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Abstract

Joint behavior of drought characteristics under climate change is evaluated using copula method which has recently attained popularity in analysis of complex hydrologic systems with correlated variables. Trivariate copulas are applied in this study to analyze the major drought variables; duration, severity, and intensity in the Upper Klamath River basin in Oregon. Results show that, among the variables, duration-severity is the most correlated pair whereas duration-intensity is the least correlated one. The impact of climate change on future droughts is evaluated using five Global Climate Models (GCMs) under one emission scenario. Comparing to the historical events, an overall decrease in drought duration and severity is estimated for the time period of 2020-2090 and the maximum duration is shown a decrease from 8 months to 5 months. Among the five GCMs employed in this study, GFDL-CM2.1 and CSIRO-MK3.0 are recognized as the wettest and driest projections, respectively. High uncertainty associated with GCM products is demonstrated in the analysis of return period by means of bivariate copulas; however, all projections result in larger return periods; i.e., less frequent droughts comparing to historical droughts during the reference period.

Drought Characteristics

National Drought Mitigation Center (NDMC) classifies drought events into four different groups: Meteorological, Agricultural, Hydrological, and Socioeconomical droughts (Fig. 1) . All droughts are initiated by Meteorological drought which itself is a result of precipitation lack, temperature increase, and humidity shortage in general.

Drought events during a time period are recognized by means of various indices which are inherently correlated and dependant to each other. Some of drought indices are shown in Figure 2.

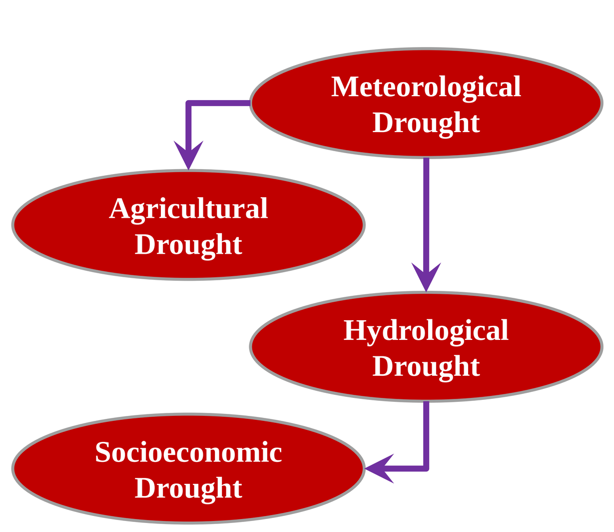

Fig. 1 Drought types by NDMC

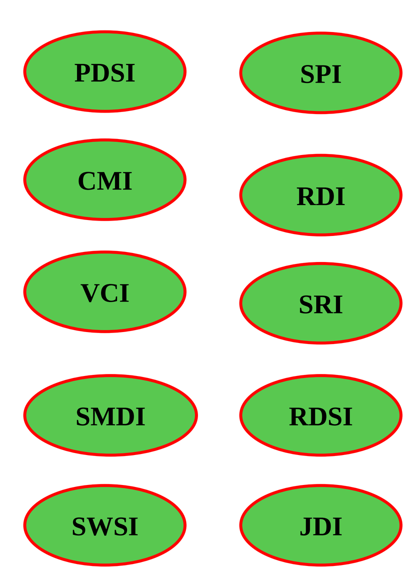

Fig. 2 Drought indices

Each drought event has three attributions: Duration, Severity, and Intensity (Fig. 3).

- **Duration** is the length of period in which the index values are less than truncation level.
- **Severity** is the absolute cumulative index values based on the duration time.
- **Intensity** is severity divided by duration.
- The time elapsed between sequential events onsets is called interarrival time.

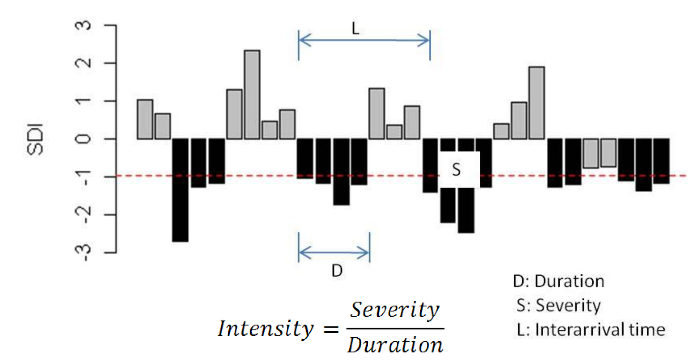

Fig. 3 Drought characteristics

Copula Application to Correlated Variables

- Drought duration, severity, and intensity are correlated to each other.
- Joint behavior of drought variables should be analyzed by multivariate probability distributions.

$$H(x_1,x_2,\dots,x_n)=P(X_1\le x_1,X_2\le x_2,\dots,X_n\le x_n)=\int_{-\infty}^{x_1}\int_{-\infty}^{x_2}\int_{-\infty}^{x_3} f(x_1,x_2,\dots,x_n)\,dx_1\,dx_2\dots dx_n$$

$$T(X_1\ge x_1,X_2\ge x_2,\dots,X_n\ge x_n)=\frac{\mu}{1-P(X_1\le x_1,X_2\le x_2,\dots,X_n\le x_n)}$$

- Copula is a multivariate distribution with uniform marginal distributions over (0,1). An n-dimensional Copula (C) is the joint probability of univariate marginal distribution *F1,…,Fn* as follows:

$$H(x_1,x_2,\dots,x_n)=C[F_1(x_1),F_2(x_2),\dots,F_n(x_n)]=C(u_1,\dots,u_n)$$

- Elliptical, archimedean, and extreme value copulas are the most applied copula families in hydrologic studies.

Trivariate Gumbel Copula:

$$C(u_1,u_2,u_3)=C_1[C_2(u_1,u_2),u_3]=\exp\{-[(-\ln u_1)^{\theta_2}+(-\ln u_2)^{\theta_2}]^{\frac{\theta_1}{\theta_2}}+(-\ln u_3)^{\theta_1}\}^{\frac{1}{\theta_1}}$$

Trivariate t-Copula:

$$f(x)=\frac{\Gamma\left(\frac{\nu+d}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{(\pi\nu)^d|R|}}\left(1+\frac{(x-\mu)'R^{-1}(x-\mu)}{\nu}\right)^{-(\nu+d)/2}$$

$$C^t_{\nu,R}(u)=t^t_{\nu,R}[t_\nu^{-1}(u_1),\dots,t_\nu^{-1}(u_n)]$$

Case Study

Historical and future droughts are evaluated for Upper Klamath River Basin in Western United States with drainage area of 21000 km^2.

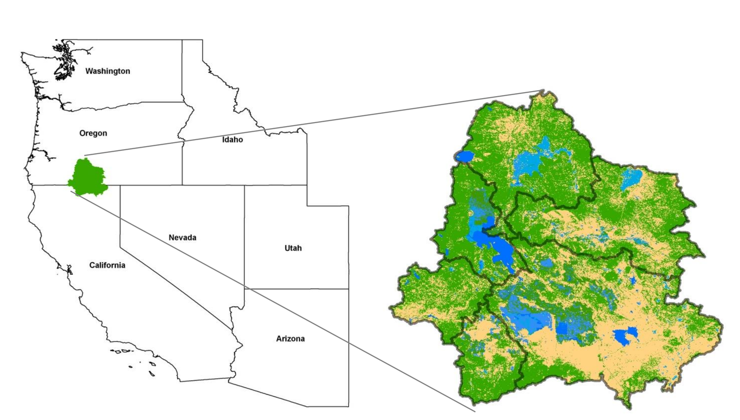

Fig. 4 Upper Klamath River Basin

- Historical droughts are investigated for the time period of 1920-2009.
- Climate change impacts on drought events are studied for the time period of 2020-2090.
- Streamflow Drought Index (SDI) is used for drought analysis:

$$SDI_{i,k}=\frac{y_{i,k}-\bar{y}_k}{s_{y,k}}$$

$$y_{i,k}=\ln(V_{i,k})\qquad i=1,2,\dots,T\quad k=1,2,3,4$$

Climate Projection

To evaluate the impact of climate change on future drought events, simulations of five Coupled General Circulation Models (CGCMs) are used in this study (Table 1) which are paired with the A1B emission scenario.

Table 1: Specification of CGCMs used in this study

Model ID	Country	Resolution	
		Atmosphere	Ocean
BCCR-BCM2.0	Norway	2.8° × 2.8°	1.0° × 1.0°
CNRM-CM3	France	2.8° × 2.8°	2.0° × 1.0°
CSIRO-MK3.0	Australia	1.9° × 1.9°	1.9° × 1.0°
GFDL-CM2.1	USA	2.5° × 2.0°	1.0° × 1.9°
UKMO-HADCM3	UK	3.8° × 3.8°	1.3° × 1.3°

Drought Analysis

To apply the copula approach in drought analysis, SDI is examined to characterize the drought events of Upper Klamath Basin. Figure 5 shows the timeseries of SDI in historical time period.

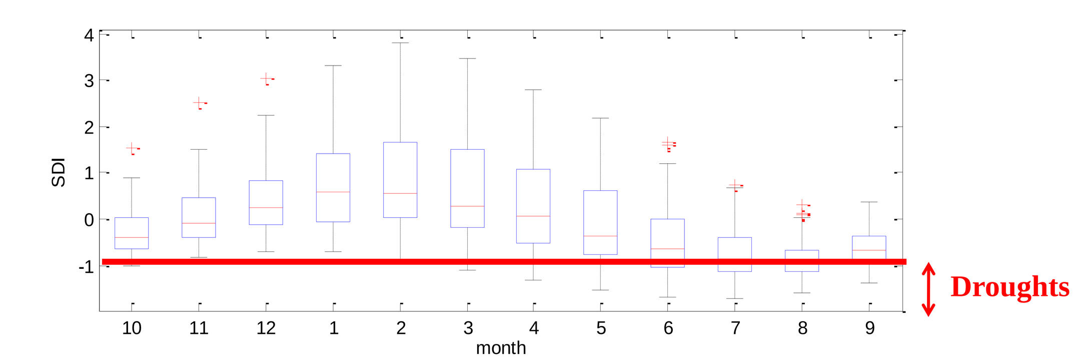

Fig. 5 Monthly variation of 6-month SDI for the time period of 1907-2009.

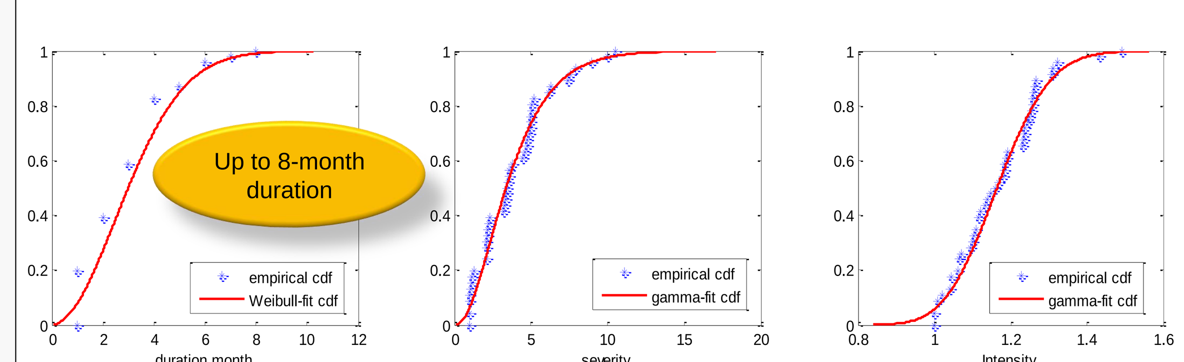

Fig. 6 Empirical and best theoretical CDFs fitted to drought variables

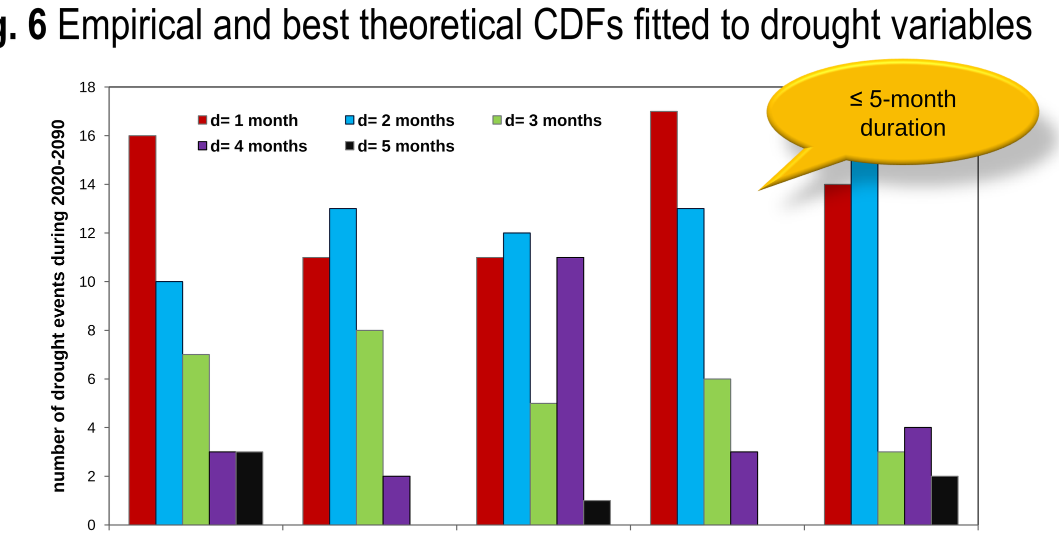

Fig. 7 Duration of future droughts in the time period of 2020-2090..

Conditional Probability

$$P(S\le s|D\ge d)=\frac{P(D\ge d,S\le s)}{P(D\ge d)}=\frac{F_S(s)-F_{DS}(d,s)}{1-F_D(d)}=\frac{F_S(s)-C(F_D(d),F_S(s))}{1-F_D(d)}$$

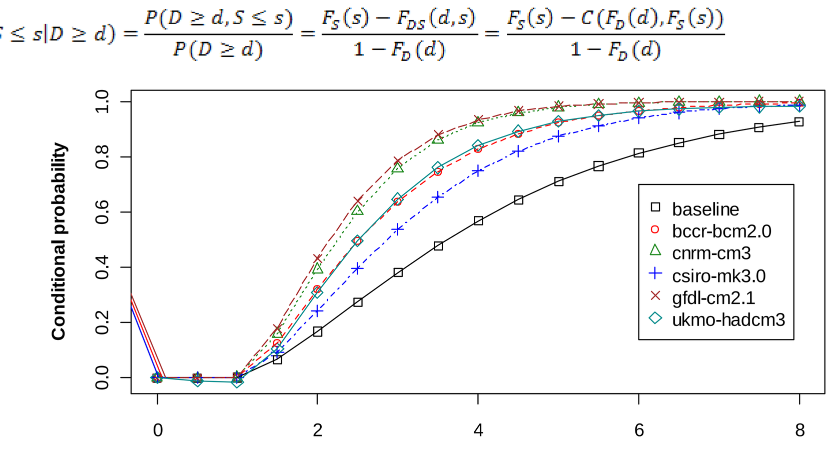

Fig. 8 Conditional drought severity distribution given 1-month duration

Joint Return Periods

Single Variable

$$T_D=\frac{E(L)}{1-F_D(d)}\qquad T_S=\frac{E(L)}{1-F_S(s)}\qquad T_I=\frac{E(L)}{1-F_I(i)}$$

Bivariate Return Periods

D>=d **AND** S>=s

$$T_{AND(DS)}=\frac{E(L)}{P(D\ge d,S\ge s)}=\frac{E(L)}{1-F_D(d)-F_S(s)+F_{DS}(d,s)}=\frac{E(L)}{1-F_D(d)-F_S(s)+C(F_D(d),F_S(s))}$$

D>=d **OR** S>=s

$$T_{OR(DS)}=\frac{E(L)}{P(D\ge d\ or\ S\ge s)}=\frac{E(L)}{1-F_{DS}(d,s)}=\frac{E(L)}{1-C(F_D(d),F_S(s))}$$

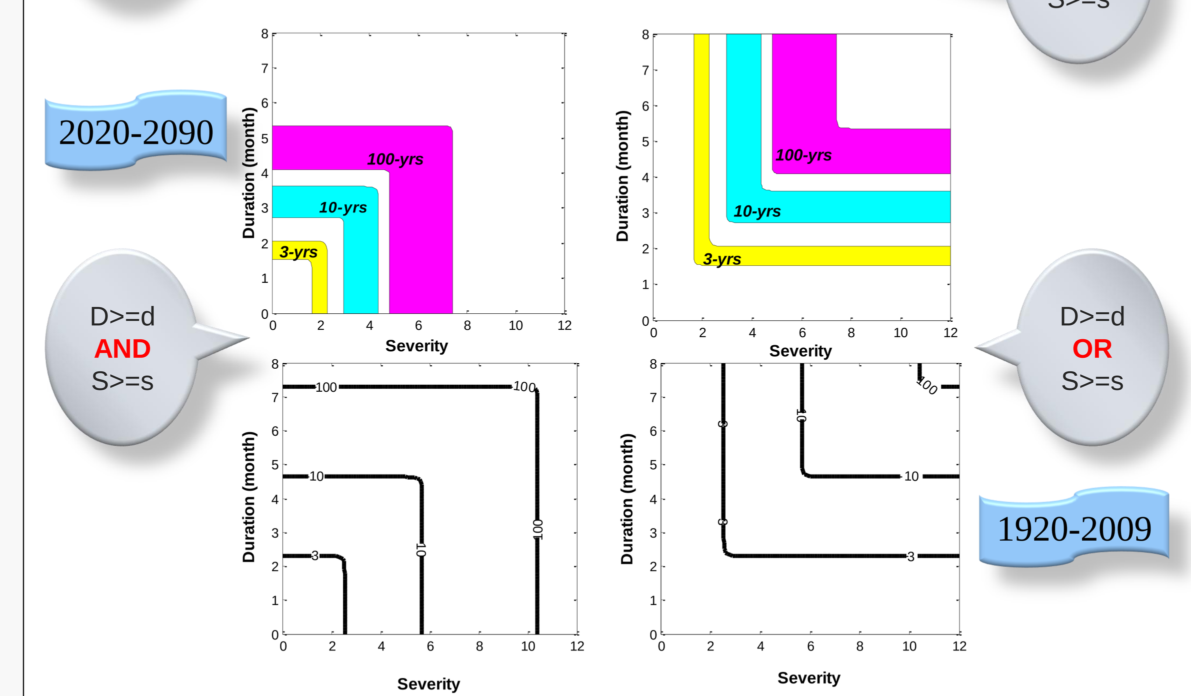

Fig. 9 Contour plots of bivariate joint return periods

Trivariate Return Periods

D>=d **AND** S>=s **AND** I>=i

$$T_{AND(DSI)}=\frac{E(L)}{P(D\ge d,S\ge s,I\ge i)}=\frac{E(L)}{1-F_D(d)-F_S(s)-F_I(i)+F_{DS}(d,s)+F_{DI}(d,i)+F_{IS}(i,s)-F_{DSI}(d,s,i)}=\frac{E(L)}{1-F_D(d)-F_S(s)-F_I(i)+F_{DS}(d,s)+F_{DI}(d,i)+F_{IS}(i,s)-C(F_D(d),F_S(s),F_I(i))}$$

D>=d **OR** S>=s **OR** I>=i

$$T_{OR(DSI)}=\frac{E(L)}{1-F_{DSI}(d,s,i)}=\frac{E(L)}{1-C(F_D(d),F_S(s),F_I(i))}$$

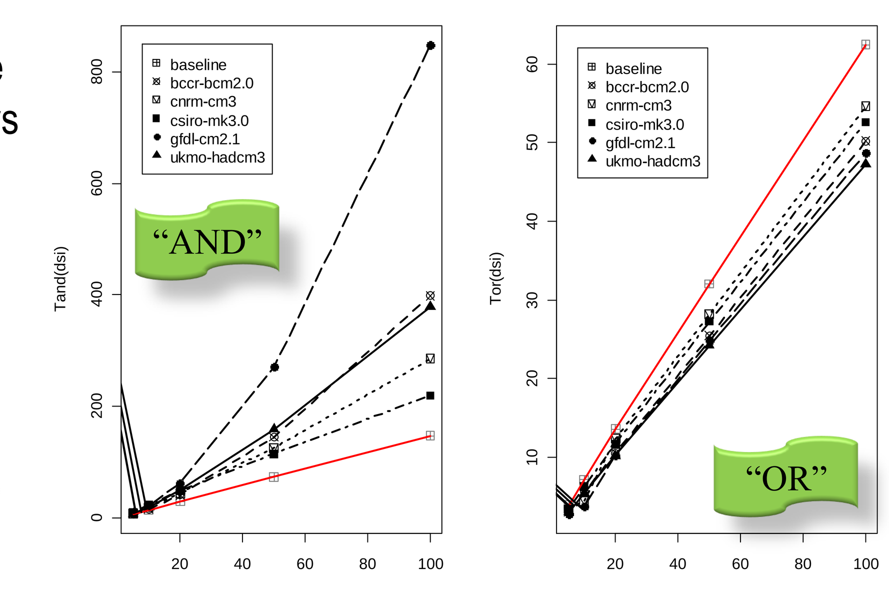

Fig. 9 Trivariate return periods vs those obtained from univariate marginal distributions for historical and future droughts from five GCMs

Findings

- Future droughts associated with 5 GCMs under A1B emission scenario have less duration and severity than the past.
- Based on the conditional probability analysis using Copula, given 1-month drought duration, CNRM-CM3 and GFDL-CM2.1 are seen as the wettest climate scenarios; whereas CSIRO-MK3.0 output show the most severe droughts.
- The return period analysis based on either bivariate or trivariate copulas showed that climate change makes an overall decline in drought severity and duration in the Upper Klamath River basin.

References

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- Shiau, J. T., 2006. Fitting drought duration and severity with two-dimensional copulas. Water Resources Management, 20(5), 795-815.