Analysis of Tire Vibration by Using a Hybrid Two-Dimensional Finite Element Based on Composite Shell Theory

J Stuart Bolton
Purdue University, bolton@purdue.edu

Yong-Joe Kim

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Analysis of Tire Vibration by Using a Hybrid Two-Dimensional Finite Element Based on Composite Shell Theory

Yong-Joe Kim and J. Stuart Bolton
Ray W. Herrick Laboratories
Purdue University
USA
Contents (2-D FE)

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Literature Review

• Richards (1991)
  – Hybrid 2-D FE based on membrane.
  – Vibration model coupled with tire’s internal acoustic cavity.
• Brockman & Champion (1992)
  – Hybrid 2-D FE based on linear elastic solid in cylindrical coordinates.
  – Rotation effects of a tire
  – Non-linear strains for inflation pressure
• Nilsson & Finnveden (2002)
  – Hybrid 2-D FE based on circular conical shell.
• Kim and Bolton (2003)
  – Hybrid 2-D FE based on composite, circular conical shell
  – Thickness variation
  – Non-linear strains for inflation pressure
Definition of Hybrid 2-D FE Model

- Circular Symmetry of Structural Characteristics
  - Same cross-sectional structure around circumference
  - Modeling of cross-section
- Response
  - Function of circumferential coordinate
- Hybrid 2-D FE model includes circular symmetric FE model which has constant cross-sectional displacement around circumference.
- The cross-section is approximated by 2-D finite elements while an analytical wave solution is assumed in the circumferential direction.

⇒ Efficient Modeling and Computation
Basic Concept

- Finite Element approximation in $x$-direction and analytical solution in $\theta$-direction

A Tire

2-D FE element (Circular Conical Shell)
Element Geometry

Node 1

Node 2

\[ x_1 \]

\[ x_3 \]

\[ x_2 \]

\[ X_1 \]

\[ X_3 \]

\[ R_2 \]

\[ A_2 \]

\[ a \]

\[ \phi \]

\[ L \]
Displacements

\[ \begin{bmatrix} u_1(x, \theta, t) \\ u_2(x, \theta, t) \\ u_3(x, \theta, t) \end{bmatrix} = \begin{bmatrix} \chi_{11}(x) & \chi_{12}(x) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \chi_{21}(x) & \chi_{22}(x) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \chi_{31}(x) & \chi_{32}(x) & \chi_{33}(x) & \chi_{34}(x) & \end{bmatrix} \begin{bmatrix} a_n(\theta, t) \\ a_{n+1}(\theta, t) \\ b_n(\theta, t) \\ b_{n+1}(\theta, t) \\ c_n(\theta, t) \\ d_n(\theta, t) \\ c_{n+1}(\theta, t) \\ d_{n+1}(\theta, t) \end{bmatrix} \]

\[ \Rightarrow u = \chi \psi \]

Nodal displacements

Shape functions

Node, \( n \)

Node, \( n+1 \)

\( x_2 = \theta \)
FE Formulation

- Displacement
  \[ u(x, \theta, t) = \chi(x)y(t, \theta) \]

- Strain and displacement (geometric relation)
  \[ e(x, \theta, t) = E_0(x)y(\theta, t) + E_1(x)\frac{\partial y(\theta, t)}{\partial \theta} + E_2(x)\frac{\partial^2 y(\theta, t)}{\partial \theta^2} + \text{(nonlinear terms)} \]

- Resultant force and strain (layered composite shell)
  \[ R = Ce \quad \text{(Eq. 5.5)} \]

- Variational principle
  \[ \delta U = \frac{1}{2} \int \int \int R^T \delta e A_x A_0 dxd\theta dt \]
  \[ \delta(U - T - W_q - W_f) = 0 \]
Initial Stresses (Inflation Pressure)

- Nonlinear strains should be considered.
  \[
  \varepsilon_{11}^0 = \varepsilon_{11}^l + \varepsilon_{11}^n \\
  \varepsilon_{22}^0 = \varepsilon_{22}^l + \varepsilon_{22}^n \\
  \varepsilon_{12}^0 = \varepsilon_{12}^l + \varepsilon_{12}^n
  \]

- Potential energy associated with initial stresses
  \[
  \delta U^n = \iiint \left( N_{11} \varepsilon_{11}^n + N_{22} \varepsilon_{22}^n + N_{12} \varepsilon_{12}^n \right) A_1 A_2 dx_1 dx_2 dt
  \approx \iiint \left( N_{11}^0 \varepsilon_{11}^n + N_{22}^0 \varepsilon_{22}^n + N_{12}^0 \varepsilon_{12}^n \right) A_1 A_2 dx_1 dx_2 dt
  \]
  \(\varepsilon_{ij}^n\) : nonlinear strains
  \(N_{ij}^0\) : initial resultant forces

- Initial resultant forces should be obtained from static analysis
Equation of Motion

- **Element equation**

\[
\sum_{m=0}^{4} K_m \frac{\partial^m y}{\partial \theta^m} + \sum_{m=0}^{2} K^0_m \frac{\partial^m y}{\partial \theta^m} + M \frac{\partial^2 y}{\partial t^2} = F^e + F^i + Q^e
\]

- **Global system equation (global matrix assembly)**

\[
\sum_{m=0}^{4} K_m \frac{\partial^m y}{\partial \theta^m} + \sum_{m=0}^{2} K^0_m \frac{\partial^m y}{\partial \theta^m} + M \ddot{y} = F^e + Q
\]
Procedure

Stiffness matrices \((K_m, m = 0, 1, \ldots, 4)\)
Distributed force for inflation pressure \((Q)\)

\[ K_m Q \]

Static Analysis:
Initial displacement \((y^0)\) and initial resultant force \((R^0)\)

\[ R^0 \]

Stiffness matrices \((K^0_m)\) associated with initial force
Mass matrix \((M)\)
Dynamic force \((F\) and \(Q)\)

\[ K_m K^0_m M F Q \]

Dynamic Analysis:
Free and forced vibration
Tire Treadband Model

- CIRCULAR CYLINDRICAL SHELL MODEL
  - Constant thickness
  - Isotropic, orthotropic, and composite materials
  - Inflation pressure
  - Simply supported at both treadband edges
Full Tire Model

- Orthotropic material properties
- Inflation pressure
- Zero translational displacements at both sidewall edges
Isotropic Treadband Model

Isotropic model with the inflation pressure of 20 psi
(a) Analytical model
(b) 3-D FE model (SHELL63)
(c) 3-D FE model (SHELL181)
(d) 2-D FE model
SHELL63 - Elastic Shell
SHELL181 - Finite Strain Layered Shell

\[ E = 4.8 \times 10^8 \text{ Pa} \]
\[ \nu_{12} = 0.45 \]
\[ h = 0.008 \text{ m} \]
\[ \rho = 1200 \text{ kg/m}^3 \]
Orthotropic Treadband Model

Orthotropic model with the inflation pressure of 20 psi
(a) 3-D FE model (SHELL63)
(b) 3-D FE model (SHELL181)
(c) 3-D FE model (SHELL99)
(d) 2-D FE

SHELL99 - Linear Layered Structural Shell

\[ E_1 = 3.2 \times 10^8 \text{ Pa} \]
\[ E_2 = 7.5 \times 10^8 \text{ Pa} \]
\[ \nu_{12} = 0.45 \]
\[ h = 0.008 \text{ m} \]
\[ \rho = 1200 \text{ kg/m}^3 \]
Composite Treadband Model (Forced Response)

Composite model with the inflation pressure of 20 psi
(a) 3-D FE model (SHELL99)
(b) 2-D FE
Static Analysis (2-D FE Full Tire Model)
Natural frequencies and circumferential mode numbers
(a) without inflation and
(b) with inflation
Forced Results (Full Tire Model)

Experiment

3-D

2-D

Without Inflation

With Inflation
Conclusions

• A hybrid 2-D finite element for a composite shell with initial stresses and thickness variation was formulated by using the Variational Principle.
• Tire treadband model and full tire model were analyzed by using the hybrid 2-D finite elements and 3-D finite elements.
• It has been shown that the results obtained from the hybrid 2-D FE analysis agree with those obtained from 3-D FE analysis.
• This procedure offers an efficient tool for analyzing the dynamics of tires at the design stage.
• Future work
  – Rotation effects
  – Coupling with tire’s internal acoustic cavity