4-1998

Optimization of Acoustical Systems Incorporating Fibrous Elements for Sound Absorption

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Optimization of Acoustical Systems Incorporating Fibrous Elements For Sound Absorption

by

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Introduction

- Porous materials are rigid, elastic or limp
- "Limp" porous material
  - in vacuo bulk stiffness less than that of air
- Fibers move as a result of viscous and inertial coupling
- The validated limp theory can be used to optimize the acoustical properties of fibrous materials
Limp Porous Material Model

- Elastic Porous Model (Based on the Biot Theory)

\[
\sigma_i = 2N\epsilon_i + Ae_s + Q\epsilon, \quad i = x, y, z
\]

\[
s = Qe_s + R\epsilon
\]

\[
\tau_{ij} = \tau_{ji} = N\gamma_{ij}, \quad i, j = x, y, z
\]
Limp Porous Material Model

- Assume
  - in vacuo bulk modulus of solid phase is zero
  - bulk modulus of fiber material much greater than that of air

\[ P = \frac{(1-\phi)^2}{\phi} K_f \quad , \quad Q = (1-\phi) K_f \quad , \quad R = \phi K_f \quad \Rightarrow \quad PR - Q^2 = 0 \]

- \( PR - Q^2 = 0 \) causes singularity in Elastic Model and reduces 2 waves equations to single 2nd order equation

- Input Parameters
  1. Flow resistivity
  2. Porosity
  3. Tortuosity (rigid and limp models)
  4. Bulk elasticity (elastic model)
Limp Porous Material Model

Wave Equations
- 4th order
- 2nd order

Single 2nd Order Wave Equation
Efficient and Stable

Air  Elastic Porous Material

Air  Limp Porous Material

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Evidence of Limp Behavior

Rigid Model

\[ \sigma = 3.5 \times 10^4 \text{ Rayls/m} \]
\[ \phi = 0.99 \]
\[ q^2 = 1.2 \]

Limp Model

\[ \rho_b = 15.97 \text{ kg/m}^3 \]

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Transfer Matrix Method

\[
\begin{bmatrix}
    p_1 \\
    v_{1x}
\end{bmatrix}_{x=0} = \begin{bmatrix}
    T_{11} & T_{12} \\
    T_{21} & T_{22}
\end{bmatrix}
\begin{bmatrix}
    p_2 \\
    v_{2x}
\end{bmatrix}_{x=d}
\]

- \([T]_{\text{composite}} = [T]_1[T]_2 \ldots [T]_n\)
- \(\alpha\) and \(TL\) may be calculated when \([T]_{\text{composite}}\) is known
- Air spaces, porous layers, limp impermeable layers, limp resistive layers, elastic panels
Optimization Examples

- Accurate models allow treatments to be optimized automatically
- Optimum?
  - Specified level of acoustical performance at minimum weight, thickness and cost
- Examples
  - optimal performance for a given thickness
  - vary flow resistivity and basis weight

\[
NRC = \frac{\alpha_{250} + \alpha_{500} + \alpha_{1000} + \alpha_{2000}}{4}
\]

Incident Sound

Scrim
Fibrous layer
Hard termination

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Absorption - *NRC Contours*

Homogeneous Layer + Resistive Scrim

1. resistive scrim
2. fibrous material
   - 3.50 cm
   - 400 g/m²

Scrim Flow Resistance (Rayls) vs. Scrim Basis Weight (g/m²)

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Absorption Coefficients

(a) Scrim 1, Frequency (Hz) vs. Absorption Coefficient \( \alpha \)

(b) Scrim 2, Frequency (Hz) vs. Absorption Coefficient \( \alpha \)

(c) Scrim 3, Frequency (Hz) vs. Absorption Coefficient \( \alpha \)

(d) Scrim 4, Frequency (Hz) vs. Absorption Coefficient \( \alpha \)
# Scrim Properties

<table>
<thead>
<tr>
<th>Scrim</th>
<th>Weight (g/m²)</th>
<th>Flow resistance (Rayls)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scrim 1</td>
<td>26</td>
<td>26.8</td>
</tr>
<tr>
<td>Scrim 2</td>
<td>46</td>
<td>106</td>
</tr>
<tr>
<td>Scrim 3</td>
<td>92</td>
<td>212</td>
</tr>
<tr>
<td>Scrim 4</td>
<td>294</td>
<td>3000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scrim + Fibrous Layer</th>
<th>Measured NRC</th>
<th>Predicted NRC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scrim 1</td>
<td>0.47</td>
<td>0.49</td>
</tr>
<tr>
<td>Scrim 2</td>
<td>0.50</td>
<td>0.55</td>
</tr>
<tr>
<td>Scrim 3</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>Scrim 4</td>
<td>0.52</td>
<td>0.52</td>
</tr>
</tbody>
</table>
Absorption Coefficients

- $W_b = 50 \text{ g/m}^2$
- $W_b = 100 \text{ g/m}^2$
- $W_b = 200 \text{ g/m}^2$
- $W_b = 250 \text{ g/m}^2$

- $FR = 50 \text{ Rayls}$
- $FR = 280 \text{ Rayls}$
- $FR = 2000 \text{ Rayls}$
- $FR = 10^4 \text{ Rayls}$
Conclusions

- Light, fibrous materials can be modeled as limp
- An efficient, stable and accurate theoretical model has been developed for this purpose
- Theory combined with transfer matrix approach offers system optimization tool
- The flow resistance of a scrim used with a layer of fibrous material has more significant effect on absorption coefficient than does the basis weight
- This work provides a useful tool to identify optimal system parameters