Optimization of Acoustical Systems Incorporating Fibrous Elements for Sound Absorption

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Optimization of Acoustical Systems Incorporating Fibrous Elements For Sound Absorption

by

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Outline

- Introduction
- Limp Porous Material Model
- Experimental Verification
- Optimization Examples
- Conclusions
Introduction

- Porous materials are rigid, elastic or limp

- "Limp" porous material
  - in vacuo bulk stiffness less than that of air

- Fibers move as a result of viscous and inertial coupling

- The validated limp theory can be used to optimize the acoustical properties of fibrous materials
Limp Porous Material Model

- Elastic Porous Model (Based on the Biot Theory)

\[ \sigma_i = 2Ne_i + Ae_s + Q\epsilon, \quad i = x, y, z \]

\[ s = Qe_s + R\epsilon \]

\[ \tau_{ij} = \tau_{ji} = N\gamma_{ij}, \quad i, j = x, y, z \]
Limp Porous Material Model

- Assume
  - in vacuo bulk modulus of solid phase is zero
  - bulk modulus of fiber material much greater than that of air

\[ P = \frac{(1-\phi)^2}{\phi} K_f , \quad Q = (1-\phi)K_f , \quad R = \phi K_f \quad \Rightarrow \quad PR - Q^2 = 0 \]

- \(PR - Q^2 = 0\) causes singularity in Elastic Model and reduces 2 waves equations to single 2nd order equation

- Input Parameters
  1. Flow resistivity
  2. Porosity
  3. Tortuosity
  4. Bulk elasticity

(rigid and limp models)

(elastic model)
Limp Porous Material Model

Wave Equations
- 4th order
- 2nd order

Single 2nd Order Wave Equation
Efficient and Stable

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Evidence of Limp Behavior

Rigid Model
\[ \sigma = 3.5 \times 10^4 \text{ Rayls/m} \]
\[ \phi = 0.99 \]
\[ q^2 = 1.2 \]

Limp Model
\[ \rho_b = 15.97 \text{ kg/m}^3 \]

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Transfer Matrix Method

\[
\begin{bmatrix}
  p_1 \\
  v_{1x} \bigg|_{x=0}
\end{bmatrix}
= \begin{bmatrix}
  T_{11} & T_{12} \\
  T_{21} & T_{22}
\end{bmatrix}
\begin{bmatrix}
  p_2 \\
  v_{2x} \bigg|_{x=d}
\end{bmatrix}
\]

- \([T]_{\text{composite}} = [T]_1[T]_2 \ldots [T]_n\)
- \(\alpha\) and \(TL\) may be calculated when \([T]_{\text{composite}}\) is known
- Air spaces, porous layers, limp impermeable layers, limp resistive layers, elastic panels

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Optimization Examples

- Accurate models allow treatments to be optimized automatically
- Optimum?
  - Specified level of acoustical performance at minimum weight, thickness and cost
- Examples
  - optimal performance for a given thickness
  - vary flow resistivity and basis weight

\[
NRC = \frac{\alpha_{250} + \alpha_{500} + \alpha_{1000} + \alpha_{2000}}{4}
\]

Incident Sound
- Scrim
- Fibrous layer
- Hard termination
Absorption - NRC Contours

Homogeneous Layer + Resistive Scrim

1. resistive scrim
2. fibrous material
   - 3.50 cm
   - 400 g/m²

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Absorption Coefficients

(a) Scrim 1, Frequency (Hz)

(b) Scrim 2, Frequency (Hz)

(c) Scrim 3, Frequency (Hz)

(d) Scrim 4, Frequency (Hz)
### Scrim Properties

<table>
<thead>
<tr>
<th>Scrim</th>
<th>Weight (g/m²)</th>
<th>Flow resistance (Rayls)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scrim 1</td>
<td>26</td>
<td>26.8</td>
</tr>
<tr>
<td>Scrim 2</td>
<td>46</td>
<td>106</td>
</tr>
<tr>
<td>Scrim 3</td>
<td>92</td>
<td>212</td>
</tr>
<tr>
<td>Scrim 4</td>
<td>294</td>
<td>3000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scrim + Fibrous Layer</th>
<th>Measured NRC</th>
<th>Predicted NRC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scrim 1</td>
<td>0.47</td>
<td>0.49</td>
</tr>
<tr>
<td>Scrim 2</td>
<td>0.50</td>
<td>0.55</td>
</tr>
<tr>
<td>Scrim 3</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>Scrim 4</td>
<td>0.52</td>
<td>0.52</td>
</tr>
</tbody>
</table>
Optimal Flow Resistance

Scrim Flow Resistance (Rayls)

1 3.5cm Fibrous Layer
2 3.5cm Fibrous Layers

Scrim Basis Weight (g/m²)

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Absorption Coefficients

- \( W_b = 50 \text{ g/m}^2 \)
- \( W_b = 100 \text{ g/m}^2 \)
- \( W_b = 200 \text{ g/m}^2 \)
- \( W_b = 250 \text{ g/m}^2 \)

- \( FR = 50 \text{ Rayls} \)
- \( FR = 280 \text{ Rayls} \)
- \( FR = 2000 \text{ Rayls} \)
- \( FR = 10^4 \text{ Rayls} \)

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Conclusions

- Light, fibrous materials can be modeled as limp
- An efficient, stable and accurate theoretical model has been developed for this purpose
- Theory combined with transfer matrix approach offers system optimization tool
- The flow resistance of a scrim used with a layer of fibrous material has more significant effect on absorption coefficient than does the basis weight
- This work provides a useful tool to identify optimal system parameters