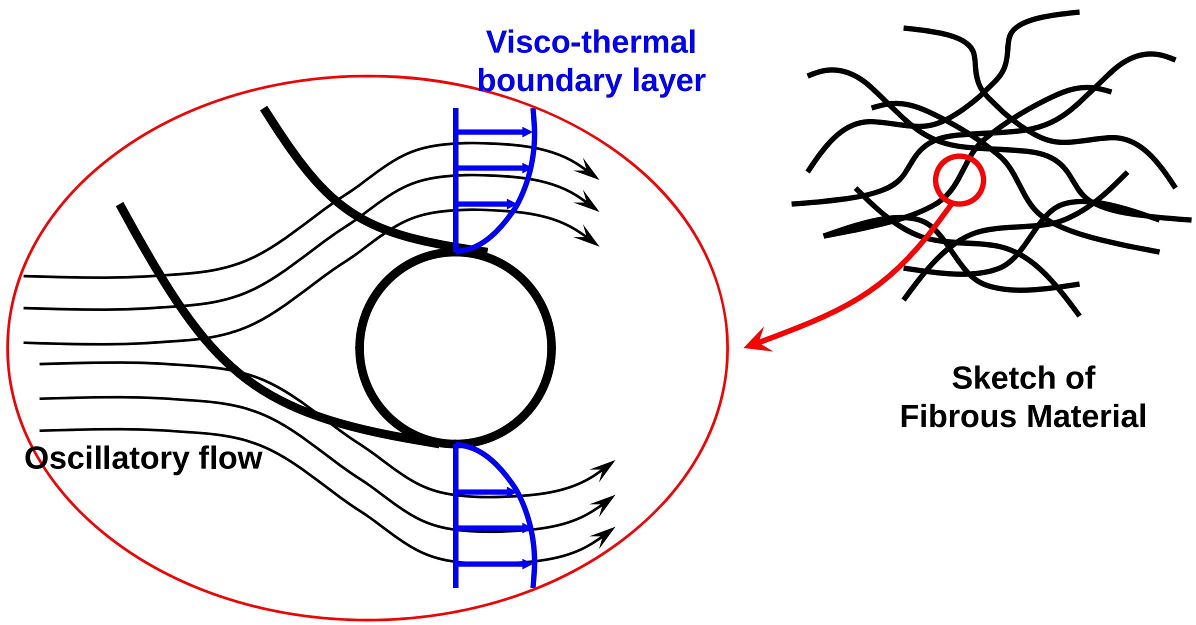
Purdue University

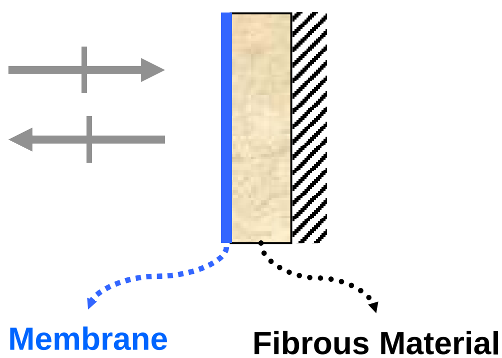
## Introduction - Background

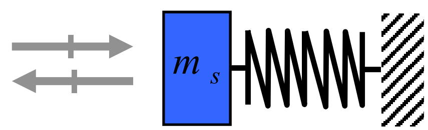
- **“Conventional” Sound Absorbing Material**
  - Open cell Foams/Glass fibers/Polymeric fibers
  - Sound energy dissipation by thermal and viscous interaction of sound field and material fibers

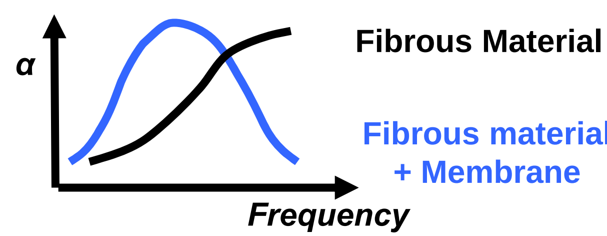


# Introduction - Background

- **“Conventional” Nonfibrous Material Usages**
  - Some environmental needs
    - - Healthy Surroundings/Ease of Maintenance
    - - Recycling
    - - Moisture-Resistance
  - Fibrous materials can be used with impermeable membranes to “tune” their performance



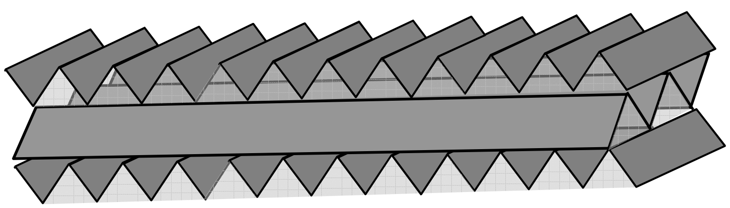

- Conventionally, membrane does not dissipate any energy

# Introduction – Motivation & Objectives

- Recently, it has been observed that

  *stacked sheets of accordion-folded, impermeable membranes (e.g., mylar sheets) offer substantial levels of low frequency absorption even though such arrays feature no obvious dissipative elements.*

Alternating layers of folded mylar



- How does this sound absorption arise?

- How do you model this effect?

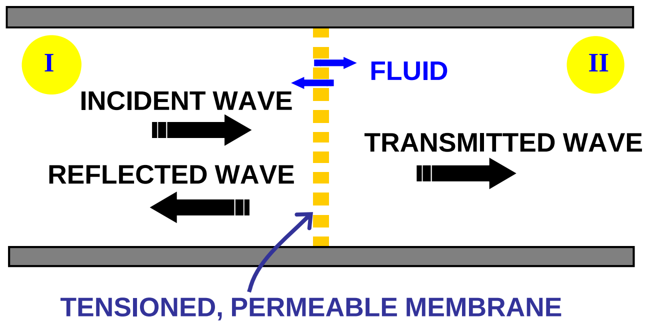
**OBJECTIVES:**

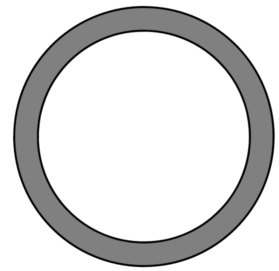
- Identify the origin of the sound absorption and behavior capacity of such a treatment
  - Develop models of those processes
  - Use those models to optimize the acoustical performance

**HYPOTHESIS:**

- Dissipation results from losses due to local flexing of membranes stiffened by curvature (i.e., by folding) or tension

# Theoretical Model - Permeable Membrane



## Assumed Solutions

- Sound Pressures in Acoustic Cavities:

$$P_{\mathrm{I}}(r,z) = e^{-jkz} + \sum_n B_n J_{\mathrm{o}}(k_{r_n} r) e^{jk_{z_n} z}$$

$$P_{\mathrm{II}}(r,z) = \sum_n C_n J_{\mathrm{o}}(k_{r_n} r) e^{-jk_{z_n} z}$$

- Membrane Displacement (Solid Component):

$$y(r,t) = \sum_n A_n J_o(k_{0_n} r)$$

- Membrane Displacement (Fluid Component):

$$u(r,t) = \sum_n F_n J_o(k_{0_n} r)$$

# Theoretical Model – Solution Method

## Boundary Conditions

- The Continuities of Velocity at the Both Side of a Membrane:

$$-\frac{1}{j\omega\rho_o}\frac{\partial P_{\mathrm{I}}}{\partial z}\bigg|_{z=0} = (1-\Omega)\frac{\partial y}{\partial t} + \Omega\frac{\partial u}{\partial t}$$

$$-\frac{1}{j\omega\rho_o}\frac{\partial P_{\mathrm{II}}}{\partial z}\bigg|_{z=0} = (1-\Omega)\frac{\partial y}{\partial t} + \Omega\frac{\partial u}{\partial t}$$

- The Force Equilibrium Equation in the Membrane:

$$\nabla^2 y - \frac{\rho_s}{T}\frac{\partial^2 y}{\partial t^2} - \frac{R_f}{T}\frac{\partial (y-u)}{\partial t} = -\frac{(1-\Omega)}{T}(P_{\mathrm{I}} - P_{\mathrm{II}})$$

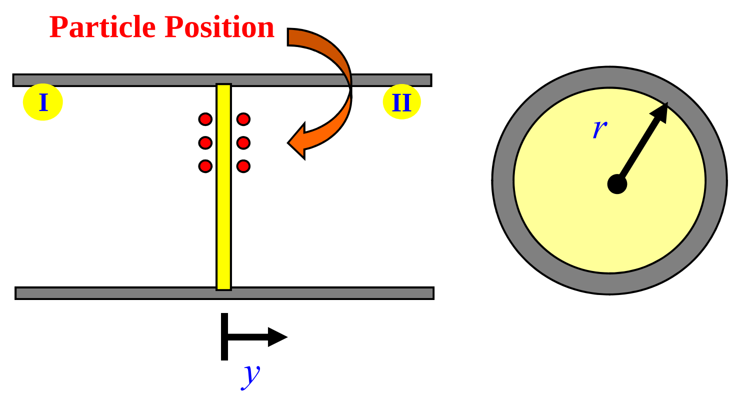
$$\rho_o \Omega h \frac{\partial^2 u}{\partial t^2} - R_f \frac{\partial (y-u)}{\partial t} = \Omega (P_{\mathrm{I}} - P_{\mathrm{II}})$$

where

$T = T_o(1 + j\eta)$ $\qquad \Omega = N\pi a^2 / A$ $\qquad P_{front} - P_{back} = R_f v_f$

## Solution Method

Apply four boundary conditions on a point-by-point basis across the membrane




~ Number of Point at which B.C.'s applied

$$\left[\begin{array}{c} Coefficient \\ Matrix \end{array}\right] \begin{Bmatrix} A_1 \\ \cdots \\ A_N \\ B_0 \\ \cdots \\ B_N \\ C_0 \\ \cdots \\ C_N \end{Bmatrix} = \begin{Bmatrix} forcing \\ Vector \end{Bmatrix}$$

# Theoretical Model – Model Verification

- Sound Power Calculation

Compare power calculated by using Acoustical Solution with power calculated using Membrane-based solution

Sound Field Based Solution

$$W_I = \frac{1}{2}\mathrm{Re}\{\int_0^a P_I(u_I{}^*)(2\pi r)dr\} \qquad W_{II} = \frac{1}{2}\mathrm{Re}\{\int_0^a P_{II}(u_{II}{}^*)(2\pi r)dr\}$$

Power Dissipated : $\boxed{W_{d,a} = W_I - W_{II}}$

Membrane Based Solution

$$W_{d,m_{solid}} = \frac{1}{2}\mathrm{Re}\{\int_0^a T\left[(\nabla^2 y + k_f^2 y - \frac{R_f}{T}\frac{\partial(y-u)}{\partial t})\right]\left[\frac{\partial y}{\partial t}\right]^*(2\pi r)dr\}$$

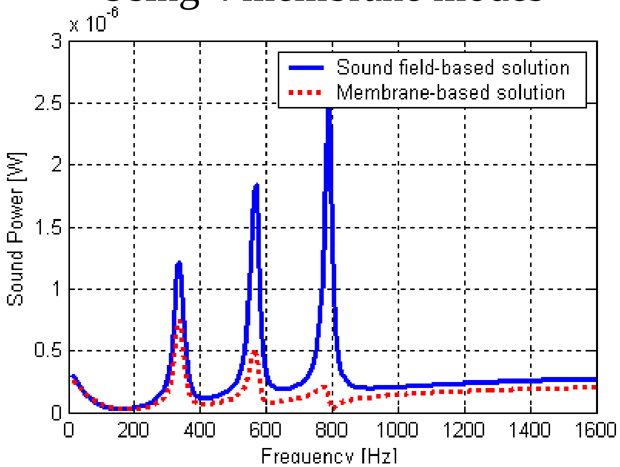
$$W_{d,m_{fluid}} = \frac{1}{2}\mathrm{Re}\{\int_0^a \left[(\rho_0 \Omega h\frac{\partial^2 u}{\partial t^2} - R_f\frac{\partial(y-u)}{\partial t})\right]\left[\frac{\partial u}{\partial t}\right]^*(2\pi r)dr\}$$

Power Dissipated : $\boxed{W_{d,m} = W_{d,m_{solid}} + W_{d,m_{fluid}}}$

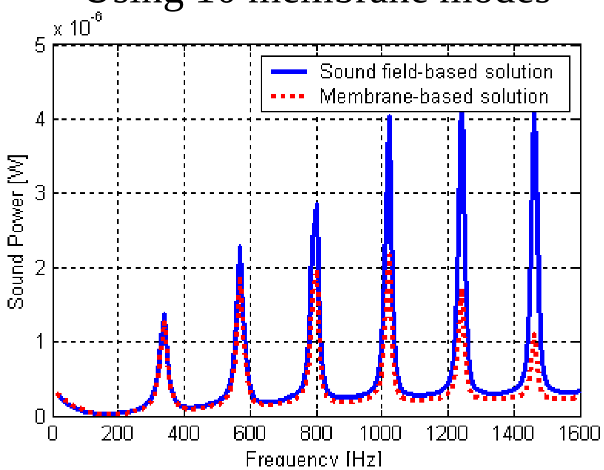
Should be equal if model works properly

# Theoretical Model – Model Verification

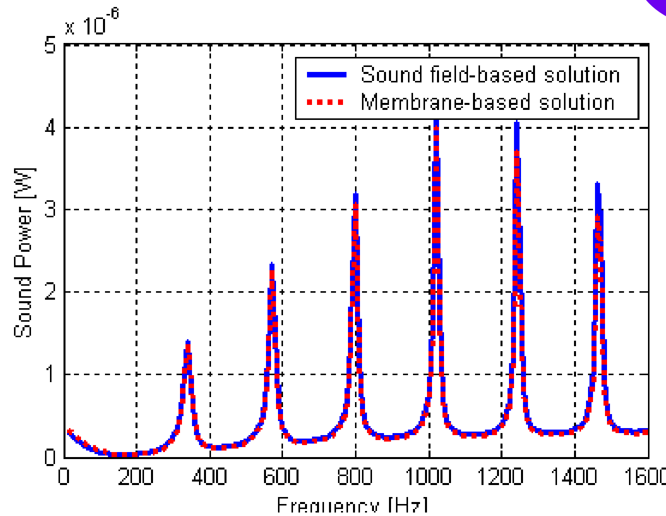
Using 4 membrane modes




Using 10 membrane modes




Using 30 membrane modes

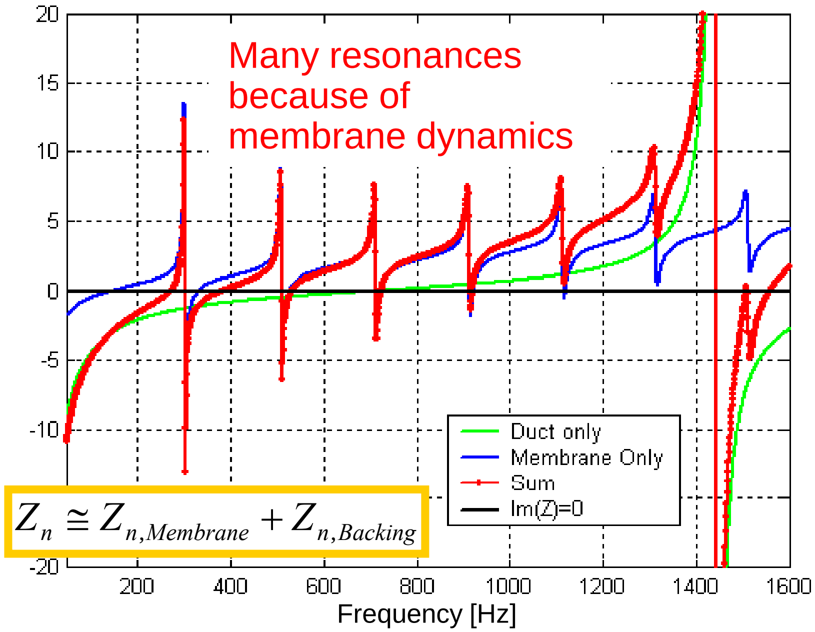




$a$=0.05 m, $\rho_s$=0.174 kg/m2, $To$=85 N/m, $\eta$=0.005, $\Omega$=0.0018, $R_f$=0.15 Rayls, $h$=0.2 mm

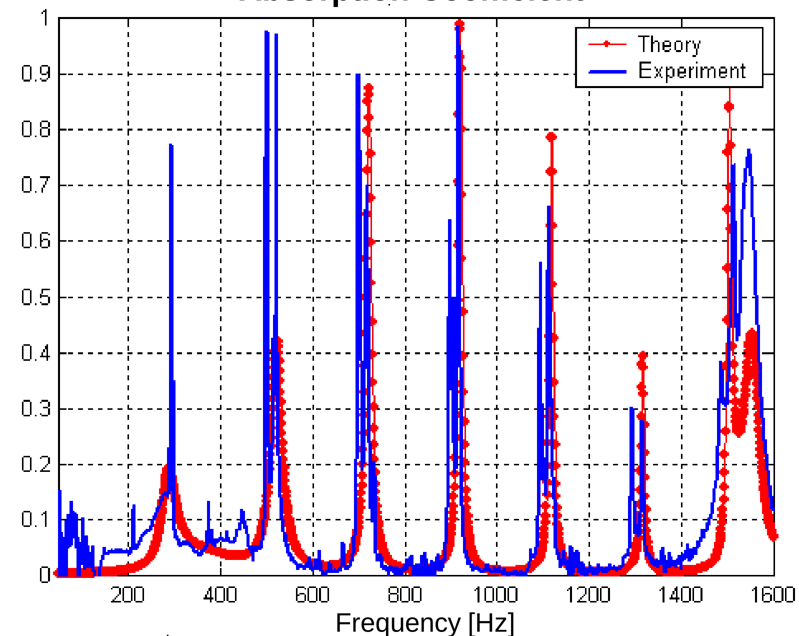
# Sound Dissipation Mechanism – Impermeable Membrane

- Membrane with finite-depth backing space

Imaginary part of Impedance




Absorption Coefficient

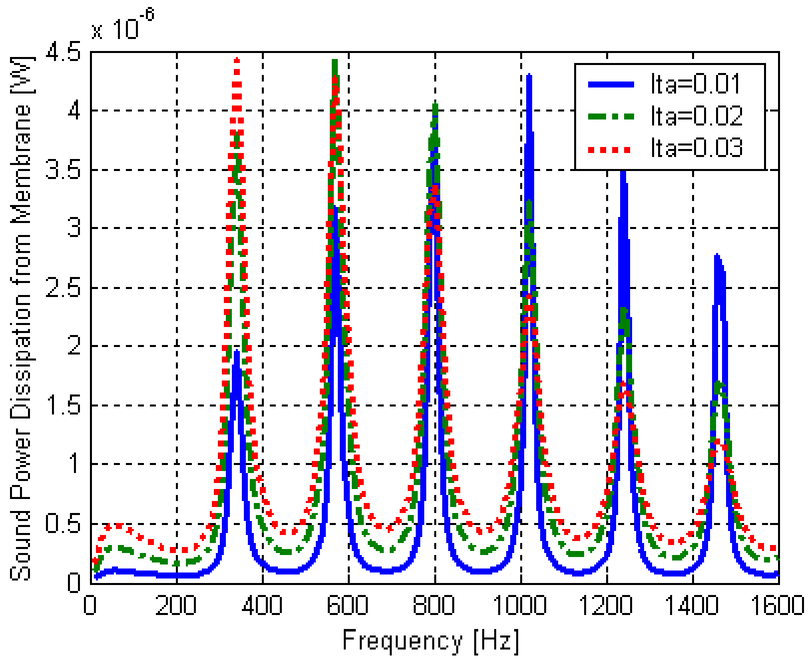



$T = 75Pa \quad \eta = 0.0063 \quad \rho_s = 0.19 \, {}^{kg}/_{m^2} \quad l = 0.1192m$

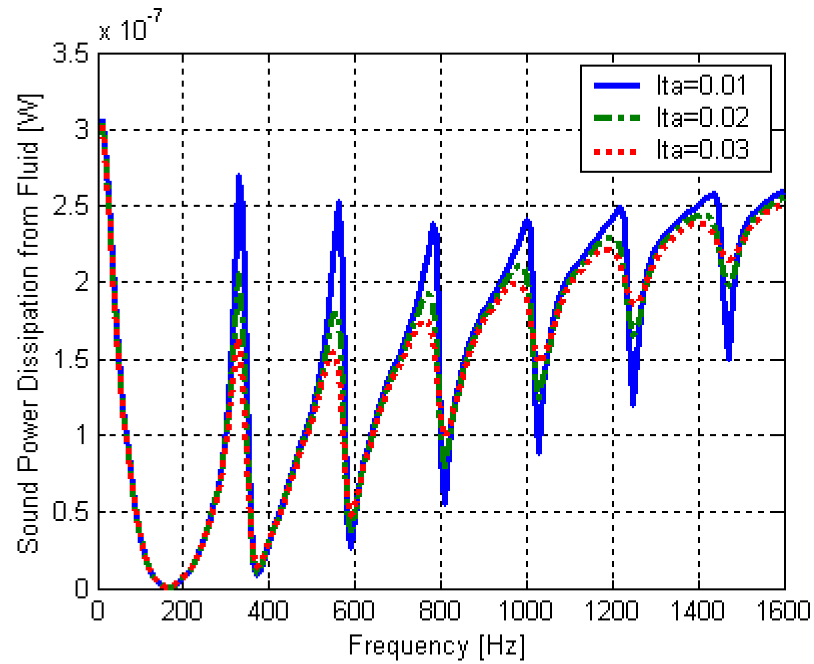
- Absorption peaks when Im{ $Z_n$ } = 0
- Significant sound absorption in narrow frequency regions produced by dissipation in the membrane

# Sound Dissipation Mechanism –
# Effect of Membrane Loss Factor

Sound Power Dissipation from Membrane



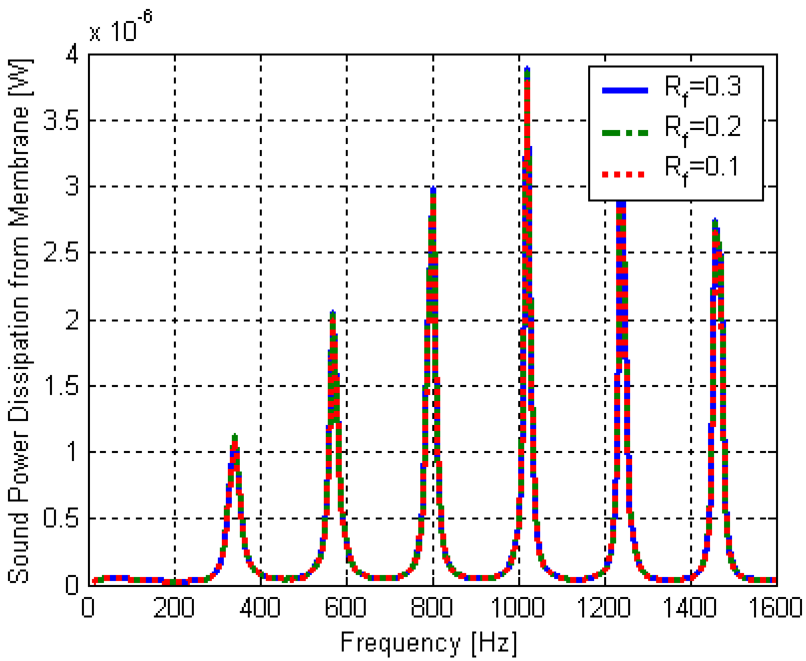

Sound Power Dissipation from Fluid



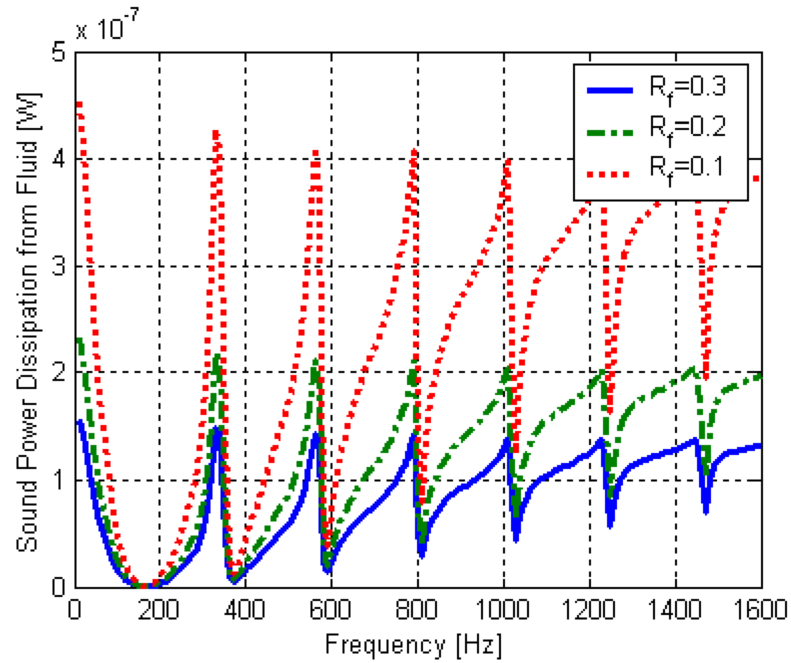

$a$=0.05 m, $\rho_s$=0.174 kg/m*2, $T_o$=85 N/m, Ω*=0.0018, $R_f$=0.15 Rayls, $h$=0.2 mm

# Sound Dissipation Mechanism –
# Effect of Flow Resistance

Sound Power Dissipation from Membrane



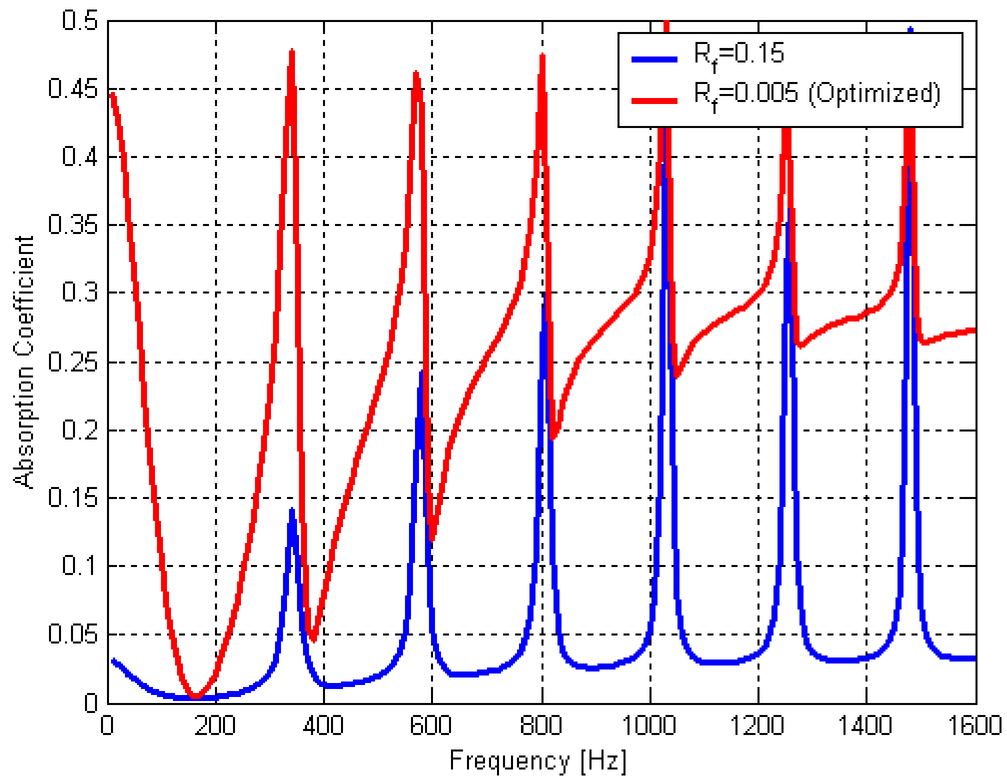

Sound Power Dissipation from Fluid




$a$=0.05 m, $\rho_s$=0.174 kg/m2, $T_o$=85 N/m, $\eta$=0.005, $\Omega$=0.0018, $h$=0.2 mm

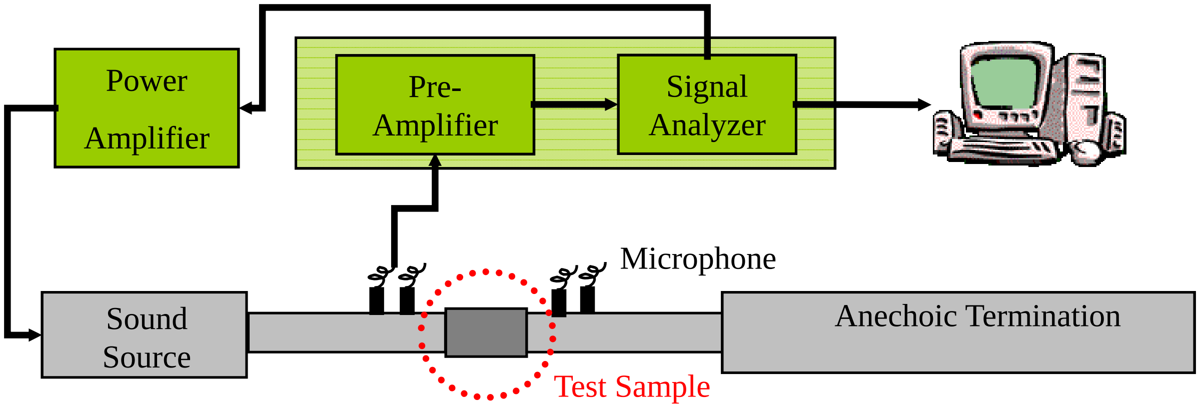
## Sound Dissipation Mechanism –
# "Optimized" Flow Resistance

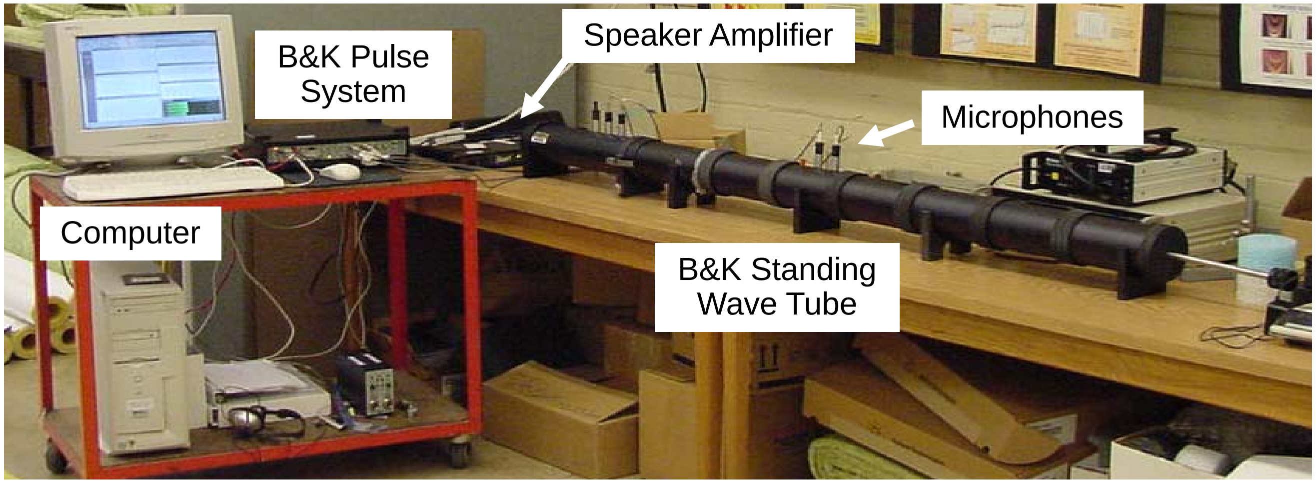
* The maximum sound power dissipation in the higher frequency range can be designed by choosing an "optimized" flow resistance.




$a$=0.05 m, $\rho_s$=0.174 kg/m2, $T_o$=85 N/m, $\eta$=0.005, $\Omega$=0.0018, $h$=0.2 mm

# Experimental Set-up



# Acoustical Properties : Theory & Test

Measuring Sound Pressure : $P_1$ to $P_4$

Calculation of Normal Incident Pressure Transmission Coeff. & Reflection Coeff.: T, R

**Experimental Results**

- Transmission Loss:

  $TL_t$=20 log10 (1/|T|)

- Absorption Coeff.: $\alpha_t=1-|R|^2$
- Dissipation Coeff.: $\alpha_{d,t}=1-|R|^2-|T|^2$

Given experimental results as input, find appropriate material properties (e.g., $T_o$ , $\rho_s$ , $\eta$ )

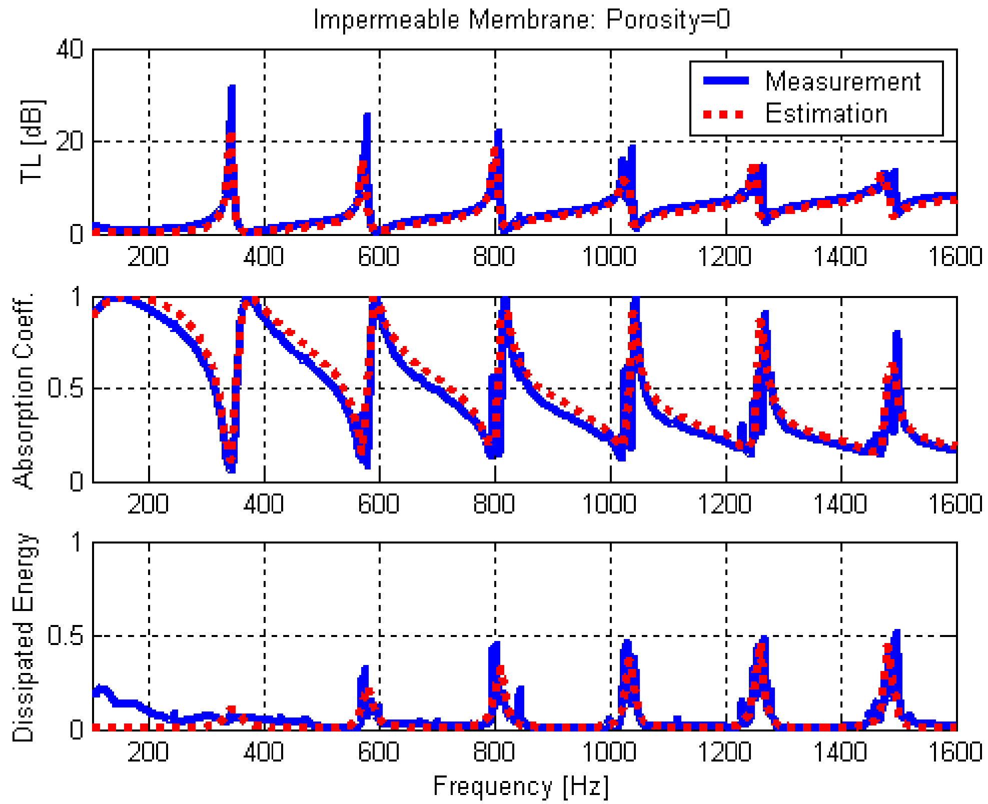
**Theoretical Estimation**

- Transmission Loss: $TL_e$
- Absorption Coeff.: $\alpha_e$
- Dissipation Coeff.: $\alpha_{d,e}$

# Model Optimization

- Given experimental results as input, Find appropriate material properties ($T_o$, $\rho_s$, $\eta$ )

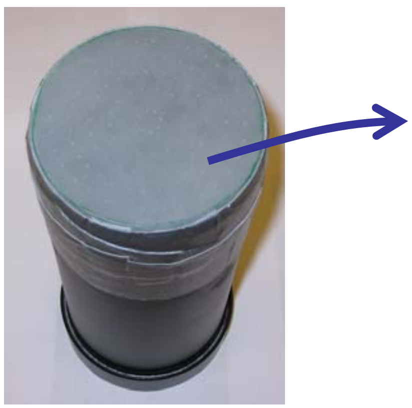
Note : Most "absorption" results from transmission through membrane in anechoic termination case



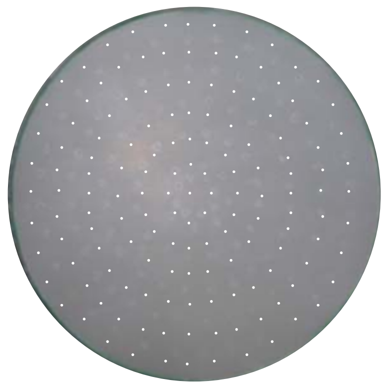

$\rho_s$=0.174 kg/m*2, $T_o$=85.41 N/m,* $\eta$=0.005

- Objective function : $$f=\sum_{n=1}^{N}\left[\left(TL_e-TL_t\right)^2+\kappa\left(\alpha_e-\alpha_t\right)^2\right]=f(T_o,\rho_s,\eta)$$

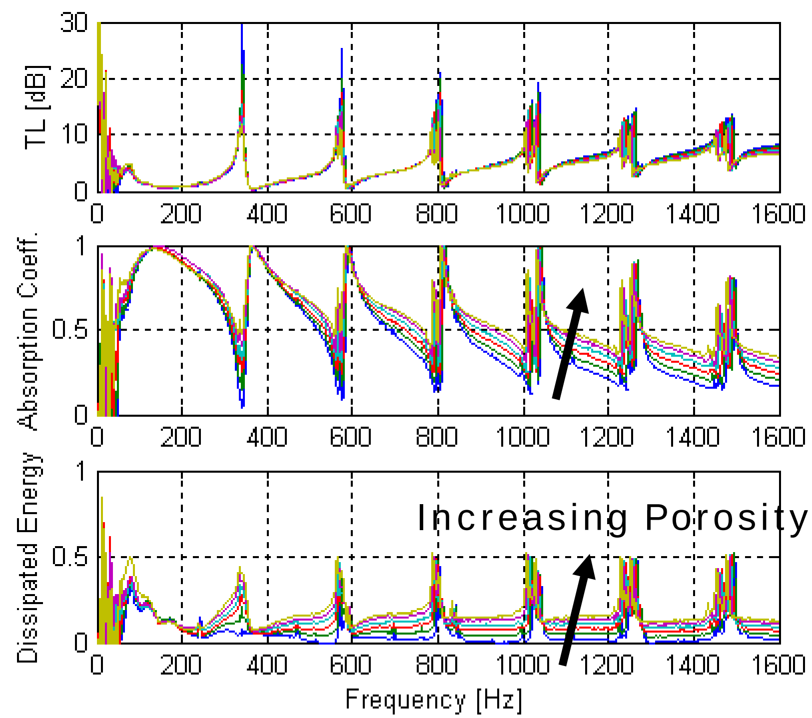
# Membrane Permeability: Measurement



A duct with a tensioned, permeable membrane

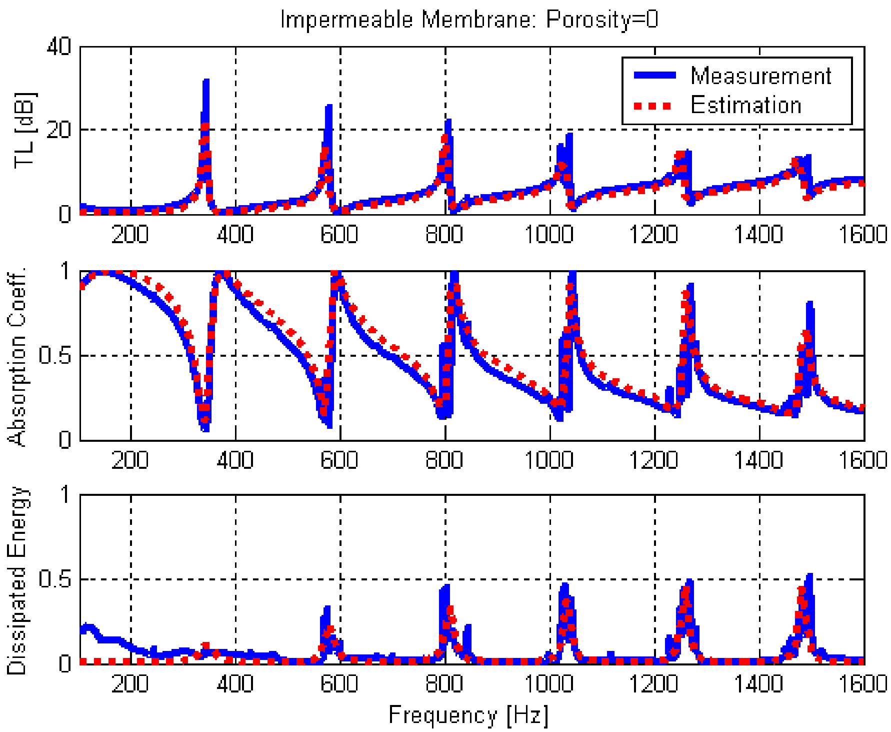


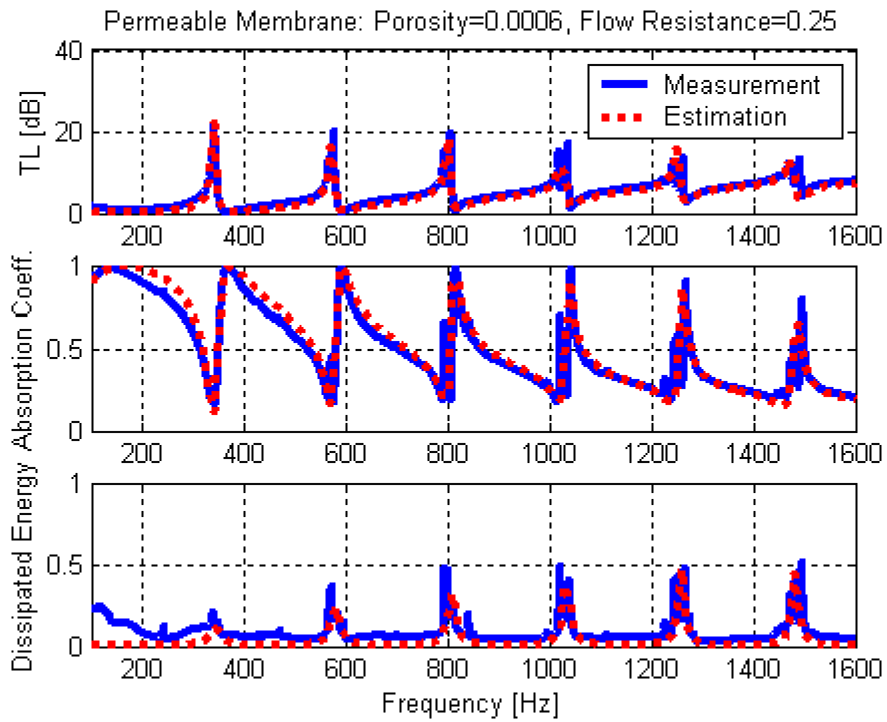
Membrane



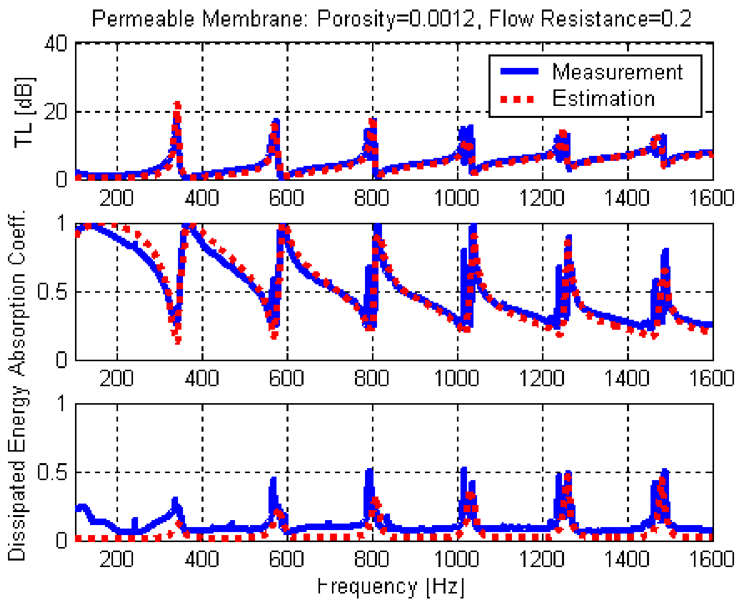

- Each pore had a diameter of approximately 0.5 mm and the total number of pores ranged from 24 to 120 increasing in steps of 24: the pores were uniformly distributed over the membrane.

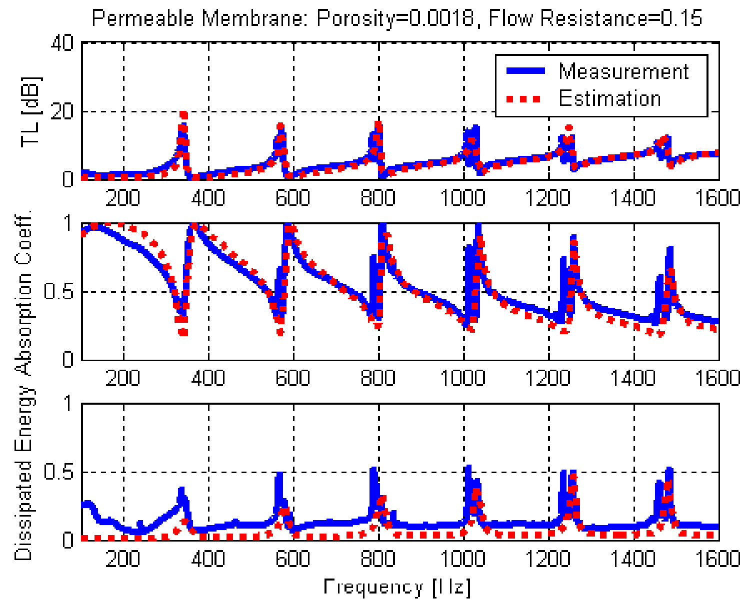
# Membrane Permeability: Comparison of Measurement and Prediction

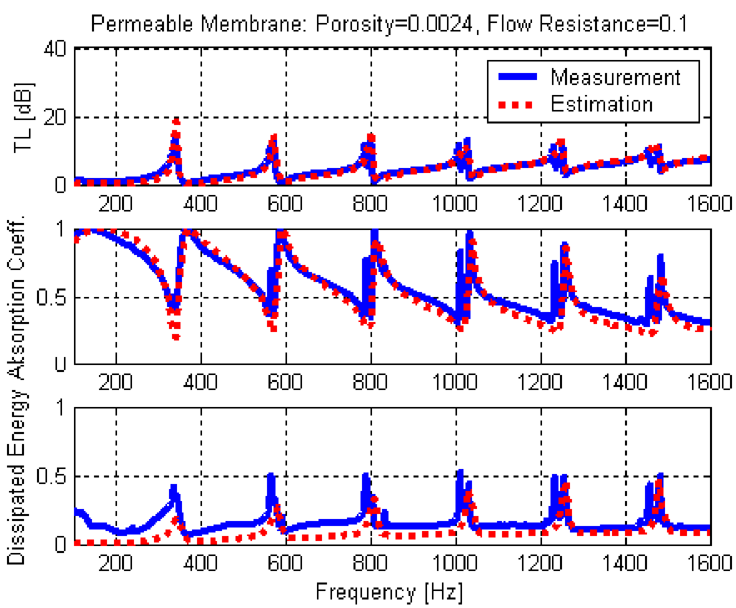


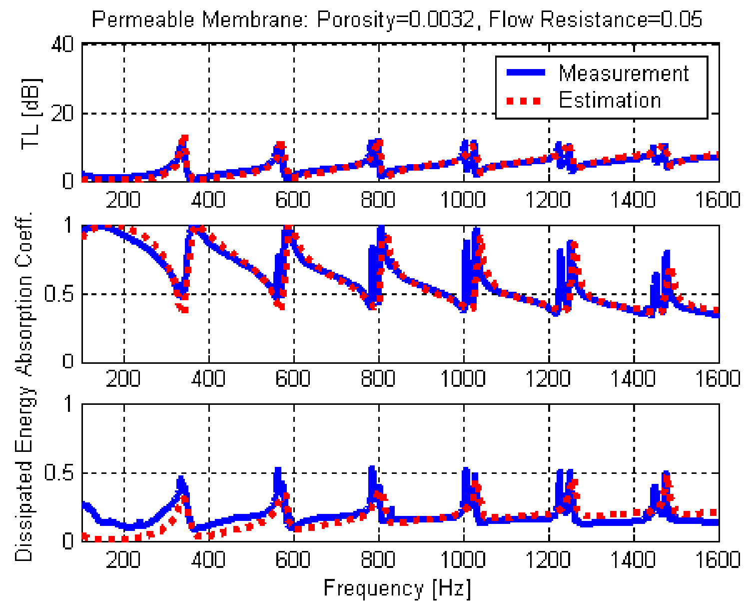




* The acoustical properties were predicted by using estimated porous material parameters (i.e., porosity, flow resistance, and membrane thickness) in combination with the impermeable membrane parameters (i.e., tension, loss factor, and membrane surface density) found by the optimization procedure

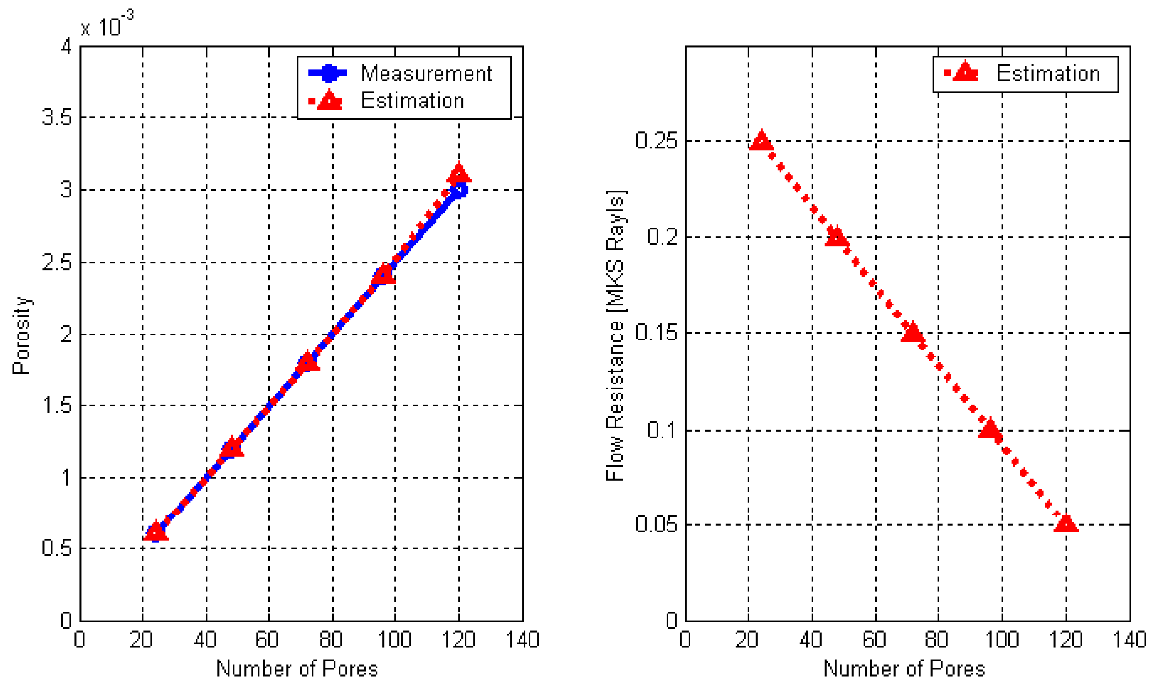

Permeable Membrane: Porosity=0.0012, Flow Resistance=0.2
Measurement
Estimation
TL [dB]
Dissipated Energy Absorption Coeff.
Frequency [Hz]


Permeable Membrane: Porosity=0.0018, Flow Resistance=0.15
Measurement
Estimation
TL [dB]
Dissipated Energy Absorption Coeff.
Frequency [Hz]


Permeable Membrane: Porosity=0.0024, Flow Resistance=0.1
Measurement
Estimation
TL [dB]
Dissipated Energy Absorption Coeff.
Frequency [Hz]


Permeable Membrane: Porosity=0.0032, Flow Resistance=0.05
Measurement
Estimation
TL [dB]
Dissipated Energy Absorption Coeff.
Frequency [Hz]

# Membrane Permeability: Porosity & Flow Resistance

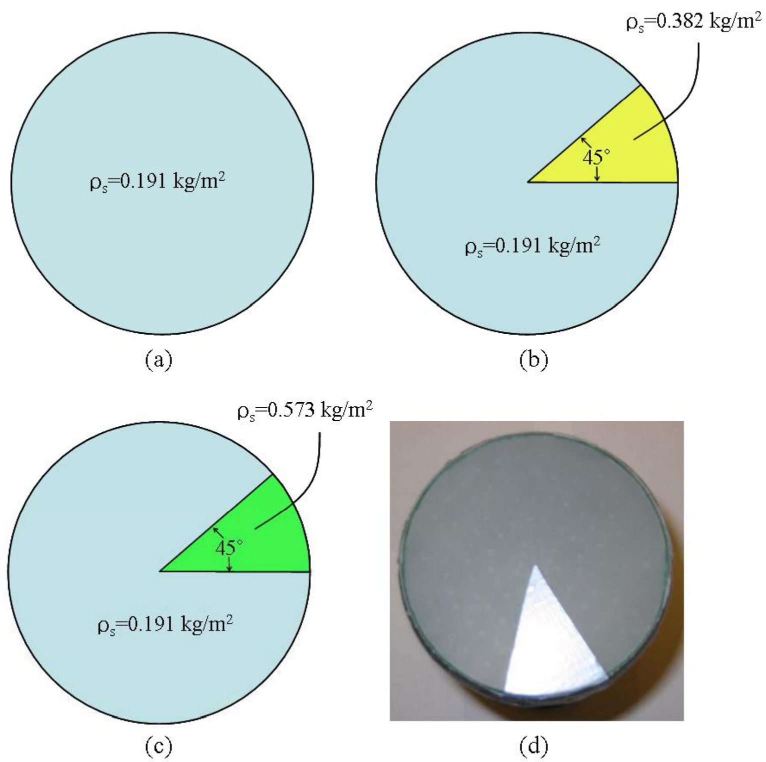



* Measured Porosity = Membrane Pore Area / Membrane Total Area

- Consistent Permeable Membrane Models
- Physically reasonable parameter estimates

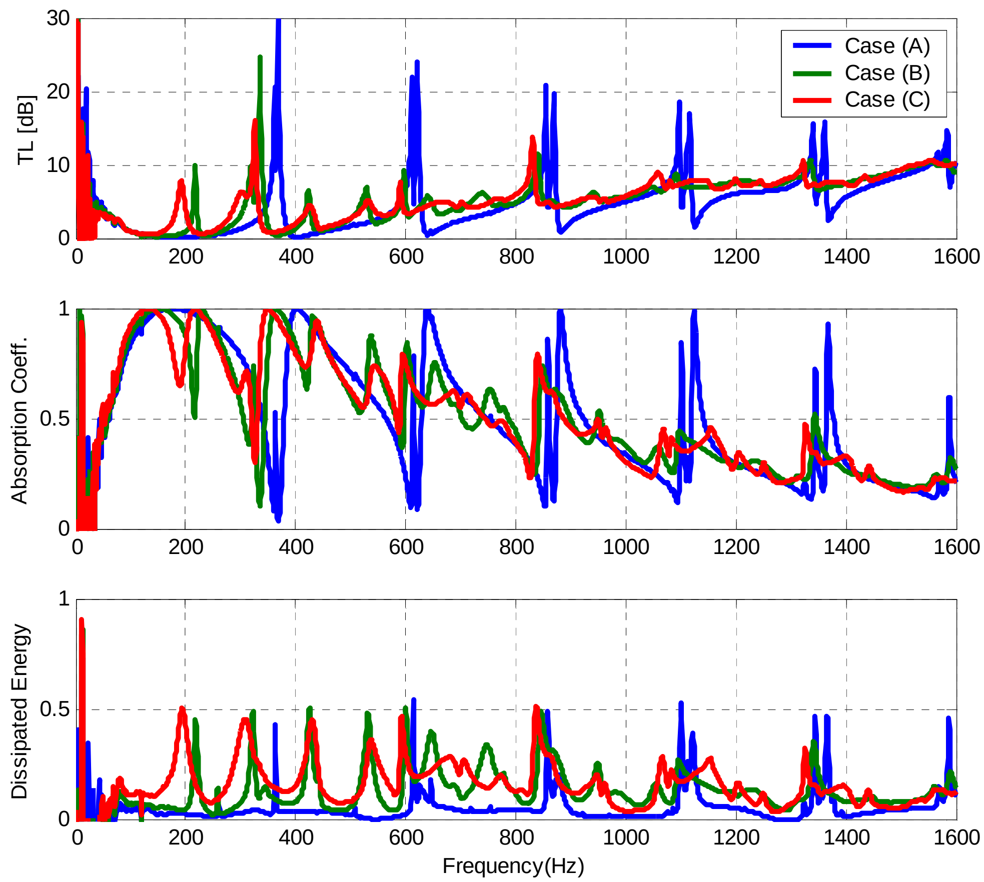
# Mode Splitting Effect: Measurement

## Non-Uniform Membrane



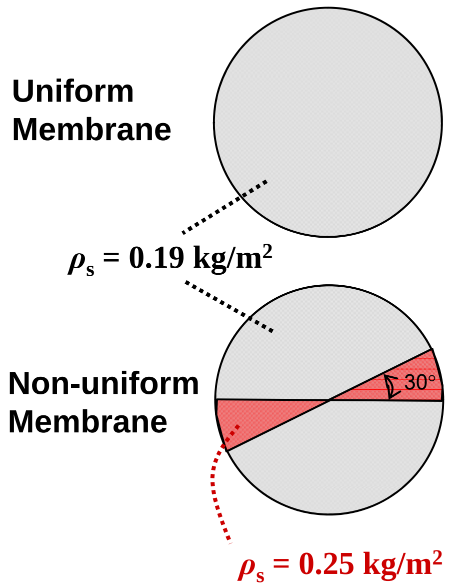

- The attached tape affected both the membrane density and the loss factor because of the viscoelastic nature of the tape.

## Comparison of Acoustical Properties

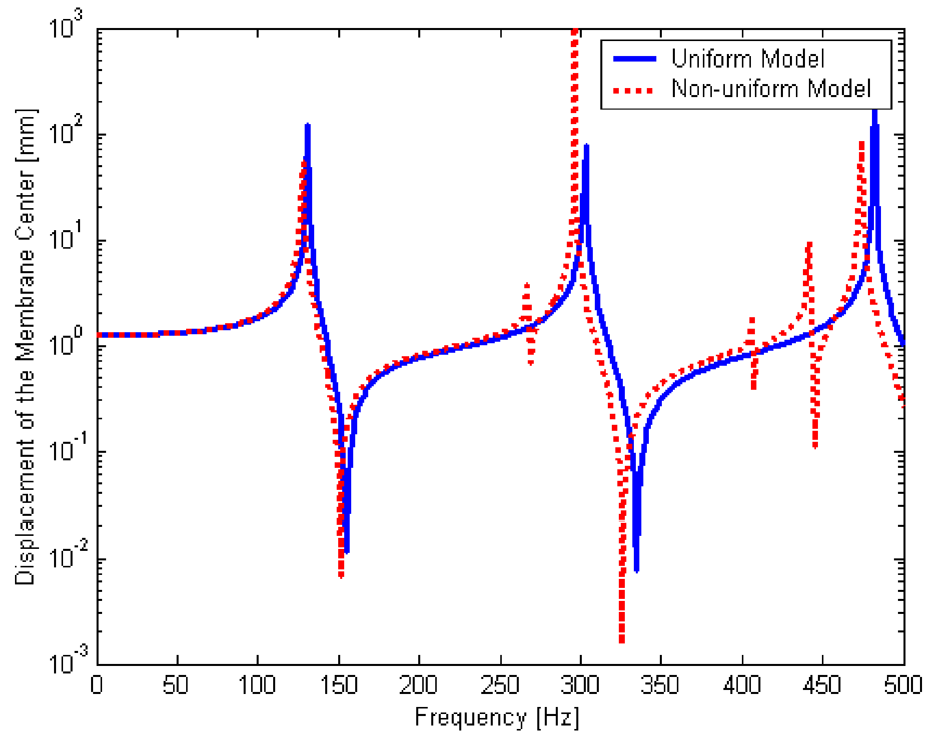


# Mode Splitting Effect: ANSYS

ANSYS Membrane Models




Comparison of Forced Responses




- As the non-uniformity of the membrane density becomes larger, additional peaks occur, which can be accounted for by the mode splitting phenomenon
- This work supports the assumption that the mode splitting phenomenon that occurred in the various measured data resulted from membrane inhomogeniety.

# Conclusions

- In the present work, **an acoustical model for a tensioned, permeable membrane was presented.** Specifically, the effect of membrane porosity on the sound absorption was investigated both theoretically and experimentally. Excellent agreement was found between measured and predicted values.

- It was found that **permeable membranes can dissipate a significant amount of energy due both to vibration of the membrane and oscillatory flow through the membrane**. The energy dissipation related to membrane vibration occurs in narrow frequency regions associated with membrane resonances while the energy dissipation associated with oscillatory fluid flow through the membrane occurs over a broad range of frequencies and thus supplements the former energy dissipation mechanism.

- The model presented here can provides **the foundation necessary to design membrane-based sound absorbing systems** having enhanced sound dissipation capacity.

- It was verified that **the mode splitting visible in the measured results is consistent with slight membrane inhomogeneity**. The latter indicated that any resonance mechanism that creates family of resonances could account for energy dissipation seen in membrane systems.