Sound Absorption Characteristics of Membrane-Based Sound Absorbers

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Sound Absorption Characteristics of Membrane-Based Sound Absorbers

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Motivation

- Recently, it has been observed that
  - Macro-cellular polyolefin foams (e.g., Quash-like) absorb sound energy even though the foams are mostly closed-celled and the average cell size is very large.

- How does this sound absorption arise?
- How do you model this effect?


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Quash Membrane Model

- To this point, model is based on tensioned membranes
- Stiffness of this model is provided by tension of membrane

Top View of Quash
Foam Modeling Procedure

3-D Model

Unit cell

Membrane

Rigid Frame

2-D Model

Sound Energy Loss Mechanisms

- Energy dissipation by membrane flexure
- Viscous loss through perforation
- Thermo-viscous boundary layer effect

Material Properties
- Tension
- Loss factor
- Membrane Density
- Surface Film Density
- Porosity
- Flow Resistance

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Theoretical Model - Permeable Membrane

Assumed Solutions

- Sound Pressures in Acoustic Cavities:
  \[ P_I(r, z) = e^{-jkz} + \sum_{n} B_n J_0(k_{r_n} r)e^{jk_{z_n}z} \]
  \[ P_{II}(r, z) = \sum_{n} C_n J_0(k_{r_n} r)e^{-jk_{z_n}z} \]

- Membrane Displacement (Solid Component):
  \[ y(r, t) = \sum_{n} A_n J_0(k_{0_n} r) \]

- Membrane Displacement (Fluid Component):
  \[ u(r, t) = \sum_{n} F_n J_0(k_{0_n} r) + F_o \]
Theoretical Model – Solution Method

- **Boundary Conditions**

  - The Continuities of Velocity at the Both Side of a Membrane:
    
    \[
    - \frac{1}{j\omega\rho_o} \left. \frac{\partial P_i}{\partial z} \right|_{z=0} = (1 - \Omega) \frac{\partial v}{\partial t} + \Omega \frac{\partial u}{\partial t}
    \]
    
    \[
    - \frac{1}{j\omega\rho_o} \left. \frac{\partial P_{II}}{\partial z} \right|_{z=0} = (1 - \Omega) \frac{\partial v}{\partial t} + \Omega \frac{\partial u}{\partial t}
    \]

  - The Force Equilibrium Equation in the Membrane:
    
    \[
    \nabla^2 y - \frac{\rho_s \partial^2 y}{T \partial t^2} - R_f \frac{\partial (y-u)}{\partial t} = \frac{1}{T} (P_i - P_{II})
    \]
    
    \[
    \rho_o \Omega h \frac{\partial^2 u}{\partial t^2} - R_f \frac{\partial (y-u)}{\partial t} = \Omega (P_i - P_{II})
    \]

    where
    
    \[ T = T_o (1 + j \eta) \quad \Omega = N \pi a^2 / A \quad P_{\text{front}} - P_{\text{back}} = R_f v_f \]

- **Solution Method**

  Apply four boundary conditions on a point-by-point basis across the membrane

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Energy dissipation by Membrane

Measurement

Sound Pressure from microphones

Estimation

Fourier Bessel coeff. of Incident / Transmitted Waves

Normal Incident Reflection coeff.: \( R \)

Transfer Impedance of Membrane:

\[
Z_m = \frac{(1+R)}{(1-R)} - \rho_o c
\]

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Transfer Matrix Method

• Membrane & Air Cavity Transfer Matrix

\[
[T_m] = \begin{pmatrix} 1 & Z_m \\ 0 & 1 \end{pmatrix} \quad [T_a] = \begin{pmatrix} \cos(k \cdot (l_2 - l_1)) & i\rho_o c \sin(k \cdot (l_2 - l_1)) \\ \frac{i}{\rho_o c} \sin(k \cdot (l_2 - l_1)) & \cos(k \cdot (l_2 - l_1)) \end{pmatrix}
\]

• Total Transfer Matrix

\[
\left\{ \begin{array}{c} P_{Top} \\ u_{Top} \end{array} \right\} = [T_{total}] \left\{ \begin{array}{c} P_{Bottom} \\ u_{Bottom} \end{array} \right\} \quad [T_{total}] = [T_m] [T_a] [T_m] [T_a] \quad \ldots \ldots \quad [T_m] [T_a]
\]

• Reflection Coefficient

\[
R_{Top} = \frac{Z_{Top} - \rho_o c_o}{Z_{Top} + \rho_o c_o}
\]

• Absorption Coefficient

\[
\alpha = 1 - |R_{Top}|^2
\]
Experimental Set-up

Power Amplifier

B&K Pulse System
  Signal Analyzer
  Pre-Amplifier

Computer

Sound Source

Microphone

Standing Wave Tube

Quash Sample
Experimental Set-up

Loading Quash Sample

Measurement Set-up

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Comparison of Measurement and Prediction

\[ \rho_m = 0.0885 \text{ kg/m}^2, \ T_o = 0.13 \text{ N/m}, \ \eta = 1.6, \ m_s = 0.1586 \text{ kg/m}^2, \ \Omega = 0.0085, \]
\[ R_f = 0.286 \text{ Rayls}, \ t = 0.0002 \text{ m}, \ d_o = 0.00486 \text{ m}, \ h = 0.05832 \text{ m}, \ N = 12. \]
Parameter Effects on Sound Absorption

Foam Thickness

Membrane Size

\[ \rho_{m} = 0.07 \text{ kg/m}^2, \ T_0 = 0.5 \text{ N/m}, \ \eta = 0.7, \ m_s = 0.2 \text{ kg/m}^2, \ \Omega = 0.01, \ R_f = 0.2 \text{ Rayls}, \ t = 0.0002 \text{ m}, \ d_o = 0.0059 \text{ m}, \]

\[ \rho_{m} = 0.07 \text{ kg/m}^2, \ T_0 = 0.5 \text{ N/m}, \ \eta = 0.7, \ m_s = 0.2 \text{ kg/m}^2, \ \Omega = 0.01, \ R_f = 0.2 \text{ Rayls}, \ t = 0.0002 \text{ m}, \ N = 10 \]
Parameter Effects on Sound Absorption

Surface Mass

Membrane Porosity

\[ \rho_m = 0.07 \text{ kg/m}^2, \; T_o = 0.5 \text{ N/m}, \; \eta = 0.7, \; \Omega = 0.01, \; R_f = 0.2 \text{ Rayls}, \; t = 0.0002 \text{ m}, \; d_o = 0.0059 \text{ m}, \; N = 10 \]

\[ \rho_m = 0.07 \text{ kg/m}^2, \; T_o = 0.5 \text{ N/m}, \; \eta = 0.7, \; m_s = 0.07 \text{ kg/m}^2, \; R_f = 0.2 \text{ Rayls}, \; t = 0.0002 \text{ m}, \; d_o = 0.0059 \text{ m}, \; N = 10 \]
Parameter Effects on Sound Absorption

$m_s=0.2135 \text{ kg/m}^2$, $\Omega=0.0$, $R_f=0.2$ Rayls, $t=0.0002 \text{ m}$, $d_o=0.0056 \text{ m}$, $h=0.056 \text{ m}$, $N=12$

(1/3-octave band normal incidence absorption averaged from 125 Hz to 1600 Hz)
Some High Absorption Designs

New Design 1
\[ \rho_m = 0.2213 \text{ kg/m}^2 (\uparrow), \]
\[ T_o = 0.065 \text{ N/m (\downarrow), } \eta = 1.6, \]
\[ m_s = 0.1941 \text{ kg/m}^2 (\uparrow), \Omega = 0.02 (\uparrow), \]
\[ R_f = 0.286 \text{ Rayls, } t = 0.0002 \text{ m, } \]
\[ d_o = 0.00583 \text{ m (\uparrow), } h = 0.0583 \text{ m, } \]
\[ N = 10 (\downarrow) \]

New Design 2
\[ \rho_m = 0.1770 \text{ kg/m}^2 (\uparrow), \]
\[ T_o = 0.13 \text{ N/m, } \eta = 1.6, \]
\[ m_s = 0.0970 \text{ kg/m}^2 (\downarrow), \Omega = 0.03 (\uparrow), \]
\[ R_f = 0.286 \text{ Rayls, } t = 0.0002 \text{ m, } \]
\[ d_o = 0.00583 \text{ m (\uparrow), } h = 0.0583 \text{ m, } \]
\[ N = 10 (\downarrow) \]
Conclusions & Future Work

- An acoustical model for membrane-based sound absorbing materials was presented and was verified experimentally on the basis of acoustical measurements.

- It has been found that the theoretical model can accurately reproduce the acoustical behavior of the particular foam studied here.

- It was shown that the choice of particular combinations of material properties can result in improved sound absorption.

- The present work can provide the foundation necessary to design membrane-based sound absorbing materials having enhanced sound absorption capacity.

- The present work implies that alternative stiffness mechanisms of membrane systems such as flexural stiffness, membrane curvature, bulk elasticity, and membrane inhomogeneity, can also result in sound dissipation in membrane-based foams; this work will be presented in the future.

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