Purdue University
Purdue e-Pubs

GTAP Technical Papers | Agricultural Economics

9-1-2003

A New Regional Household Demand System for GTAP

Robert McDougall
Purdue University

Follow this and additional works at: http://docs.lib.purdue.edu/gtaptp

McDougall, Robert , "A New Regional Household Demand System for GTAP" (2003). *GTAP Technical Papers.* Paper 20.
http://docs.lib.purdue.edu/gtaptp/20

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact epubs@purdue.edu for additional information.

A New Regional Household Demand System for GTAP

Robert McDougall

GTAP Technical Paper No. 20

Revision 1

September 2003

Robert McDougall: Deputy Director, Center for Global Trade Analysis, Department of Agricultural Economics, Purdue University, 1145 Krannert Building, West Lafayette, IN 47907

A New Regional Household Demand System for GTAP

Robert A. McDougall

Abstract

The GTAP model, versions 4.1 and lower, suffers from some defects in the implementation of the regional household demand system. Most seriously, the upper level of the demand system assumes that each regional household faces a fixed price for utility from private consumption. But with a private consumption demand system of the CDE form, the price of utility from private consumption depends on the level of private consumption expenditure. With no fixed price for utility from private consumption, the familiar Cobb-Douglas demand system does not apply. Accordingly, the upper-level demand equations are in error.

Furthermore, utility and equivalent variation are wrongly computed in simulations with non-standard settings for the CDE expansion parameters. Even with the standard settings, in multi-step simulations the utility and equivalent variation computations are inexact. The welfare decomposition inherits the defects of the equivalent variation computation.

In removing these defects we revise in passing some minor misfeatures of the old treatment: Firstly, we treat the entire final demand system as the demand system of a representative household, rather than a conglomeration of representative and region-wide demand systems (subsection 2.6). Secondly, we provide a new facility for shifting the allocation of regional income exogenously by modifying rather than overriding the final demand system (subsection 2.14). Finally, we eliminate an uninterpretable "nuisance term" from the decomposition of equivalent variation (subsection 4.3).

Contents

1 Introduction **1**

2 The upper level of the regional household demand system **2**
2.1 The old treatment . . . 2
2.2 Defects in the old treatment: identification . . . 6
2.3 Revised theory . . . 9
2.4 Defects in the old treatment: assessment . . . 14
2.5 Possible remedies . . . 16
2.6 A new treatment . . . 16
2.7 Shared variables . . . 17
2.8 Government consumption . . . 18
2.9 Utility from private consumption . . . 18
2.10 Saving . . . 20
2.11 Regional household preliminaries . . . 20
2.12 Regional household demands . . . 21
2.13 Regional household utility . . . 22
2.14 Shifting income allocation without affecting the utility elasticity . . . 25
2.15 Changes to the data file . . . 26

3 Equivalent variation **27**
3.1 The old treatment . . . 27
3.2 Defects in the old treatment . . . 29
3.3 A new treatment . . . 29
3.4 Equivalent variation with preference change . . . 30
3.5 Shared objects . . . 31
3.6 The shadow government consumption demand system . . . 32
3.7 The shadow private consumption demand system . . . 32
3.8 The shadow upper-level regional household demand system . . . 34
3.9 Computing the equivalent variation . . . 37

4 Decomposing the equivalent variation **37**
4.1 The old treatment . . . 37
4.2 Defects in the old treatment . . . 39
4.3 A revised treatment . . . 39
4.4 Decomposing real *per capita* income . . . 41
4.5 Implementation . . . 43

5 Properties and behavior of the final demand system **45**

6 Future work **48**

References **49**

Appendices:

A The demand system and the Gorman conditions **51**

B Alternative private consumption demand systems **53**

C Accompanying Software **55**

D Revision History **57**

1 Introduction

The GTAP model, versions 4.1 and lower, suffers from some defects in the implementation of the regional household demand system:

- The upper level of the demand system assumes that each regional household faces a fixed price for utility from private consumption. But with a private consumption demand system of the CDE form, the price of utility from private consumption depends on the level of private consumption expenditure. With no fixed price for utility from private consumption, the familiar Cobb-Douglas demand system does not apply. Accordingly, the upper-level demand equations are in error.

- Utility and equivalent variation are wrongly computed in simulations with non-standard settings for the CDE expansion parameters. Even with the standard settings, in multi-step simulations the utility and equivalent variation computations are inexact.

- The welfare decomposition inherits the defects of the equivalent variation computation.

In removing these defects we revise in passing some minor misfeatures of the old treatment:

- We treat the entire final demand system as the demand system of a representative household, rather than a conglomeration of representative and region-wide demand systems (subsection 2.6).

- We provide a new facility for shifting the allocation of regional income exogenously by modifying rather than overriding the final demand system (subsection 2.14).

- We eliminate an uninterpretable "nuisance term" from the decomposition of equivalent variation (subsection 4.3).

The main disadvantage of the new treatment relative to the old is that its implementation and properties are somewhat more complex. It requires more equations and variables, mostly to support an exact calculation of the equivalent variation. Also, whereas the old treatment allocated regional income in fixed shares between private consumption expenditure, government household expenditure, and saving, the new treatment allows the shares to vary in response to changes in income and consumer prices.

This paper describes the new treatment. In its electronic form, it should be accompanied by several program files—source code, `gtap.tab`, for the revised solution program, a Tablo stored input file `gtap.fts` showing a typical model condensation, and a GTAP test simulation command file `ghom.cmf` showing a typical model closure.

We include extensive listings of new and old source code. Listings of old source code come from a GTAP model version 5 prerelease used in the August 2000 GTAP short course, incorporating Ken Itakura's reorganization of the code structure but no relevant changes in the model theory over version 4.1. We do not describe but rather take as given the standard GTAP model notation; the paper should accordingly be read

in conjunction with the source code or the original GTAP model documentation (Hertel and Tsigas [5]). The source code, together with command files for certain illustrative simulations (section 5), is included in an accompanying software package (appendix C).

We adopt the convention that a lower-case symbol denotes percentage change in the corresponding upper-case symbol; so for a variable X, x denotes percentage change in X, that is, $x = (1/100)(dX/X)$.

2 The upper level of the regional household demand system

2.1 The old treatment

In the GTAP model as originally implemented (Hertel and Tsigas [5]), in each region a *regional household* allocates regional income so as to maximize *per capita* aggregate utility according to a Cobb-Douglas utility function. The maximand is described as "aggregate" utility because it comprises both government and private sector behavior. The arguments in the utility function are *per capita* utility from private consumption, *per capita* utility from government consumption, and *per capita* real saving. We refer to these as the *upper-level commodities* of the final demand system.

Real saving is a single commodity, defined as saving deflated by a saving price. Utility from government consumption is a Cobb-Douglas aggregate of government consumption of individual commodities. *Per capita* utility from private consumption is aggregated from *per capita* private consumption of individual commodities following Hanoch's ([3]) constant difference elasticity (*CDE*) demand system.

We note that in the private consumption demand system, unlike the government consumption demand system, the variable maximized is a *per capita* rather than an economy-wide utility. This is necessary because the private consumption demand system is non-homothetic. The allocation of private consumption expenditure across commodities depends on the sum to be allocated, and the appropriate sum variable is not economy-wide but *per capita* private consumption expenditure.

The CDE demand system is characterized by an implicit expenditure-*cum*-indirect-utility function,

$$1 = \sum_i B_i U^{\Upsilon_i R_i} \left(\frac{P_i}{X}\right)^{\Upsilon_i}, \tag{2.1}$$

where U denotes utility, P_i, the price of commodity i, X, expenditure, and B_i, Υ_i, and R_i, various parameters. Following Hanoch [3], we call the B_i distribution parameters, the Υ_i substitution parameters, and the R_i expansion parameters. Constraints on the parameters are:

$$\forall i,\ B_i > 0,$$
$$\forall i,\ R_i > 0,$$

and either

$$\forall i,\ \Upsilon_i < 0$$

or

$$\forall i,\ 0 < \Upsilon_i < 1.$$

Although we are not required to do so by theory, in standard GTAP data bases we normalize the expansion parameters so that their share-weighted sum is equal to one,

$$\sum_i S_i^P R_i = 1,$$

where S_i^P denotes the share of commodity i in private consumption expenditure.

This completes the specification the final demand system; it remains to work out the implications of the specification. This is done briefly in Hertel and Tsigas ([5]), but to support later discussion (subsection 2.2) we provide here a more detailed derivation for the upper level of the system.

We write the upper-level utility function as

$$U = CU_P^{B_P} U_G^{B_G} U_S^{B_S}, \tag{2.2}$$

where U denotes *per capita* aggregate utility, U_P, *per capita* utility from private consumption, U_G, *per capita* government consumption, and U_S, *per capita* real saving, and B_P, B_G, and B_S are distribution parameters.

We define a saving price P_S, and postulate the existence of suitable price indices P_G and P_P for utility from government and private consumption. Then given income Y, the regional household maximizes U subject to the budget constraint

$$N(P_P U_P + P_G U_G + P_S U_S) = Y, \tag{2.3}$$

where N denotes population.

Since the utility function is Cobb-Douglas, we expect the regional household to allocate regional income in fixed shares between the upper-level commodities:

$$Y_P = \frac{B_P}{B} Y, \tag{2.4}$$

$$Y_G = \frac{B_G}{B} Y, \tag{2.5}$$

$$Y_S = \frac{B_S}{B} Y, \tag{2.6}$$

where B denotes the sum of the distribution parameters, $B = B_P + B_G + B_S$, Y_P private consumption expenditure, $Y_P = NP_P U_P$, Y_G government consumption expenditure,

$Y_G = NP_GU_G$, and Y_S saving, $Y_S = NP_SU_S$. Then

$$NP_PU_P = \frac{B_P}{B}Y,$$
$$NP_GU_G = \frac{B_G}{B}Y,$$
$$NP_SU_S = \frac{B_S}{B}Y.$$

Putting

$$Q_P = NU_P, \tag{2.7}$$
$$Q_G = NU_G, \tag{2.8}$$
$$Q_S = NU_S, \tag{2.9}$$

where Q_P denotes private consumption, Q_G government consumption, and Q_S saving, we obtain

$$P_PQ_P = \frac{B_P}{B}Y,$$
$$P_GQ_G = \frac{B_G}{B}Y,$$
$$P_SQ_S = \frac{B_S}{B}Y.$$

To allow for exogenous shocks in the allocation of saving, we define "slack variables" K_S and K_G for saving and utility from government consumption, initially equal to one. We insert these into the corresponding demand equations:

$$P_GQ_G = K_G\frac{B_G}{B}Y,$$
$$P_SQ_S = K_S\frac{B_S}{B}Y.$$

Differentiating and rearranging, we obtain

$$q_G = y - p_G + \kappa_G, \tag{2.10}$$
$$q_S = y - p_S + \kappa_S. \tag{2.11}$$

These appear in the old code as:

```
Equation GOVERTU
# computation of utility from government consumption (HT 39) #
(all,r,REG)
    ug(r) = y(r) - pgov(r) + govslack(r);
```

and

```
Equation SAVINGS
# regional demand for savings (HT 38) #
```

```
(all,r,REG)
    qsave(r) = y(r) - psave(r) + saveslack(r) ;
```

In the presence of shocks to the slack variables, the upper-level demand system is no longer operative; the budget constraint however must still be observed. Accordingly, we include in the model not the demand equation for utility from private consumption but instead the budget constraint

$$Y_P = Y - Y_G - Y_S.$$

We express government consumption expenditure Y_G as the sum of expenditures on individual commodities, $Y_G = \sum_i Y_{Gi}$, where Y_{Gi} denotes government consumption expenditure on commodity i, $Y_{Gi} = P_{Gi}Q_{Gi}$, where P_{Gi} denotes the price of commodity i when purchased for government consumption, and Q_{Gi} government consumption of commodity i. Then

$$\begin{aligned} Y_P &= Y - \sum_i Y_{Gi} - Y_S \\ &= Y - \sum_i P_{Gi}Q_{Gi} - P_S Q_S, \end{aligned}$$

or, in percentage change form,

$$Y_P y_P = Y y - \sum_i Y_{Gi}(p_{Gi} + q_{Gi}) - Y_S(p_S + q_S).$$

This appears in the old code as:

```
Equation PRIVATEXP
# private consumption expenditure (HT 8) # (all,r,REG)
    PRIVEXP(r)*yp(r)
  = INCOME(r)*y(r)
  - SAVE(r)*[psave(r) + qsave(r)]
  - sum(i,TRAD_COMM, VGA(i,r)*[pg(i,r) + qg(i,r)])
  ;
```

Finally, we compute utility. Substituting for U_G and U_S from equations (2.8) and (2.9) into equation (2.2), we have

$$U = CU_P^{B_P}\left(\frac{Q_G}{N}\right)^{B_G}\left(\frac{Q_S}{N}\right)^{B_S}.$$

Differentiating, we obtain

$$\begin{aligned} u &= B_P u_P + B_G(q_G - n) + B_S(q_S - n) \qquad (2.12) \\ &= B\left[\frac{B_P}{B}u_P + \frac{B_G}{B}(q_G - n) + \frac{B_S}{B}(q_S - n)\right] \\ &= B\left[\frac{Y_P}{Y}u_P + \frac{Y_G}{Y}(q_G - n) + \frac{Y_S}{Y}(q_S - n)\right], \end{aligned}$$

using equations (2.4–2.6). Then, setting $B = 1$, we have

$$Yu = Y_P u_P + Y_G(q_G - n) + Y_S(q_S - n). \tag{2.13}$$

This appears in the old code as:

```
Equation UTILITY
# computation of per capita regional utility (HT 37) #
(all,r,REG)
    INCOME(r)*u(r)
  = PRIVEXP(r)*up(r)
  + GOVEXP(r)*[ug(r) - pop(r)]
  + SAVE(r)*[qsave(r) - pop(r)]
  ;
```

2.2 Defects in the old treatment: identification

While the old treatment has proven serviceable in many GTAP applications, it is not without defects. We identify three, of very different magnitude:

- It is slightly confusing in formulation, shifting unnecessarily between unitary and representative households, and economy-wide and *per capita* utilities.
- In setting saving or government consumption exogenously, the user cannot adjust preferences within the upper-level demand system, but must override them. There are some advantages to maintaining a working upper-level demand system even when some upper-level demands are exogenized.
- The underlying theory (subsection 2.1) is invalid; the model equations do not logically follow from the theory's premises.

The first, and very minor, objection to the old treatment is that in formulation it is slightly incoherent. The upper-level utility function is attributed to a unitary "regional household", that is, a notional single agent that takes all the income and does all the consumption in the region. But its arguments are *per capita* variables. That would suggest that the utility function should be attributed to a "representative household", that is, to any one of a large notional collection of identical small households which together absorb the income and perform the consumption of their region. The government consumption variable in the upper-level utility function is *per capita* government consumption, but in the government consumption demand system the variable is economy-wide government consumption. Utility from private consumption pertains to a representative private household, and utility from government consumption to a "government household", both distinct from the "regional household" that enjoys aggregate utility.

Taking a sympathetic view of the old system, we may note that it is based informally on the representative household concept, but employs some plausible simplifications for homothetic sub-systems (in particular, government consumption). Taking it literally, however, it contains some slight disconnections between the upper and lower levels.

Since we need (later in this subsection) to examine closely the interactions between the levels, it is useful to identify and remove these disconnections, even at the expense of belaboring a small issue.

Taking the old treatment literally, we are not entitled to talk about upper and lower levels of the demand system. To do so would imply that they are part of the same agent's demand system, whereas logically they pertain to different agents. More specifically, the aggregate utility function pertains to a unitary regional household that displays an altruistic interest in the welfare of the representative private household, and also cares about a variable, *per capita* government consumption, that is related to but distinct from the welfare of the government household. We deliberately slur over these niceities in deriving the old system (subsection 2.1). In discussing below (in this subsection) the more substantive defects of the old system, we override them, treating all the demand subsystems as components of a representative regional household demand system. Finally, in presenting the new treatment, we consistently follow the representative agent approach (subsection 2.6), and implement the associated minor substantive changes (subsection 2.8).

The second limitation of the old treatment, also minor, is that the saving and government consumption slack variables, K_S and K_G, override rather than modify the upper-level demand system. We should be able to represent exogenous shifts in income disposition as shifts in preferences in the upper-level demand system. This would have three advantages:

- It would let us shock demand for any of the three upper-level commodities. The old treatment lets us shock either saving or government consumption but not private consumption.

- It would let the upper-level demand system do some work even when some external outcomes are imposed. For example, while exogenizing saving, we could let the demand system allocate remaining income between private and government consumption. In the old treatment, with saving exogenous, the demand system determines government consumption expenditure, but private consumption expenditure is determined residually; since if the demand system were allowed to determine it, expenditure and saving would not sum to income.

- It would allow us to obtain meaningful welfare results even when some upper-level income allocations are set exogenously.

The main defect in the old treatment is that the demand equations are invalid. The error is in the old upper-level budget constraint (2.3), $N(P_P U_P + P_G U_G + P_S U_S) = Y$. In adopting this formulation for the constraint, the old treatment assumes that the regional household can obtain utility from private consumption at some fixed price P_P. This assumption is non-trivial and in fact unwarranted.

We rewrite the old upper-level budget constraint as

$$P_P U_P + P_G U_G + P_S U_S = X, \tag{2.14}$$

where X denotes *per capita* income. Recalling that utility from private consumption and utility from government consumption are defined within the private and government consumption demand subsystems, we obtain the general form of the constraint,

$$E_P(\mathbf{P_P}, U_P) + E_G(\mathbf{P_G}, U_G) + P_S U_S = X, \tag{2.15}$$

where E_P and E_G are *per capita* expenditure functions, and $\mathbf{P_P}$ and $\mathbf{P_G}$ price vectors, for private and government consumption. It might so happen that the expenditure functions were of the form

$$\begin{aligned} E_P(\mathbf{P_P}, U_P) &= \Pi_P(\mathbf{P_P})U_P, \\ E_G(\mathbf{P_G}, U_G) &= \Pi_G(\mathbf{P_G})U_G \end{aligned} \tag{2.16}$$

for some functions $\Pi_P(\mathbf{P_P})$ and $\Pi_G(\mathbf{P_G})$. If so, we could could set $P_P = \Pi_P(\mathbf{P_P})$ and $P_G = \Pi_G(\mathbf{P_G})$, and replace the general budget constraint (2.15) with the simpler form (2.14). In fact, the government consumption expenditure function is of the required form, but the private consumption expenditure function is not; so we cannot use the simpler budget constraint.

To show that the private consumption expenditure function cannot be written in the form (2.16), we employ the general proposition (cf., e.g., Deaton and Muellbauer [1] p. 143):

Proposition 1 *For any demand system, the expenditure function is of the form* $E(\mathbf{P}, U) = \Pi(\mathbf{P})F(U)$ *for some monotonic increasing function* F *if and only if the system is homothetic.*

Proof. For sufficiency, note that if the system is homothetic, there exists a strictly increasing function F such that for all consumption vectors $\mathbf{Q}$, for all positive K, $F \circ U(K\mathbf{Q}) = KF \circ U(\mathbf{Q})$. Let U_0 be some arbitrary utility level; and for all price vectors $\mathbf{P}$, let $\Pi(\mathbf{P}) = E(\mathbf{P}, U_0)/F(U_0)$. Then for any utility level U_1 and any price vector $\mathbf{P}$, $E(\mathbf{P}, U_1) = \Pi(\mathbf{P})F(U_1)$. For suppose that with prices $\mathbf{P}$, consumption $\mathbf{Q_0}$ yields utility U_0 at minimum cost. Then consumption $(F(U_1)/F(U_0))\mathbf{Q_0}$ yields utility U_1 at minimum cost; so $E(\mathbf{P}, U_1) = \mathbf{P} \bullet (F(U_1)/F(U_0))\mathbf{Q_0} = (F(U_1)/F(U_0))E(\mathbf{P}, U_0) = \Pi(\mathbf{P})F(U_1)$. Hence the expenditure function is of the specified form. For necessity, note that by Shephard's lemma, the budget share of each commodity i is equal to the elasticity of Π with respect to the price of i; so the budget shares are independent of utility; so the system is homothetic.■

Now as Hanoch [3] shows, the CDE is in general non-homothetic. Indeed, this is a requirement for any empirically satisfactory demand system (see, for example, Deaton and Muellbauer [1] p. 144), and part of the reason for adopting the CDE in GTAP (Hertel and Tsigas [5], p. 49). So in GTAP, the private consumption expenditure function is not of the form (2.16), and the budget constraint is not equation (2.14).

This shows that the old theory is defective, in that it contains an invalid derivation; it does not show how or whether the relevant results are in error. An appendix discusses how the defects in the theory relate to the Gorman conditions for two-stage budgeting

(appendix A). The main text proceeds to correct the theory (subsection 2.3) and compare the corrected with the old results (subsection 2.4).

2.3 Revised theory

We find above (subsection 2.2) that we need to revise the budget constraint in the upper-level demand system from the special form (2.14), $P_P U_P + P_G U_G + P_S U_S = X$, to the more general form (2.15), $E_P(\mathbf{P_P}, U_P) + E_G(\mathbf{P_G}, U_G) + P_S U_S = X$. We now derive the demand equations, an equation for utility, and some auxiliary equations under this more general assumption.

As an aid to the reader, we distinguish these derived equations by enclosing them in $\boxed{\text{boxes}}$.

We begin by obtaining a general solution for the Cobb-Douglas demand system in the absence of fixed prices.

Proposition 2 *In the Cobb-Douglas demand system*

$$max\ U = C \prod_i U_i^{B_i} \ \ subject\ to \sum_i E_i(U_i) = X, \tag{2.17}$$

with expenditures $X_i = E_i(U_i)$ on individual commodities convex in quantities U_i, the budget share

$$\frac{X_i}{X} = \frac{\Phi_i^{-1} B_i}{\sum_j \Phi_j^{-1} B_j}, \tag{2.18}$$

where Φ_i denotes the elasticity of expenditure on commodity i with respect to quantity of commodity i. In the corresponding cost minimization problem, the elasticity of expenditure with respect to utility, Φ, is given by:

$$\Phi^{-1} = \sum_i \Phi_i^{-1} B_i. \tag{2.19}$$

Proof. The Lagrangean,

$$\mathcal{L} = C \prod_i U_i^{B_i} - \Lambda \left(\sum_i E_i(U_i) - X \right),$$

where Λ denotes the Lagrange multiplier. Differentiating with respect to subutility i, we obtain

$$0 = \frac{\partial \mathcal{L}}{\partial U_i} = \frac{B_i U}{U_i} - \Lambda E_i'(U_i). \tag{2.20}$$

Also, by definition, the elasticity of expenditure on subutility i with respect to the subutility level,

$$\Phi_i = \frac{U_i E_i'(U_i)}{X_i}. \tag{2.21}$$

Combining equations (2.20) and (2.21), we obtain

$$X_i = \frac{\Phi_i^{-1} B_i U}{\Lambda}. \tag{2.22}$$

Also total expenditure,

$$X = \sum_i X_i \tag{2.23}$$

Combining equations (2.22) and (2.23) and solving for Λ, we obtain

$$\Lambda = \frac{U}{X} \sum_i \Phi_i^{-1} B_i. \tag{2.24}$$

Substituting from equation (2.24) into (2.22), we obtain the first required result, equation (2.18):

$$\frac{X_i}{X} = \frac{\Phi_i^{-1} B_i}{\sum_j \Phi_j^{-1} B_j}.$$

Furthermore, by definition,

$$\Phi = \frac{U}{X} E'(U), \tag{2.25}$$

where $E(U)$ denotes the overall expenditure function (considered as a function of utility only). Also, by the envelope theorem,

$$\Lambda = \frac{1}{E'(U)}. \tag{2.26}$$

Combining equations (2.25), (2.26), and (2.24), we obtain the second required result, equation (2.19):

$$\Phi^{-1} = \sum_i \Phi_i^{-1} B_i.$$

■

Note that with expenditures proportional to quantities, the elasticities Φ_i are unity, so the equation reduces to the standard Cobb-Douglas fixed-shares equation. In general however the expenditure shares are variable.

Now the government consumption demand system is Cobb-Douglas, so it is homothetic, so we can cardinalize utility from government consumption so that $\Phi_G \equiv 1$. Also saving is a single commodity, so $\Phi_S \equiv 1$. Applying then proposition 2 to the GTAP

demand system, we have

$$\frac{X_P}{X} = \left(\frac{\Phi_P}{\Phi}\right)^{-1} B_P,$$
$$\frac{X_G}{X} = \Phi B_G,$$
$$\frac{X_S}{X} = \Phi B_S,$$

or, in percentage change form,

$$\boxed{x_P - x = -(\phi_P - \phi)} \tag{2.27}$$

$$\boxed{x_G - x = \phi} \tag{2.28}$$

$$\boxed{x_S - x = \phi} \tag{2.29}$$

For percentage change in the utility elasticity of income, ϕ, we have

$$\begin{aligned}
\phi &= \sum_i \frac{\Phi_i^{-1} B_i}{\sum_j \Phi_j^{-1} B_j} \phi_i && \text{differentiating (2.19),} \\
&= \sum_i \frac{X_i}{X} \phi_i && \text{substituting from (2.18),} \\
&= \sum_i S_i \phi_i && \text{putting } S_i = X_i/X, \\
&= S_P \phi_P + S_G \phi_G + S_S \phi_S.
\end{aligned}$$

Since $\phi_G = \phi_S = 0$, this reduces to

$$\boxed{\phi = S_P \phi_P} \tag{2.30}$$

As we see from these equations, the utility elasticity of income, Φ, is a weighted average of the lower-level utility elasticities Φ_P, Φ_G, and Φ_S. Since Φ_G and Φ_S are fixed, changes in Φ depend only on changes in the utility elasticity of private consumption expenditure, Φ_P. An increase in Φ_P, a shift so to speak towards decreasing returns from private consumption, leads to a budget reallocation away from private consumption toward government consumption and saving.

We now develop an equation for changes in Φ_P. As shown by Hanoch [3], with the CDE form for the private consumption demand system, the utility elasticity is a weighted average of the expansion parameters:

$$\Phi_P = \sum_i S_i^P R_i. \tag{2.31}$$

Differentiating, we obtain

$$\phi_P = \sum_i S_{Ri} s_{Pi}$$

where S_{Ri} denotes the expansion-parameter-weighted budget share of commodity i,

$$S_{Ri} = \frac{S_i^P R_i}{\sum_j S_j^P R_j} = \frac{S_i^P R_i}{\Phi_P}. \tag{2.32}$$

Then writing p_{Pi} for the price of commodity i in private consumption, and u_{Pi} for *per capita* private consumption of commodity i, we obtain

$$\boxed{\phi_P = \sum_i S_{Ri}(p_{Pi} + u_{Pi} - x_P)} \tag{2.33}$$

We see from this equation that shifts in private expenditure allocation toward commodities with high expansion parameters R_i tend to be associated with increases in the private expenditure utility elasticity, while shifts towards commodities with low expansion parameters tend to be associated with decreases.

For aggregate utility we use the general result:

Proposition 3 *For the upper level of a weakly separable demand system,*

$$max\ U(U_1, \ldots, U_G) \text{ subject to } \sum_i E_i(\mathbf{P_i}, U_i) = X,$$

where $E_i(\mathbf{P_i}, U_i)$ denotes the expenditure function for the i'th lower-level demand system, we have

$$x = p + \Phi u,$$

where p is an expenditure-share-weighted index of commodity group price indices, $p = \sum_i S_i p_i$, where S_i denotes the share of expenditure on group i in total expenditure, $S_i = X_i/X$, and p_i is an expenditure-weighted index of prices of commodities in group i, $p_i = \sum_j S_j^i p_{ij}$, where S_j^i denotes the share of commodity j from group i in total expenditure on group i, $S_j^i = X_{ij}/X_i$, where X_{ij} denotes expenditure on commodity j from group i, and p_{ij} denotes the price of commodity j from group i.

Proof. Define the Lagrangean

$$\mathcal{L} = U(U_1, \ldots, U_G) - \Lambda \left(\sum_i E_i(\mathbf{P_i}, U_i) - X \right). \tag{2.34}$$

Then the elasticity of utility with respect to income,

$$\begin{aligned}
\frac{\partial \log U}{\partial \log X} &= \frac{X}{U}\frac{\partial \mathcal{L}}{\partial X} && \text{by the envelope theorem} \\
&= \frac{X}{U}\Lambda && \text{differentiating (2.34)} \\
&= \Phi^{-1},
\end{aligned}$$

and the elasticity of utility with respect to the price of the j'th commodity in the i'th commodity group, that is, with respect to the j'th component of $\mathbf{P_i}$,

$$\begin{aligned}
\frac{\partial \log U}{\partial \log P_{ij}} &= \frac{P_{ij}}{U}\frac{\partial \mathcal{L}}{\partial P_{ij}} && \text{by the envelope theorem} \\
&= -\frac{P_{ij}}{U}\Lambda\frac{\partial X_i}{\partial P_{ij}} && \text{differentiating (2.34)} \\
&= -\Lambda\frac{P_{ij}Q_{ij}}{U} && \text{by Shephard's lemma} \\
&= -\frac{X}{U}\Lambda\frac{P_{ij}Q_{ij}}{X} \\
&= -\Phi^{-1}S_{ij},
\end{aligned}$$

where Q_{ij} denotes consumption, and S_{ij} the share in total expenditure, of commodity j in commodity group i (for the envelope theorem see e.g. Varian [8]). Then totally differentiating the indirect utility function, we have

$$\begin{aligned}
u &= \sum_i \sum_j \frac{\partial \log U}{\partial \log P_{ij}} p_{ij} + \frac{\partial \log U}{\partial \log X} x \\
&= -\Phi^{-1} \sum_i \sum_j S_{ij} p_{ij} + \Phi^{-1} x \\
&= \Phi^{-1}(x - p),
\end{aligned}$$

where p is the expenditure-share-weighted index of commodity group price indices,

$$\begin{aligned}
p &= \sum_i \sum_j S_{ij} p_{ij} \\
&= \sum_i \sum_j \frac{X_{ij}}{X} p_{ij} = \sum_i \frac{X_i}{X} \sum_j \frac{X_{ij}}{X_i} p_{ij} = \sum_i S_i \sum_j S^i_j p_{ij} \\
&= \sum_i S_i p_i,
\end{aligned}$$

as in the statement of the proposition. Solving for x, we obtain

$$x = p + \Phi u,$$

as was to be shown.■

Moving from the general formulation of proposition 3 to the specific case of the GTAP upper-level demand system, we copy the utility equation verbatim:

$$\boxed{x = p + \Phi u} \tag{2.35}$$

but write the disposition price index equation in the more specific form

$$\boxed{p = S_P p_P + S_G p_G + S_S p_S} \tag{2.36}$$

where p_P and p_G denote expenditure-weighted price indices for private and government consumption, and p_S denotes the price of saving.

2.4 Defects in the old treatment: assessment

Having identified an error in the derivation of the old theory (subsection 2.2), and revised the theory to remove that defect (subsection 2.3), we now compare the results of the revised theory with the original.

From equations (2.27)–(2.29), we see that under the revised theory, the upper-level income disposition shares are not in general fixed. They are fixed in the special case $\phi = \phi_P = 0$; from equation (2.30), this condition reduces to $\phi_P = 0$; from equation (2.31), this is satisfied with fixed private consumption expenditure shares S_i^P or uniform expansion parameters $R_i \equiv R$; that is, in the special case of a homothetic system. In general however, the old top-level demand equations, which assume fixed income disposition shares, are in error.

For utility, the old and new treatments use rather different approaches, so we cannot directly compare the two equations. Instead we derive a new utility equation consistent with the new theory but similar in approach to the old equation. We follow the old derivation as far as equation (2.12),

$$u = B_P u_P + B_G(q_G - n) + B_S(q_S - n).$$

Then, instead of the old $S_i = B_i/B$ (from equations (2.4)–(2.6)), we use the new equation (2.18), $S_i = \Phi_i^{-1} B_i / \sum_j \Phi_j^{-1} B_j$. From this and equation (2.19), $\Phi^{-1} = \sum_i \Phi_i^{-1} B_i$, we obtain

$$B_i = \frac{\Phi_i}{\Phi} S_i,$$

for $i = P, G, S$. Substituting into equation (2.12), and setting $\Phi_G = \Phi_S = 1$, we obtain

$$u = \Phi^{-1}[\Phi_P S_P u_P + S_G(q_G - n) + S_S(q_S - n)].$$

Comparing this with the old utility equation (2.13),

$$\begin{aligned} & Yu = Y_P u_P + Y_G(q_G - n) + Y_S(q_S - n), \\ \Leftrightarrow \quad & u = S_P u_P + S_G(q_G - n) + S_S(q_S - n), \end{aligned} \tag{2.37}$$

we note that the old computation is invalid in general, but valid in the special case

$\Phi = 1$, $\Phi_P = 1$. As we now show, standard GTAP data bases fall within the special case.

Proposition 4 *Under the old treatment, the utility elasticity of income is equal to one if and only if the expenditure-share-weighted sum of the CDE expansion parameters is equal to one.*

Proof. We have, from general theory, and from the treatment of saving and government consumption,

$$\begin{aligned} u_S &= x_S - p_S, \\ u_G &= x_G - p_G, \\ u_P &= \Phi_P^{-1}(x_P - p_P). \end{aligned}$$

With the fixed-expenditure-shares upper-level demand equations in the old system, this simplifies to

$$\begin{aligned} u_S &= x - p_S, \\ u_G &= x - p_G, \\ u_P &= \Phi_P^{-1}(x - p_P). \end{aligned}$$

Then recalling equation (2.37), we have

$$\begin{aligned} u &= S_P u_P + S_G(q_G - n) + S_S(q_S - n) \\ &= S_P u_P + S_G u_G + S_S u_S \\ &= \Phi_P^{-1} S_P(x - p_P) + S_G(x - p_G) + S_S(x - p_S), \end{aligned}$$

so the elasticity of income with respect to utility is

$$\left[\Phi_P^{-1} S_P + S_G + S_S\right]^{-1},$$

which (assuming $S_P \neq 0$) is equal to one if and only if $\Phi_P = 1$. But by equation (2.31), Φ_P is the expenditure-share-weighted sum of the CDE expansion parameters. So $\Phi = 1$ if and only if the share-weighted sum of the CDE expansion parameters is equal to one; as was to be shown.■

In constructing standard GTAP data bases, we have normalized the expansion parameters so that their expenditure-share-weighted sum is indeed equal to one. Then, from equation (2.31) and proposition 4, both the utility elasticity of private consumption expenditure and the utility elasticity of income are equal to one; so the old utility equation is valid locally. Since however normalization is not a theoretical requirement of the CDE, users may legitimately construct data bases with non-normalized parameters; and with those data bases, the utility equation is invalid. Furthermore, in multi-step simulations, initially normalized expansion parameters do not generally remain normalized; so even with initially normalized parameters, the utility equation is not exact.

The old utility equation (2.37) then is exactly accurate in Johansen simulations with data bases in which $\Phi = \Phi_P = 1$ (including standard GTAP data bases); accurate to first order in multi-step simulations with data bases in which $\Phi = \Phi_P = 1$; and inaccurate otherwise.

We note also that in GTAP simulations with the old treatment, the results for utility are slightly wrong even in Johansen simulations with standard GTAP data bases. This is because, although the utility equation itself is exact, the upper-level demand equations are wrong. In practice however, with standard data bases, errors in the utility results are likely to be small (see further section 5).

2.5 Possible remedies

There are several different approaches we might take to remedy the defects of the old treatment.

1. We might retain the basic premises of the old treatment, in particular, the CDE form for the private consumption demand system, while correcting the errors in the derived equations, adopting the revised theory expounded above (subsection 2.3).

2. We might seek a new functional form for the private consumption demand system, that would allow us to retain fixed budget shares in the upper-level system. Though it is not immediately obvious that such a form can be found, appendix B shows that this approach is indeed feasible.

3. We might abandon the concept of an upper-level demand system. Rather than representing the allocation of regional income as optimizing behavior by a fictitious regional household, we might simply impose some arbitrary rule. There would not necessarily be a concept of regional welfare, but instead a purely descriptive treatment of macroeconomic behavior. This might be a simple rule such as the fixed shares rule, or some more complex empirically motivated treatment.

Option (1) has the advantage of maximizing theoretical consistency with the old treatment. Its disadvantage is that the upper-level demand equations become more complex, so that the upper-level budget shares are no longer fixed. Options (2) and (3) let us keep the fixed budget shares property, but require changes in the basic theory. Option (3) also entails abandoning or radically revising the welfare measurement and decomposition theory, one of the special strengths of the GTAP model.

In this paper we do not assess the relative merits of these approaches, but explore only the most conservative approach, option (1). This provides part of the basis for a broader assessment of the alternatives, and offers an interim solution pending that assessment. The rest of this section is devoted to working out that interim solution.

2.6 A new treatment

We now develop a new treatment for the upper-level demand system. As discussed above (subsection 2.5), we correct errors in the old theory without changing its basic

premises. In particular, we retain the CDE form for the demand system for private consumption.

We do change one minor feature of the old framework: we redefine utility from government consumption, `ug`, as a *per capita* utility, so that it depends on *per capita* rather than total government consumption. Since saving and utility from private consumption are already *per capita* variables, this change allows us to treat the entire regional household demand system as the demand system of a representative regional household, rather than as a conglomeration of demand systems of different households (subsection 2.2).

To allow for exogenous shifts in the upper-level budget allocation, we treat the Cobb-Douglas distribution parameters B_i as variables. This allows the model to simulate exogenous budget shifts within the demand system, rather than (as with the old treatment) by overriding the demand system. With this addition, we use the revised theory derived above (subsection 2.3).

We modify the module structure within the GTAP model source code, to bring within the regional household module all equations derived from the upper level of the final demand system, rather than leaving them scattered across the regional household, government household, and investment and saving modules.

2.7 Shared variables

To implement the revised system, we first define some new cross-module variables. In the new theory, the private consumption and regional household modules share the levels coefficient Φ_P for the elasticity of private consumption expenditure with respect to utility from private consumption:

```
Coefficient (all,r,REG)
    UELASPRIV(r)
    #elasticity of cost wrt utility from private consumption#;
```

the corresponding percentage variable ϕ_P:

```
Variable (all,r,REG)
    uepriv(r)
    #elasticity of cost wrt utility from private consumption#;
```

and p_P, the private consumption price index:

```
Variable (all,r,REG)
    ppriv(r)
    #price index for private consumption expenditure in region r#;
```

The government consumption and regional household modules share the variable y_G, government consumption expenditure:

```
Variable (all,r,REG)
    yg(r)
    #regional government consumption expenditure, in region r#;
```

Similarly, the saving and regional household modules share the variable y_S, net saving:

```
Variable (all,r,REG)
    ysave(r) #net regional saving in region r#;
```

At the same time, the prices of composite commodities in government consumption, pg, and the quantities of composite commodities consumed by government, qg, previously shared between the government and private consumption modules, become local to the government consumption module.

2.8 Government consumption

Following the redefinition of utility U_G from government consumption as a *per capita* variable (subsection 2.6), we make the consequential changes in the government consumption module. Specifically, we revise the government consumption utility equation:

```
Equation GOVU
# utility from government consumption in r #
(all,r,REG)
    yg(r) - pop(r) = pgov(r) + ug(r);
```

and the government consumption demand equation:

```
Equation GOVDMNDS
# government consumption demands for composite commodities (HT 41) #
(all,i,TRAD_COMM)(all,r,REG)
    qg(i,r) - pop(r) = ug(r) - [pg(i,r) - pgov(r)];
```

Besides making these substantive changes, we remove all references to the government household from comments and labels in the source code.

2.9 Utility from private consumption

Within the private consumption module, we need new code for coefficients representing the levels of three theoretical variables: the utility elasticity of private consumption expenditure, the private consumption price index, and utility from private consumption.

To implement, we compute the level of the utility elasticity Φ_P according to equation (2.31):

```
Formula (all,r,REG)
    UELASPRIV(r) = sum{i,TRAD_COMM, CONSHR(i,r)*INCPAR(i,r)};
```

the expansion-parameter-weighted budget shares S_{Ri} according to equation (2.32):

```
Coefficient (all,i,TRAD_COMM)(all,r,REG)
    XWCONSHR(i,r)
    #expansion-parameter-weighted consumption share#;
Formula (all,i,TRAD_COMM)(all,r,REG)
    XWCONSHR(i,r) = CONSHR(i,r)*INCPAR(i,r)/UELASPRIV(r);
```

and percentage change in the utility elasticity ϕ_P according to equation (2.33):

```
Equation UTILELASPRIV
#elasticity of expenditure wrt utility from private consumption#
(all,r,REG)
    uepriv(r)
  = sum{i,TRAD_COMM, XWCONSHR(i,r)*[pp(i,r) + qp(i,r) - yp(r)]};
```

For utility from private consumption, we replace the (perfectly satisfactory) computation in the old code,

```
Equation PRIVATEU
# computation of utility from private consumption in r (HT 45) #
(all,r,REG)
    yp(r)
        = sum(i,TRAD_COMM, (CONSHR(i,r) * pp(i,r)))
        + sum(i,TRAD_COMM, (CONSHR(i,r) * INCPAR(i,r))) * up(r)
        + pop(r)
        ;
```

with a more readily interpretable computation based on the following general proposition:

Proposition 5 *For a demand system,*

$$max\ U(Q_1, \ldots, Q_I)\ subject\ to\ \sum_i P_i Q_i = X,$$

we have

$$x = p + \Phi u,$$

where p is an expenditure-share-weighted index of commodity prices, $p = \sum_i p_i$.

Proof. This is a special case of proposition 3, where the lower-level demand systems each cover just one commodity and the subutilities U_i are just the commodity consumption quantities Q_i.■

Applying proposition 5 to utility from private consumption, we have

$$y_P - n = p_P + \Phi_P u_P, \tag{2.38}$$

where the price index for private consumption,

$$p_P = \sum_i S_i^P p_{Pi}, \tag{2.39}$$

We compute the private consumption price index p_P according to equation (2.39):

```
Equation PHHLDINDEX
# price index for private consumption expenditure #
(all,r,REG)
    ppriv(r) = sum{i,TRAD_COMM, CONSHR(i,r)*pp(i,r)};
```

and utility from private consumption according to equation (2.38):

```
Equation PRIVATEU
# computation of utility from private consumption in r (HT 45) #
(all,r,REG)
    yp(r) - pop(r) = ppriv(r) + UELASPRIV(r)*up(r);
```

2.10 Saving

In subsection 2.7, we introduced a new variable `ysave` representing the money value of saving. In the saving module, we now add an equation relating it to the quantity of saving:

```
Equation SAVEQUANT #quantity of saving# (all,r,REG)
    ysave(r) = psave(r) + qsave(r);
```

The reason for adding the new variable and equation is to let users target the money value of saving, just as they can target private and government consumption expenditure.

2.11 Regional household preliminaries

Within the regional household module we revise the submodules for regional household demands (subsection 2.12) and aggregate utility (subsection 2.13). We compute at the outset some coefficients common to both submodules, the upper-level shares S_i in regional income, $S_i = X_i/X$:

```
Coefficient (all,r,REG)
    XSHRPRIV(r) #private expenditure share in regional income#;
Formula (all,r,REG)
    XSHRPRIV(r) = PRIVEXP(r)/INCOME(r);

Coefficient (all,r,REG)
    XSHRGOV(r) #government expenditure share in regional income#;
Formula (all,r,REG)
    XSHRGOV(r) = GOVEXP(r)/INCOME(r);

Coefficient (all,r,REG)
    XSHRSAVE(r) #saving share in regional income#;
Formula (all,r,REG)
    XSHRSAVE(r) = SAVE(r)/INCOME(r);
```

We also declare some common variables: the distribution parameters b_i from the top-level demand equation:

```
Variable (all,r,REG)
    dppriv(r) #private consumption distribution parameter#;
Variable (all,r,REG)
    dpgov(r) #government consumption distribution parameter#;
Variable (all,r,REG)
    dpsave(r) #saving distribution parameter#;
```

and ϕ, the utility elasticity of income:

```
Variable (all,r,REG)
    uelas(r) #elasticity of cost of utility wrt utility#;
```

2.12 Regional household demands

We extend the revised theory (subsection 2.3) to treat the Cobb-Douglas distribution parameters of the upper-level demand system as variables in the simultaneous equation system. For the demand equations, we extend equations (2.27)–(2.29), and substitute aggregate for *per capita* variables, obtaining

$$y_P - y = -(\phi_P - \phi) + b_P, \tag{2.40}$$

$$y_G - y = \phi + b_G, \tag{2.41}$$

$$y_S - y = \phi + b_S. \tag{2.42}$$

For the utility elasticity of income, ϕ, we extend equation (2.30), obtaining

$$\phi = S_P \phi_P - b_{AV}, \tag{2.43}$$

where b_{AV} denotes a weighted average of the distribution parameters,

$$b_{AV} = \sum_i S_i b_i. \tag{2.44}$$

To implement this, we first declare the distribution parameters b_i:

```
Variable (all,r,REG)
    dppriv(r) #private consumption distribution parameter#;
Variable (all,r,REG)
    dpgov(r) #government consumption distribution parameter#;
Variable (all,r,REG)
    dpsave(r) #saving distribution parameter#;
```

compute the weighted average of the distribution parameters according to equation (2.44):

```
Variable (all,r,REG)
    dpav(r) #average distribution parameter shift, for EV calc.#;
Equation DPARAV #average distribution parameter shift#
    (all,r,REG)
    dpav(r)
  =   XSHRPRIV(r)*dppriv(r)
    + XSHRGOV(r)*dpgov(r)
    + XSHRSAVE(r)*dpsave(r)
    ;
```

and compute the utility elasticity of income according to equation (2.43):

```
Equation UTILITELASTIC #elasticity of cost of utility wrt utility#
    (all,r,REG)
    uelas(r) = XSHRPRIV(r)*uepriv(r) - dpav(r);
```

Finally we implement the regional household demand equations (2.40)–(2.42):

```
Equation PRIVCONSEXP #private consumption expenditure# (all,r,REG)
    yp(r) - y(r) = -[uepriv(r) - uelas(r)] + dppriv(r);

Equation GOVCONSEXP #government consumption expenditure# (all,r,REG)
    yg(r) - y(r) = uelas(r) + dpgov(r);

Equation SAVING #saving# (all,r,REG)
    ysave(r) - y(r) = uelas(r) + dpsave(r);
```

2.13 Regional household utility

Now we compute utility for the regional household. Recalling the levels equation (2.17),

$$U = C \prod_i U_i^{B_i},$$

we extend the differential equation (2.35) to treat the scaling factor C and the distribution parameters B_i as variable, obtaining

$$u = c + \sum_i B_i(\log U_i)b_i + \Phi^{-1}(x - p). \tag{2.45}$$

We remark that the initial settings of $\log U_i$ are arbitrary, in that they are not constrained by the observed state of the economy as recorded in the data base, and do not affect the positive properties of the demand system. They affect only the sensitivity of utility to changes in preferences. Once the initial settings have been made however, theory dictates how the coefficients should be updated. By adjusting the settings of $\log U_i$, we can make utility increasing in the distribution parameters, decreasing, or locally invariant. We can also make it increasing with respect to some of the distribution parameters and decreasing with respect to others.

The requirements for implementing distribution terms in the equation are somewhat onerous, in that we need to store and update both the distribution parameters B_i and the quantities U_i—even though these are not required for any positive variables. Given all this, and the doubtful meaningfulness of utility comparisons in the presence of preference changes, it may seem hardly worthwhile incorporating the distribution parameters into the utility equation. Yet we attach some importance to it. Some important macro closures involve exogenizing the balance of trade and endogenizing a distributional variable. It would be an inconvenience when using these closures to forego results for utility and equivalent variation and the welfare decomposition. Moreover it seems that most of the welfare analysis should be just as meaningful with an exogenous as with an endogenous trade balance.

Since we must have the distributional parameters but do not welcome their welfare effects, we do what we can to minimize them. We choose initial parameter values so that, in small change simulations, changes in the distributional parameters do not affect utility (subsection 2.15). And we provide, in connection with the measurement

of equivalent variation, a mechanism for minimizing the welfare effects of distributional parameter changes in large-change simulations (subsection 3.8).

To implement equation (2.45), we first declare as percentage change variables utility u:

```
Variable (all,r,REG)
    u(r) #per capita utility from aggregate hhld expend., in region r#;
```

and the constant c in the utility function:

```
Variable (all,r,REG)
    au(r) #input-neutral shift in utility function#;
```

We need next the levels values of the distributional parameters B_i. From equations (2.18) and (2.19), we find that we can calculate them as

$$B_i = \frac{\Phi_i S_i}{\Phi}, \tag{2.46}$$

given the levels value of the utility elasticity Φ. The theory however does not determine the levels value of Φ. We could store Φ in the data base, but it is slightly more convenient to store instead the sum of the distribution parameters, $B = \sum_i B_i$; since when the utility elasticity of private consumption expenditure, Φ_P, is non-unitary, it is more natural to take B than Φ as unitary, leaving the other coefficient to take a non-obvious calculated value. From equation (2.46), we obtain the formula giving Φ in terms of given B:

$$\Phi = \frac{\sum_i S_i \Phi_i}{B}. \tag{2.47}$$

To calculate the sum B of the distribution parameter in updated databases, we use the corresponding percentage change variable b. We declare this variable:

```
Variable (all,r,REG)
    dpsum(r) #sum of the distribution parameters#;
```

and define the corresponding levels coefficient:

```
Coefficient (all,r,REG)
    DPARSUM(r) #sum of distribution parameters#;
Read
    DPARSUM from file GTAPDATA header "DPS";
Update (all,r,REG)
    DPARSUM(r) = dpsum(r);
```

This lets us define the level Φ of the utility elasticity of expenditure, according to equation (2.47):

```
Coefficient (all,r,REG)
    UTILELAS(r) #elasticity of cost of utility wrt utility#;
Formula (all,r,REG)
    UTILELAS(r)
  = [UELASPRIV(r)*XSHRPRIV(r) + XSHRGOV(r) + XSHRSAVE(r)]/DPARSUM(r);
```

We define the levels coefficients B_i for the distribution parameters using equation (2.46):

```
Coefficient (all,r,REG)
    DPARPRIV(r) #private consumption distribution parameter#;
Formula (all,r,REG)
    DPARPRIV(r) = UELASPRIV(r)*XSHRPRIV(r)/UTILELAS(r);

Coefficient (all,r,REG)
    DPARGOV(r) #government consumption distribution parameter#;
Formula (all,r,REG)
    DPARGOV(r) = XSHRGOV(r)/UTILELAS(r);

Coefficient (all,r,REG)
    DPARSAVE(r) #saving distribution parameter#;
Formula (all,r,REG)
    DPARSAVE(r) = XSHRSAVE(r)/UTILELAS(r);
```

We define also the levels coefficients U_i for the goods in the top-level utility function:

```
Coefficient (all,r,REG)
    UTILPRIV(r) #utility from private consumption#;
Read
    UTILPRIV from file GTAPDATA header "UP";
Update (all,r,REG)
    UTILPRIV(r) = up(r);

Coefficient (all,r,REG)
    UTILGOV(r) #utility from government consumption#;
Read
    UTILGOV from file GTAPDATA header "UG";
Update (all,r,REG)
    UTILGOV(r) = ug(r);

Coefficient (all,r,REG)
    UTILSAVE(r) #utility from saving#;
Read
    UTILSAVE from file GTAPDATA header "US";
Update (change) (all,r,REG)
    UTILSAVE(r) = [1.0/100.0]*[qsave(r) - pop(r)]*UTILSAVE(r);
```

We compute the outlays price index p according to equation (2.36):

```
Variable (all,r,REG)
    p(r) #price index for disposition of income by regional household#;
Equation PRICEINDEXREG
    #price index for disposition of income by regional household#
    (all,r,REG)
    p(r)
  =   XSHRPRIV(r)*ppriv(r)
    + XSHRGOV(r)*pgov(r)
    + XSHRSAVE(r)*psave(r)
    ;
```

After all these preliminaries, we compute regional household utility u, according to equation (2.35):

```
Equation UTILITY #regional household utility# (all,r,REG)
  u(r) = au(r)
    + DPARPRIV(r)*loge(UTILPRIV(r))*dppriv(r)
    + DPARGOV(r)*loge(UTILGOV(r))*dpgov(r)
    + DPARSAVE(r)*loge(UTILSAVE(r))*dpsave(r)
    + [1.0/UTILELAS(r)]*[y(r) - pop(r) - p(r)];
```

One task remains, to determine the variable `dpsum` used to update the coefficient `DPARSUM`:

```
Equation DISTPARSUM #sum of the distribution parameters# (all,r,REG)
    DPARSUM(r)*dpsum(r)
  = DPARPRIV(r)*dppriv(r) + DPARGOV(r)*dpgov(r) + DPARSAVE(r)*dpsave(r);
```

2.14 Shifting income allocation without affecting the utility elasticity

As described in subsection 2.13, the new treatment minimizes the effects of changes in the distribution parameters on the current level of utility in simulations with standard GTAP data bases. We now describe a mechanism through which users can adjust the distributional parameters without affecting the utility elasticity of income. This helps to make utility results easier to interpret.

Suppose for example that we wish to fix expenditure on government consumption, y_G, and to that end exogenize it and endogenize the corresponding distribution parameter B_G. Then depending on changes in real income, the distribution parameter may either rise or fall; and depending on whether it rises or falls, the utility elasticity of income rises or falls. If it rises, then a large change in real income may generate a small change in utility; if it falls, then a small change in real income may generate a large change in utility. Since the direction of change in the utility elasticity is just a side-effect of fixing y_G, the magnitude of the utility change is somewhat arbitrary.

We avoid this by introducing a group of shift variables to drive the distribution parameters. They include both shifters specific to individual distribution parameters and a shifter common to all of them. From equation (2.43),

$$\phi = S_P \phi_P - b_{AV},$$

we note that the effect of the distribution parameters on the utility elasticity is encapsulated in the average of the distribution parameters, b_{AV}. So we can neutralize the effects of shocks in the specific shifters by endogenizing the common shifter and exogenizing b_{AV} (`dpav`).

Accordingly, we write each distribution parameter as the product of a specific and the generic scaling factor:

```
Variable (all,r,REG)
    dpfpriv(r) #private-consumption-specific distparam shift#;
Variable (all,r,REG)
```

```
    dpfgov(r) #government-consumption-specific distparam shift#;
Variable (all,r,REG)
    dpfsave(r) #saving-specific distparam shift#;
Variable (all,r,REG)
    dpshift(r) #generic distparam shift#;

Equation DISTPARPRIV #private consumption distribution parameter#
    (all,r,REG)
    dppriv(r) = dpfpriv(r) + dpshift(r);
Equation DISTPARGOV #government consumption distribution parameter#
    (all,r,REG)
    dpgov(r) = dpfgov(r) + dpshift(r);
Equation DISTPARSAVE #saving distribution parameter#
    (all,r,REG)
    dpsave(r) = dpfsave(r) + dpshift(r);
```

In the usual case, the distributional parameters are fixed, with

exogenous `dpfpriv`, `dpfgov`, `dpfsave`, `dpshift`;

endogenous `dppriv`, `dpgov`, `dpsave`, `dpav`.

Suppose however that the user wants to fix (or shock) government consumption expenditure, `yg`. She can do this by setting `yg` exogenous, and:

exogenous `dpfpriv`, `dpfsave`, `dpshift`;

endogenous `dpfgov`, `dppriv`, `dpgov`, `dpsave`, `dpav`.

If however she wishes to neutralize the effect of the distributional parameter shift on the utility elasticity of income, she may set:

exogenous `dpfpriv`, `dpfsave`, `dpav`;

endogenous `dpfgov`, `dpshift`, `dppriv`, `dpgov`, `dpsave`.

2.15 Changes to the data file

As described in subsection 2.13, we read a new coefficient `DPARSUM` from the data file. To do this we need a new data file array `DPS`, with dimension `REG`. The new array records, for each region, the sum of the distribution parameters.

The setting of this parameter has no effect on the positive variables in the model, nor on the equivalent variation, but through the top-level utility elasticity `UTILELAS` it does affect regional utility `u`. We set it initially at 1 in each region; changes in the distribution parameters `dppriv`, `dpgov`, and `dpsave` may affect its value in updated data bases.

In standard data bases, with both `UTILELASPRIV` and `DPSUM` set equal to 1, the utility elasticity `UTILELAS` of generalized expenditure is equal to 1. This means that initially, a one per cent change in regional income translates into a one per cent change in regional utility.

We also set values for three region-dimension arrays representing levels for the commodities in the top level of the regional demand system: utility from private consumption, `UP`; utility from government consumption, `UG`; and saving, `Q`. We set these all to zero to ensure that with the standard data base, changes in the distributional parameters have no first-order effect on utility (subsection 2.13).

3 Equivalent variation

The old demand system having been found defective and revised, it is natural to review the treatment of the equivalent variation. As it turns it, this too is defective, but its defects are largely independent of those of the demand system. In the remainder of this section we derive the old treatment (subsection 3.1), assess its defects (subsection 3.2), develop a new treatment (subsection 3.3), and implement it (subsections 3.5–3.9).

By definition, the equivalent variation (EV),

$$EV = Y_{EV} - \bar{Y},$$

where Y_{EV} denotes regional income required to achieve current utility at initial prices, and $\bar{Y}$ denotes initial regional income. Differentiating, we obtain:

$$d\,EV = \frac{1}{100} Y_{EV} y_{EV}. \tag{3.1}$$

This equation provides a starting point for both the old and new treatments.

3.1 The old treatment

In the old treatment, EV is computed according to the equation

```
Equation EVREG
# regional EV, the money metric welfare change (HT 67) #
(all,r,REG)
    EV(r)
  = [REGEXP(r)/100]*[URATIO(r)*POPRATIO(r)]*[u(r) + pop(r)];
```

In mathematical notation, we may write this as

$$d\,EV = \frac{1}{100} \bar{Y} U_R N_R (n + u), \tag{3.2}$$

where $U_R = U/\bar{U}$ is the ratio of current to initial utility, $N_R = N/\bar{N}$ the ratio of current to initial population, and n the percentage change in population. This equation is not covered in the original GTAP documentation (Hertel and Tsigas [5]), having been added after that was written. We now provide a derivation, in order to explore the conditions under which the equation is valid.

Proposition 6 *Equation* (3.2) *is a valid first-order approximation for small changes in U, provided that initially the utility elasticity of income, Φ, is equal to one.*

Proof. Recall equation (3.1),

$$d\,EV = \frac{1}{100} Y_{EV} y_{EV}.$$

Now

$$y_{EV} = n + x_{EV},$$

where x_{EV} denotes percentage change in *per capita* expenditure required to achieve current utility at initial prices. Also, setting the price index p equal to zero in equation (2.35), we have

$$x_{EV} = \Phi_{EV} u, \tag{3.3}$$

where Φ_{EV} denotes the utility elasticity of income, evaluated at current utility and initial prices. So

$$y_{EV} = n + \Phi_{EV} u, \tag{3.4}$$

and

$$d\,EV = \frac{1}{100} Y_{EV}(n + \Phi_{EV} u).$$

Although this equation is suitable for implementation, it does not lead directly to the GTAP 4.1 equation (3.2). To derive that we need to replace Y_{EV} with an expression involving $\bar{Y}$. Now

$$Y_{EV} = N X_{EV} = N_R \bar{N} X_{EV},$$

where X_{EV} denotes *per capita* expenditure required to achieve current utility at initial prices $\bar{\mathbf{P}}$; and

$$X_{EV} = U_R^{\Phi_{ARC}} \bar{X},$$

where Φ_{ARC} denotes the arc elasticity of income with respect to utility along the arc between $(\bar{\mathbf{P}}, \bar{U})$ and $(\bar{\mathbf{P}}, U)$; so

$$\begin{aligned} Y_{EV} &= N_R U_R^{\Phi_{ARC}} \bar{N} \bar{X} \\ &= N_R U_R^{\Phi_{ARC}} \bar{Y}, \end{aligned}$$

and

$$d\,EV = \frac{1}{100} N_R U_R^{\Phi_{ARC}} \bar{Y}(n + \Phi_{EV} u).$$

Suppose that initially Φ is equal to one. Then Φ_{EV} also is initially equal to one, since Φ_{EV} is initially equal to Φ. So, by continuity, Φ_{EV} is arbitrarily close to one for sufficiently small changes in U. Also, by the mean value theorem, Φ_{ARC} is arbitrarily close to the initial value of Φ, one, for sufficiently small changes in U. So, to a first-order approximation,

$$d\,EV \approx \frac{1}{100} N_R U_R \bar{Y}(n + u),$$

as was to be shown.■

3.2 Defects in the old treatment

As shown above (subsection 3.1), the old computation of EV is not exact, but is a valid approximation when the utility elasticity of income Φ is equal to one. Recalling proposition 4, we note that the condition is satisfied in standard GTAP data bases. Much like the old utility equation then (subsection 2.4), the EV equation is exactly accurate in Johansen simulations with data bases in which $\Phi = 1$ (including standard GTAP data bases); accurate to first order in multi-step simulations with data bases in which $\Phi = 1$; and inaccurate otherwise.

While the old treatment works well for standard GTAP data bases and small utility changes, a treatment that works well with non-standard data bases and large changes would of course be even better. This we now develop.

3.3 A new treatment

We seek a new formula for the equivalent variation that does not assume a unit elasticity of income with respect to utility, and is consistent with the new implementation of the regional household demand system.

We cannot implement equation (3.1) for EV directly, since we do not have an explicit functional form for the regional household expenditure function. Indeed, we do not have an explicit functional form even for the private consumption expenditure function. We can however compute the expenditure function indirectly, by implementing the demand system and solving for expenditure Y given utility U. It is then easy to compute EV.

The regional demand system already present in the model gives the relation between expenditure Y, current utility U, and current prices $\mathbf{P}$. To find the expenditure Y_{EV} required to achieve current utility U at initial prices $\bar{\mathbf{P}}$, we implement a *shadow* demand system with the same utility level as the ordinary system, but with prices held at initial levels. The expenditure level in this shadow system is just the Y_{EV} required to calculate EV.

Recalling the equation (3.3) for percentage change in equivalent income, $x_{EV} = \Phi_{EV} u$, we see that we can compute equivalent income provided that we track Φ_{EV}, the utility elasticity evaluated at current utility and initial prices. To track Φ_{EV} we need to compute the corresponding percentage change variable ϕ_{EV}. To do that we need to include in the shadow system most of the upper-level regional household demand system.

Furthermore, as shown by equation (2.43), $\phi = S_P \phi_P - b_{AV}$, the regional household elasticity ϕ depends on the private consumption elasticity ϕ_P. To compute that elasticity, we need to include part of the private consumption demand system. The private consumption demand system also supplies to the top level system the change variable u_P for utility from private consumption required to update the levels coefficient U_P used in the top-level utility equation. Similarly the top-level demand system requires a variable u_G to be supplied from a government consumption demand system. Altogether then the shadow demand system includes four parts: a government consumption demand system, a private consumption demand system, an upper-level regional household demand system, and equations relating income to the equivalent variation.

3.4 Equivalent variation with preference change

So far we have not considered the effect of preference change on the equivalent variation. When the top level distribution parameters B_i or the scaling constant C change, should we calculate the equivalent variation at initial preferences or at final preferences, or should we include the effects of the preference change in the equivalent variation?

Extending our earlier notation, we may write $E(\mathbf{P}, U; \mathbf{A})$ for the generalized expenditure function evaluated at prices $\mathbf{P}$, utility U, and preferences $\mathbf{A}$. Initial income, $\bar{Y}$, is equal to $E(\bar{\mathbf{P}}, \bar{U}; \bar{\mathbf{A}})$, that is, to the expenditure function evaluated at initial prices, utility, and preferences. If we calculate the equivalent variation at initial preferences, then

$$EV = E(\bar{\mathbf{P}}, U; \bar{\mathbf{A}}) - E(\bar{\mathbf{P}}, \bar{U}; \bar{\mathbf{A}}); \tag{3.5}$$

if we calculate it at final preferences, then

$$EV = E(\bar{\mathbf{P}}, U; \mathbf{A}) - E(\bar{\mathbf{P}}, \bar{U}; \mathbf{A}); \tag{3.6}$$

if we include preference change in the equivalent variation, then

$$EV = E(\bar{\mathbf{P}}, U; \mathbf{A}) - E(\bar{\mathbf{P}}, \bar{U}; \bar{\mathbf{A}}). \tag{3.7}$$

Standard theory offers no guidance here, since it considers the equivalent variation only with constant preferences. In chosing between the measures, we prefer to minimize the effects of distribution parameter changes on the equivalent variation. One might guess that the way to do this is to adopt one of the measures that holds the distribution parameters constant; but in fact, a little reflection shows that the opposite is the case, and that we should adopt the measure that incorporates the preference changes.

Suppose, to take a simple example, that there is an increase in the scaling constant C, with income and prices constant. Then utility increases, so the region needs more income to obtain the initial level of utility with the old scaling constant, and measure (3.5) of the equivalent variation is strictly positive; and likewise the region can get the old utility level more cheaply with the new scaling constant, so measure (3.6) is also strictly positive; but with measure (3.7), the increase in utility and the change in the scaling constant offset each other. More precisely, it is easy to show that with measure (3.7):

- the effect on the equivalent variation of a change in the scaling constant C is exactly zero;
- the effect of a change in the distribution parameters B_i, unaccompanied by changes in prices, through the demand system on the equivalent variation, is exactly zero (of course, there may be general equilbrium effects);
- the effect of a change in the distribution parameters B_i, whether or not accompanied by changes in prices, through the demand system on the equivalent variation, is zero in a linearized model solution.

(See further subsection 4.3 and equation 4.5.)

Accordingly, we adopt measure (3.7), defining the equivalent variation to include changes in preferences.

3.5 Shared objects

We begin by declaring the coefficients and variables that the EV module contributes to the rest of the model. The module's primary function is to compute the regional and world-wide equivalent variations:

```
Variable (Change)(all,r,REG)
    EV(r) #equivalent variation, $ US million#;
Variable (Change)
    WEV #equivalent variation for the world#;
```

But it also contributes several coefficients needed for the EV decomposition (section 4): the utility elasticity of generalized expenditure Φ_{EV} in the EV shadow system:

```
Coefficient (all,r,REG)
    UTILELASEV(r)
    #utility elasticity of generalized expenditure, for EV calcs#;
```

and the quantities U_i of the goods in the shadow top-level demand system:

```
Coefficient (all,r,REG)
    UTILPRIVEV(r) #utility from private consumption, for EV calcs#;

Coefficient (all,r,REG)
    UTILGOVEV(r) #utility from private consumption, for EV calcs#;

Coefficient (all,r,REG)
    UTILSAVEEV(r) #utility from private consumption, for EV calcs#;
```

Finally we declare several variables shared between different parts of the EV module. The government consumption shadow demand system computes utility from government consumption, `ugev`, for use in the upper-level shadow demand system. The private consumption shadow demand system computes utility from private consumption, `upev`, and the elasticity of private consumption expenditure with respect to utility from private consumption, `ueprivev`, for use in the shadow upper-level demand system. The shadow upper-level demand system computes government consumption expenditure, `ygev`, for use in the shadow government consumption demand system; private consumption expenditure, `ypev`, for use in the shadow private consumption demand system; and equivalent income, `yev` and `INCOMEEV`, for use in the equivalent variation calculation:

```
Variable (all,r,REG)
    ugev(r) #per capita utility from gov't expend., shadow#;
Variable (all,r,REG)
    upev(r) #per capita utility from private expend., shadow#;
Variable (all,r,REG)
    ueprivev(r)
```

```
    #utility elasticity of private consn expenditure, shadow#;
Variable (all,r,REG)
    ygev(r)
    #government consumption expenditure, in region r, shadow#;
Variable (all,r,REG)
    ypev(r)
    #private consumption expenditure, in region r, shadow#;
Variable (all,r,REG)
    yev(r) #equivalent income, for EV#;

Coefficient (all,r,REG)
    INCOMEEV(r) #equivalent income, for EV#;
```

3.6 The shadow government consumption demand system

The task of the shadow government consumption demand system is to compute shadow values for the change variable u_G for utility from government consumption. It contains just one equation, a simplified version of the equation from the ordinary government consumption module (subsection 2.8) relating utility from government consumption to government consumption expenditure, with the price variable omitted:

```
Equation GOVUEV
# utility from government consumption in r #
(all,r,REG)
    ygev(r) - pop(r) = ugev(r);
```

3.7 The shadow private consumption demand system

The task of the shadow private consumption demand system is to compute shadow values for the change variables u_P for utility from private consumption, and ϕ_P for the elasticity of private consumption expenditure with respect to utility from private consumption.

Recalling equation (2.33), we have, with fixed prices,

$$\phi_P = \sum_i S_{Ri}(u_{Pi} - x_P), \tag{3.8}$$

where S_{Ri} denotes the expansion-parameter-weighted budget share, $S_i R_i / \Phi_P$, of commodity i in the shadow private consumption demand system. So to compute the shadow elasticity, we need shadow system values for the budget shares S_{Ri} and the private consumption demands u_{Pi}. To compute the private consumption demands we need the expenditure elasticities, and to compute them and the expansion-parameter-weighted budget shares, we need the ordinary budget shares. To compute the ordinary budget shares, we need to record shadow private consumption expenditures for individual composite commodities.

We implement as a shadow system as much of the private consumption demand system as we need to compute the shadow private consumption budget shares. Since the shadow system uses the same theory as the ordinary private consumption demand

system, we do not provide derivations, but instead refer the reader to the original documentation (Hertel and Tsigas [5]).

We begin by declaring the shadow private consumption demand variable:

```
Variable (all,i,TRAD_COMM)(all,r,REG)
    qpev(i,r)
    #private hhld demand for commodity i in region r, shadow#;
```

We then define the shadow private consumption expenditure levels:

```
Coefficient (all,i,TRAD_COMM)(all,r,REG)
    VPAEV(i,r)
    #private hhld expend. on i in r valued at agent's prices, shadow#;
Formula (initial) (all,i,TRAD_COMM)(all,r,REG)
    VPAEV(i,r) = VPA(i,r);
Update (all,i,TRAD_COMM)(all,r,REG)
    VPAEV(i,r) = qpev(i,r);
```

and the shadow private consumption budget shares:

```
Coefficient (all,r,REG)
    VPAREGEV(r) #private consumption expenditure in region r, shadow#;
Formula (all,r,REG)
    VPAREGEV(r) = sum{i,TRAD_COMM, VPAEV(i,r)};

Coefficient (all,i,TRAD_COMM)(all,r,REG)
    CONSHREV(i,r)
    #share of private hhld consn devoted to good i in r, shadow#;
Formula (all,i,TRAD_COMM)(all,r,REG)
    CONSHREV(i,r) = VPAEV(i,r)/VPAREGEV(r);
```

We compute the expenditure elasticities as in the ordinary demand system, but using the shadow budget shares `CONSHREV` instead of the ordinary shares `CONSHR`:

```
Coefficient (all,i,TRAD_COMM)(all,r,REG)
    EYEV(i,r)
    #expend. elast. of private hhld demand for i in r, shadow#;
Formula (all,i,TRAD_COMM)(all,r,REG)
    EYEV(i,r)
  =     [1.0/[sum{n, TRAD_COMM, CONSHREV(n,r)*INCPAR(n,r)}]]
      * [
            INCPAR(i,r)*[1.0 - ALPHA(i,r)]
          + sum{n, TRAD_COMM, CONSHREV(n,r)*INCPAR(n,r)*ALPHA(n,r)}
        ]
    + ALPHA(i,r)
    - sum{n, TRAD_COMM, CONSHREV(n,r) * ALPHA(n,r)}
    ;
```

We can now compute the shadow private consumption demands, required as shown above to update the levels coefficients for private consumption expenditure:

```
Equation PRIVDMNDSEV
    #private hhld demands for composite commodities, shadow#
    (all,i,TRAD_COMM)(all,r,REG)
    qpev(i,r) - pop(r) = EYEV(i,r)*[ypev(r) - pop(r)];
```

Next we compute the utility elasticity Φ_P of private consumption expenditure:

```
Coefficient (all,r,REG)
    UELASPRIVEV(r)
    #elast. of cost wrt utility from private consn, shadow#;
Formula (all,r,REG)
    UELASPRIVEV(r) = sum{i,TRAD_COMM, CONSHREV(i,r)*INCPAR(i,r)};
```

This appears both in the equation for utility u_P from private consumption, a simplified version of equation (2.38):

```
Equation PRIVATEUEV
# computation of utility from private consumption in r (HT 45) #
(all,r,REG)
    ypev(r) - pop(r) = UELASPRIVEV(r)*upev(r);
```

and as the denominator in the formula for the expansion-parameter-weighted budget shares S_{Ri}:

```
Coefficient (all,i,TRAD_COMM)(all,r,REG)
    XWCONSHREV(i,r)
    #expansion-parameter-weighted consumption share, shadow#;
Formula (all,i,TRAD_COMM)(all,r,REG)
    XWCONSHREV(i,r) = CONSHREV(i,r)*INCPAR(i,r)/UELASPRIVEV(r);
```

With these shares, as shown in equation (3.8), we compute the change variable ϕ_P for the utility elasticity of private consumption expenditure:

```
Equation UTILELASPRIVEV
    #elasticity of cost wrt utility from private consn, shadow#
    (all,r,REG)
    ueprivev(r)
  = sum{i,TRAD_COMM, XWCONSHREV(i,r)*[qpev(i,r) - ypev(r)]};
```

3.8 The shadow upper-level regional household demand system

The tasks of the shadow upper-level regional household demand system are to compute shadow income and shadow private consumption expenditure, given utility. To compute shadow income, it tracks the elasticity Φ_{EV} of shadow income with respect to utility. Recalling equations (2.43), $\phi = S_P\phi_P - b_{AV}$, and (2.44), $b_{AV} = \sum_i S_i b_i$, we see that it must also compute shadow values for the upper level income disposition shares S_i, $i = P, G, S$. That in turn requires shadow values for the upper-level components of income disposition, Y_P, Y_G, and Y_S; and those, shadow results for the related percentage change variables y_P, y_G, and q_S.

We begin by declaring a change variable for change in real saving:

```
Variable (all,r,REG)
    qsaveev(r) #regional demand for NET saving, shadow#;
```

We then compute the level of income in the shadow system:

```
Formula (initial) (all,r,REG)
    INCOMEEV(r) = INCOME(r);
Update (all,r,REG)
    INCOMEEV(r) = yev(r);
```

levels for the upper-level components of income disposition:

```
Coefficient (all,r,REG)
    PRIVEXPEV(r)
    #private consumption expenditure in region r, shadow#;
Formula (initial) (all,r,REG)
    PRIVEXPEV(r) = PRIVEXP(r);
Update (all,r,REG)
    PRIVEXPEV(r) = ypev(r);
!< PRIVEXPEV should agree with VPAREGEV.>!

Coefficient (all,r,REG)
    GOVEXPEV(r)
    #government consumption expenditure in region r, shadow#;
Formula (initial) (all,r,REG)
    GOVEXPEV(r) = GOVEXP(r);
Update (all,r,REG)
    GOVEXPEV(r) = ygev(r);

Coefficient (all,r,REG)
    SAVEEV(r) #saving in region r, shadow#;
Formula (initial) (all,r,REG)
    SAVEEV(r) = SAVE(r);
Update (all,r,REG)
    SAVEEV(r) = qsaveev(r);
```

and upper-level income disposition shares:

```
Coefficient (all,r,REG)
    XSHRPRIVEV(r)
    #private expenditure share in regional income, shadow#;
Formula (all,r,REG)
    XSHRPRIVEV(r) = PRIVEXPEV(r)/INCOMEEV(r);

Coefficient (all,r,REG)
    XSHRGOVEV(r)
    #government expenditure share in regional income, shadow#;
Formula (all,r,REG)
    XSHRGOVEV(r) = GOVEXPEV(r)/INCOMEEV(r);

Coefficient (all,r,REG)
    XSHRSAVEEV(r) #saving share in regional income, shadow#;
```

```
Formula (all,r,REG)
    XSHRSAVEEV(r) = SAVEEV(r)/INCOMEEV(r);
```

This enables us to compute the weighted average of the distribution parameters, following equation (2.44):

```
Variable (all,r,REG)
    dpavev(r) #average distribution parameter shift, shadow#;
Equation DPARAVEV #average distribution parameter shift, shadow#
    (all,r,REG)
    dpavev(r)
  =   XSHRPRIVEV(r)*dppriv(r)
    + XSHRGOVEV(r)*dpgov(r)
    + XSHRSAVEEV(r)*dpsave(r)
    ;
```

and the utility elasticity of income, following equation (2.43):

```
Variable (all,r,REG)
    uelasev(r)
    #elasticity of cost of utility wrt utility, shadow#;
Equation UTILITELASTICEV
    #elasticity of cost of utility wrt utility, shadow#
    (all,r,REG)
    uelasev(r) = XSHRPRIV(r)*ueprivev(r) - dpavev(r);
```

This in turn enables us to implement the upper-level demand equations, following equations (2.40)–(2.42):

```
Equation PCONSEXPEV #private consumption expenditure, shadow#
    (all,r,REG)
    ypev(r) - yev(r) = -[ueprivev(r) - uelasev(r)] + dppriv(r);

Equation GOVCONSEXPEV #government consumption expenditure#
    (all,r,REG)
    ygev(r) - yev(r) = uelasev(r) + dpgov(r);

Equation SAVINGEV #saving# (all,r,REG)
    qsaveev(r) - yev(r) = uelasev(r) + dpsave(r);
```

and to compute the level of the utility elasticity of income:

```
Formula (initial) (all,r,REG)
    UTILELASEV(r) = UTILELAS(r);
Update (all,r,REG)
    UTILELASEV(r) = uelasev(r);
```

We also define levels coefficients for the goods in the top-level utility function:

```
Formula (initial) (all,r,REG)
    UTILPRIVEV(r) = UTILPRIV(r);
Update (all,r,REG)
```

```
    UTILPRIVEV(r) = upev(r);

Formula (initial) (all,r,REG)
    UTILGOVEV(r) = UTILGOV(r);
Update (all,r,REG)
    UTILGOVEV(r) = ugev(r);

Formula (initial) (all,r,REG)
    UTILSAVEEV(r) = UTILSAVE(r);
Update (change) (all,r,REG)
    UTILSAVEEV(r) = [1.0/100.0]*[qsaveev(r) - pop(r)]*UTILSAVEEV(r);
```

Finally we compute the percentage change in shadow income, following equation (2.45):

```
Equation INCOME_SHAD #shadow income# (all,r,REG)
  u(r) = au(r)
    + DPARPRIV(r)*loge(UTILPRIVEV(r))*dppriv(r)
    + DPARGOV(r)*loge(UTILGOVEV(r))*dpgov(r)
    + DPARSAVE(r)*loge(UTILSAVEEV(r))*dpsave(r)
    + [1.0/UTILELASEV(r)]*[yev(r) - pop(r)];
```

3.9 Computing the equivalent variation

Implementing equation (3.1), we compute regional equivalent variation EV:

```
Equation EVREG #regional EV (HT 67)# (all,r,REG)
    EV(r) = [INCOMEEV(r)/100.0]*yev(r);
```

We also compute a world equivalent variation, `WEV`, as the sum of the regional equivalent variations:

```
Equation EVWLD #EV for the world (HT 68)#
    WEV = sum{r, REG, EV(r)};
```

4 Decomposing the equivalent variation

We describe the old decomposition of the equivalent variation (subsection 4.1), discuss its defects (subsection 4.2), and derive (subsection 4.3, 4.4) and implement (subsection 4.5) a new decomposition.

In the derivations below, we derive each EV decomposition formula from two simpler formulae: a lengthy formula decomposing some income-related variable, such as real income or real *per capita* income, and a *decomposition scheme* relating the income variable to EV. Substituting the decomposition of the income-related variable into the decomposition scheme yields the full EV decomposition.

4.1 The old treatment

The old derivation (Huff and Hertel [6]) uses a decomposition of real income,

$$D = Y(y - p), \tag{4.1}$$

where D stands for a rather lengthy decomposition (reproduced with minor changes in subsection 4.3) of real regional income into components related to factor endowments, technological change, allocative efficiency, and terms of trade. The relation between real income and EV is given by the decomposition scheme

$$d\,EV = \frac{1}{100} U_R N_R \frac{\bar{Y}}{Y} \left[D - \left(\sum_i Y_{Pi}(R_i - 1) \right) u_P \right], \tag{4.2}$$

where Y_{Pi} denotes private consumption expenditure on commodity i.

The problems with the old decomposition relate not to the real income decomposition but to the decomposition scheme. Accordingly, we do not derive here the real income decomposition, but refer the reader to the original documentation. We do provide a new derivation for the decomposition scheme, in order to identify sources of error in the old decomposition, and also to explain why the old decomposition is consistent with the old computation of EV.

We use the old utility equation (2.37),

$$\hat{u} = S_P u_P + S_G(q_G - n) + S_S(q_S - n). \tag{4.3}$$

We recall (from subsection 2.4) that the old computation is invalid in general, but valid in the special case $\Phi = 1,\ \Phi_P = 1$, and that standard GTAP data bases fall within the special case. We use for this derivation the notation $\hat{u}$ for utility computed according to equation (4.3).

Recalling equations (2.10) and (2.11), and dropping the government consumption and saving slack variables κ_G and κ_S, we have

$$q_G = y - p_G,$$
$$q_S = y - p_S.$$

Also, from equation (2.38), we have $\Phi_P u_P = y_P - n - p_P$. Adding $u_P - \Phi_P u_P$ to both sides, and putting y for y_P (consistent with the old treatment provided the slack variables are zero), we obtain

$$u_P = y - n - p_P - (\Phi_P - 1)u_P.$$

Substituting into equation (4.3), we obtain

$$\hat{u} = y - n - (S_P p_P + S_G p_G + S_S p_S) - S_P(\Phi_P - 1)u_P.$$

Then substituting from equation (2.36), $p = S_P p_P + S_G p_G + S_S p_S$, we obtain

$$\hat{u} = y - n - p - S_P(\Phi_P - 1)u_P.$$

Substituting into the old EV equation (3.2), $d\,EV = (1/100)U_R N_R \bar{Y}(n + \hat{u})$, we obtain

$$d\,EV = \frac{1}{100} U_R N_R \bar{Y}[y - p - S_P(\Phi_P - 1)u_P].$$

Substituting for D from equation (4.1), we obtain

$$d\,EV = \frac{1}{100} U_R N_R \frac{\bar{Y}}{Y} [D - Y_P(\Phi_P - 1)u_P].$$

Substituting for Φ_P from equation (2.31), $\Phi_P = \sum_i S_i^P R_i$, we obtain finally the old EV decomposition scheme,

$$d\,EV = \frac{1}{100} U_R N_R \frac{\bar{Y}}{Y} \left[D - \left(\sum_i Y_{Pi}(R_i - 1) \right) u_P \right].$$

4.2 Defects in the old treatment

The old welfare decomposition has two defects: it contains a nuisance term, the term in u_P in equation (4.2); and it is in general invalid.

As shown in subsection 3.1, the old decomposition relies on the old utility equation (4.3), and inherits its validity conditions. Accordingly, it is valid in Johansen simulations with data bases in which $\Phi = \Phi_P = 1$ (including standard GTAP data bases); approximate in non-linear simulations in which initially $\Phi = \Phi_P = 1$; and invalid otherwise.

While this is the major defect of the old decomposition, it is also in a way a merit, since it allows the decomposition to be consistent with the old EV computation. More specifically, the old EV computation and decomposition are consistent because they use the same equation (4.3) for aggregate utility.

4.3 A revised treatment

Hanslow [4] presents a general welfare decomposition applicable to many CGE models. For convenience, we base our derivation on the GTAP-specific Huff and Hertel ([6]) approach. As revised, the results are consistent with the Hanslow decomposition.

In revising the decomposition, we at first assume no changes in preferences, and then extend our results to incorporate preference changes. Rearranging equation (2.35), $x = p + \Phi u$, we obtain

$$\begin{aligned} u &= \Phi^{-1}(x - p) \\ &= \Phi^{-1}(y - p - n) && \text{by defn. of } x \\ &= \Phi^{-1}(Y^{-1}D - n) && \text{from (4.1)} \end{aligned}$$

Substituting into equation (3.4), $y_{EV} = \Phi_{EV} u + n$, we obtain

$$y_{EV} = \frac{\Phi_{EV}}{\Phi}(Y^{-1}D - n) + n.$$

Then substituting into equation (3.1), $d\,EV = \frac{1}{100} Y_{EV} y_{EV}$, we obtain the decomposi-

tion scheme

$$d\,EV = \frac{1}{100}\frac{\Phi_{EV}}{\Phi}\frac{Y_{EV}}{Y}D - \frac{1}{100}\left[\frac{\Phi_{EV}}{\Phi} - 1\right]Y_{EV}n.$$

This scheme suffers from one objectionable feature, the presence of a nuisance term involving population growth n. In simulations with standard data bases (with $\Phi_{EV} = \Phi = 1$ initially), the term would typically be small but non-zero. We can remove this nuisance by modifying the income decomposition, to decompose not real regional income $y - p$ but real *per capita* income $x - p$. Accordingly we write

$$Y(x - p) = D^*, \tag{4.4}$$

where D^* represents a decomposition of real *per capita* income. Then proceeding as before, we obtain

$$\begin{aligned} u &= \Phi^{-1}Y^{-1}D^*, \\ y_{EV} &= \frac{\Phi_{EV}}{\Phi}Y^{-1}D^* + n, \end{aligned}$$

and

$$d\,EV = \frac{1}{100}\frac{\Phi_{EV}}{\Phi}\frac{Y_{EV}}{Y}D^* + \frac{1}{100}Y_{EV}n.$$

Now instead of a nuisance term, we have an interpretable term in population growth n.

Finally, we incorporate preference changes. Instead of the simpler equation (2.35), we begin with the more complete equation (2.45),

$$\begin{aligned} u &= c + \sum_i B_i(\log U_i)b_i + \Phi^{-1}(x - p) \\ &= c + \sum_i B_i(\log U_i)b_i + \Phi^{-1}Y^{-1}D^*, \end{aligned}$$

substituting from equation (4.4). Also, adapting equation (2.45), we have

$$y_{EV} = \Phi_{EV}\Big(-c - \sum_i B_i(\log U_{EVi})b_i + u\Big) + n,$$

where U_{EVi} denotes the level of good i in the top-level utility function, in the shadow demand system with initial prices but current utility and preferences. Then proceeding as before, we obtain

$$\begin{aligned} d\,EV = &-\frac{1}{100}\Phi_{EV}Y_{EV}\sum_i B_i\left(\log\frac{U_{EVi}}{U_i}\right)b_i \\ &+ \frac{1}{100}\frac{\Phi_{EV}}{\Phi}\frac{Y_{EV}}{Y}D^* + \frac{1}{100}Y_{EV}n. \end{aligned} \tag{4.5}$$

We note that changes c in the utility scaling factor do not affect the equivalent variation, and that changes in the distribution parameters affect it only when corre-

lated with differences between the actual and shadow subutilities U_i and U_{EVi} (since $\sum_i B_i \log U_{EVi} = \log(U/C) = \sum_i B_i \log U_i$). If both distribution parameter changes and price changes favor usage of top-level good i, then the effect of the distribution parameter changes on utility is more favorable with final prices than with initial prices, so expenditure in the shadow system needs to be higher than it would otherwise, so the contribution to the equivalent variation is positive. Conversely, if the distribution parameter for good i increases while price changes operate to discourage its consumption, the contribution to the equivalent variation is negative.

4.4 Decomposing real *per capita* income

Based on Huff and Hertel [6], we have a decomposition of real regional income:

```
(all,r,REG)
    INCOME(r)*[y(r) - p(r)]
=   sum{i, ENDW_COMM, VOA(i,r)*qo(i,r)} - VDEP(r)*kb(r)
  + sum{j, PROD_COMM, VOA(j,r)*ao(j,r)}
  + sum{j, PROD_COMM, VVA(j,r)*ava(j,r)}
  + sum{j, PROD_COMM, sum{i, ENDW_COMM, VFA(i,j,r)*afe(i,j,r)}}
  + sum{j, PROD_COMM, sum{i, TRAD_COMM, VFA(i,j,r)*af(i,j,r)}}
  + sum{s, REG, sum{i, TRAD_COMM, sum{m, MARG_COMM,
      VTMFSD(m,i,s,r)*atmfsd(m,i,r,s)
    }}}
  + sum{i, NSAV_COMM, PTAX(i,r)*qo(i,r)}
  + sum{i, ENDW_COMM, sum{j, PROD_COMM, ETAX(i,j,r)*qfe(i,j,r)}}
  + sum{j, PROD_COMM, sum{i, TRAD_COMM, IFTAX(i,j,r)*qfm(i,j,r)}}
  + sum{j, PROD_COMM, sum{i, TRAD_COMM, DFTAX(i,j,r)*qfd(i,j,r)}}
  + sum{i, TRAD_COMM, IPTAX(i,r)*qpm(i,r)}
  + sum{i, TRAD_COMM, DPTAX(i,r)*qpd(i,r)}
  + sum{i, TRAD_COMM, IGTAX(i,r)*qgm(i,r)}
  + sum{i, TRAD_COMM, DGTAX(i,r)*qgd(i,r)}
  + sum{i, TRAD_COMM, sum{s, REG, XTAXD(i,r,s)*qxs(i,r,s)}}
  + sum{i, TRAD_COMM, sum{s, REG, MTAX(i,s,r)*qxs(i,s,r)}}
  + sum{i, TRAD_COMM, sum{s, REG, VXWD(i,r,s)*pfob(i,r,s)}}
  + sum{m, MARG_COMM, VST(m,r)*pm(m,r)}
  - sum{i, TRAD_COMM, sum{s, REG, VXWD(i,s,r)*pfob(i,s,r)}}
  - sum{m, MARG_COMM, VTMD(m,r)*pt(m)}
  + NETINV(r)*pcgds(r) - SAVE(r)*psave(r)
  ;
```

This is a equation from Huff and Hertel [6], modified to conform to the new treatment of international margins and new notation for tax revenue coefficients introduced in release 5 of the GTAP model. The right hand side is the expression represented above as D. Rearranging, and subtracting `INCOME(r)*pop(r)` from both sides, we obtain a decomposition for real *per capita* income:

```
(all,r,REG)
    INCOME(r)*[y(r) - pop(r) - p(r)]
=   sum{i,ENDW_COMM, VOA(i,r)*[qo(i,r) - pop(r)]}
  - VDEP(r)*[kb(r) - pop(r)]
```

```
+ sum{j,PROD_COMM, VOA(j,r)*ao(j,r)}
+ sum{j,PROD_COMM, VVA(j,r)*ava(j,r)}
+ sum{j,PROD_COMM, sum{i,ENDW_COMM, VFA(i,j,r)*afe(i,j,r)}}
+ sum{j,PROD_COMM, sum{i,TRAD_COMM, VFA(i,j,r)*af(i,j,r)}}
+ sum{s,REG, sum{i,TRAD_COMM, sum{m,MARG_COMM,
    VTMFSD(m,i,s,r)*atmfsd(m,i,r,s)
  }}}
+ sum{i,NSAV_COMM, PTAX(i,r)*[qo(i,r) - pop(r)]}
+ sum{i,ENDW_COMM, sum{j,PROD_COMM,
      ETAX(i,j,r)*[qfe(i,j,r) - pop(r)]
  }}
+ sum{j,PROD_COMM, sum{i,TRAD_COMM,
      IFTAX(i,j,r)*[qfm(i,j,r) - pop(r)]
  }}
+ sum{j,PROD_COMM, sum{i,TRAD_COMM,
      DFTAX(i,j,r)*[qfd(i,j,r) - pop(r)]
  }}
+ sum{i,TRAD_COMM, IPTAX(i,r)*[qpm(i,r) - pop(r)]}
+ sum{i,TRAD_COMM, DPTAX(i,r)*[qpd(i,r) - pop(r)]}
+ sum{i,TRAD_COMM, IGTAX(i,r)*[qgm(i,r) - pop(r)]}
+ sum{i,TRAD_COMM, DGTAX(i,r)*[qgd(i,r) - pop(r)]}
+ sum{i,TRAD_COMM, sum{s,REG,
      XTAXD(i,r,s)*[qxs(i,r,s) - pop(r)]
  }}
+ sum{i,TRAD_COMM, sum{s,REG,
      MTAX(i,s,r)*[qxs(i,s,r) - pop(r)]
  }}
+ sum{i,TRAD_COMM, sum{s,REG, VXWD(i,r,s)*pfob(i,r,s)}}
+ sum{m,MARG_COMM, VST(m,r)*pm(m,r)}
- sum{i,TRAD_COMM, sum{s,REG, VXWD(i,s,r)*pfob(i,s,r)}}
- sum{m,MARG_COMM, VTMD(m,r)*pt(m)}
+ NETINV(r)*pcgds(r)
- SAVE(r)*psave(r)
;
```

Here the right hand side is the expression referred to above as D^*.

Unlike for example Hanslow [4], we do not introduce into the decomposition a new term involving population. Instead we incorporate the population variable into the terms involving quantity variables. We prefer this approach for several reasons.

- Looking forward to the equivalent variation decomposition, it does not create there a nuisance term involving population growth. There is indeed still a population growth term. It is however no longer a nuisance term but an interpretable term, expressing the intuition that in the absence of imbalances in growth, income grows equiproportionally with population.

- It does lead to a redefinition of the endowment terms. We recognize now an increase in utility arising not from growth in total endowments, but from growth in endowments *per capita*. While change admittedly is bad, this change is not

very bad, since the new endowment terms are as readily interpretable as the old ones.

- It leads also to a redefinition of the allocative efficiency effects, but here the change is for the better. With balanced growth in a distorted economy, the old decomposition reported an allocative efficiency improvement associated with every taxed flow, and an allocative efficiency deterioration associated with every subsidised flow. Intuitively however, balanced growth involves no change in allocative efficiency. The new decomposition here conforms to intuition better than the old.

4.5 Implementation

To implement the new treatment, we need to define the new population growth term in the decomposition, and revise the old terms. The old terms included a factor representing $\frac{U}{U}\frac{N}{N}\bar{Y}/Y$. The new terms include instead a factor representing $(\Phi_{EV}/\Phi)(Y_{EV}/Y)$. Since the numerator $\frac{U}{U}\frac{N}{N}\bar{Y}$ in the old factor is an approximation to Y_{EV} (provided that the elasticity of income with respect to utility is initially equal to one), and since Φ_{EV}/Φ in the new factor is (for small changes) approximately equal to one, the old factor may be considered an approximation to the new one.

To implement the new treatment, we first compute the equivalent variation scaling factor $(\Phi_{EV}/\Phi)(Y_{EV}/Y)$:

```
Coefficient (all,r,REG)
    EVSCALFACT(r) #equivalent variation scaling factor#;
Formula (all,r,REG)
    EVSCALFACT(r)
  = [UTILELASEV(r)/UTILELAS(r)]*[INCOMEEV(r)/INCOME(r)];
```

We then revise the decomposition-based computation of equivalent variation, using equation (4.5) and the real *per capita* income decomposition obtained in subsection 4.4.

```
Variable (Linear,Change)(all,r,REG)
    EV_ALT(r) # regional EV computed in alternative way #;
Equation EV_DECOMPOSITION
# decomposition of Equivalent Variation #
(all,r,REG)
    EV_ALT(r)
  = - 0.01*UTILELASEV(r)*INCOMEEV(r)*[
          DPARPRIV(r)*loge(UTILPRIVEV(r)/UTILPRIV(r))*dppriv(r)
        + DPARGOV(r)*loge(UTILGOVEV(r)/UTILGOV(r))*dpgov(r)
        + DPARSAVE(r)*loge(UTILSAVEEV(r)/UTILSAVE(r))*dpsave(r)
      ]
    +   [0.01*EVSCALFACT(r)]
      * [
            sum{i,NSAV_COMM, PTAX(i,r)*[qo(i,r) - pop(r)]}
          + sum{i,ENDW_COMM, sum{j,PROD_COMM,
              ETAX(i,j,r)*[qfe(i,j,r) - pop(r)]
            }}
```

```
            + sum{j,PROD_COMM, sum{i,TRAD_COMM,
                IFTAX(i,j,r)*[qfm(i,j,r) - pop(r)]
              }}
            + sum{j,PROD_COMM, sum{i,TRAD_COMM,
                DFTAX(i,j,r)*[qfd(i,j,r) - pop(r)]
              }}
            + sum{i,TRAD_COMM, IPTAX(i,r)*[qpm(i,r) - pop(r)]}
            + sum{i,TRAD_COMM, DPTAX(i,r)*[qpd(i,r) - pop(r)]}
            + sum{i,TRAD_COMM, IGTAX(i,r)*[qgm(i,r) - pop(r)]}
            + sum{i,TRAD_COMM, DGTAX(i,r)*[qgd(i,r) - pop(r)]}
            + sum{i,TRAD_COMM, sum{s,REG,
                XTAXD(i,r,s)*[qxs(i,r,s) - pop(r)]
              }}
            + sum{i,TRAD_COMM, sum{s,REG,
                MTAX(i,s,r)*[qxs(i,s,r) - pop(r)]
              }}
            + sum{i,ENDW_COMM, VOA(i,r)*[qo(i,r) - pop(r)]}
            - VDEP(r)*[kb(r) - pop(r)]
            + sum{i,PROD_COMM, VOA(i,r)*ao(i,r)}
            + sum{j,PROD_COMM, VVA(j,r)*ava(j,r)}
            + sum{i,ENDW_COMM, sum{j,PROD_COMM, VFA(i,j,r)*afe(i,j,r)}}
            + sum{j,PROD_COMM, sum{i,TRAD_COMM, VFA(i,j,r)*af(i,j,r)}}
            + sum{m,MARG_COMM, sum{i,TRAD_COMM,
                sum{s,REG, VTMFSD(m,i,s,r)*atmfsd(m,i,s,r)}}}
            + sum{i,TRAD_COMM, sum{s,REG, VXWD(i,r,s)*pfob(i,r,s)}}
            + sum{m,MARG_COMM, VST(m,r)*pm(m,r)}
            + NETINV(r)*pcgds(r)
            - sum{i,TRAD_COMM, sum{s,REG, VXWD(i,s,r)*pfob(i,s,r)}}
            - sum{m,MARG_COMM, VTMD(m,r)*pt(m)}
            - SAVE(r)*psave(r)
          ]
    + 0.01*INCOMEEV(r)*pop(r);
```

Consistency between this and the standard equivalent variation computation is a check on the validity of the decomposition.

Finally we compute various components of the change in equivalent variation. We compute first the distributional parameter component:

```
Variable (Linear,Change) (all,r,REG) CNTdpar(r)
    # contribution to EV of change in distribution parameters#;
Equation CNT_WEV_dpar (all,r,REG)
    CNTdpar(r) = -0.01*UTILELASEV(r)*INCOMEEV(r)*[
          DPARPRIV(r)*loge(UTILPRIVEV(r)/UTILPRIV(r))*dppriv(r)
        + DPARGOV(r)*loge(UTILGOVEV(r)/UTILGOV(r))*dpgov(r)
        + DPARSAVE(r)*loge(UTILSAVEEV(r)/UTILSAVE(r))*dpsave(r)
      ];
```

and the population component:

```
Variable (Linear,Change) (all,r,REG) CNTpop(r)
    #contribution to EV in region r of change in population#;
```

```
Equation CONT_WEV_pop (all,r,REG)
    CNTpop(r) = 0.01*INCOMEEV(r)*pop(r);
```

The other components derive from the real *per capita* income decomposition. They are generally similar to the corresponding components of the old decomposition, but with the new scaling factor replacing the old. For instance, for the allocative efficiency effect associated with production subsidies and income taxes, we have corresponding to the first term in the decomposition:

```
Variable (Linear,Change) (all,r,REG) CNTqor(r)
    #contribution to EV in region r of output changes#;
Equation  CONT_WEV_qor (all,r,REG)
    CNTqor(r)
  = sum{i,NSAV_COMM, 0.01*EVSCALFACT(r)*PTAX(i,r)*[qo(i,r) - pop(r)]};
```

The code for the remaining components is not reproduced here but may be found in the associated program source file.

5 Properties and behavior of the final demand system

Having described the implementation of the new demand system, we now consider its behavior and properties. There are certain properties that the revised model should display, that can be precisely specified and mathematically demonstrated. We are also concerned to develop a practical feeling for its behavior: how different its results are from the old system, what new kinds of behavior can be observed, and how greatly they are likely to affect simulation results. In discussing these matters, we refer to results from various illustrative simulations from the software package accompanying this documentation (see appendix C). Except where otherwise stated, these are based on a trade liberalisation scenario involving removal of import barriers within the *APEC* (Asia Pacific Economic Cooperation) group of countries (experiment 1 in [9]).

In the new treatment, the model should display several readily checked properties:

- All variables except utility from private consumption (u_P) and overall utility (u) are invariant with respect to rescalings of the CDE expansion parameters (R_i or `INCPAR`). The software package includes a pair of simulations with the old theory, `old` with the standard `INCPAR`, and `oincpar` with rescaled `INCPAR`. Most variables are the same in the two simulations (to the numerical accuracy of the solution), but some variables, including not only `u` and `up` but also `EV`, `EV_ALT`, and `WEV` are about one order of magnitude greater in `oincpar`. The package also includes a similar pair of simulations with the new theory, `new` with the standard `INCPAR` and `nincpar` with the rescaled `INCPAR`. Now we find that `u` and `up` differ in the two simulations, but `EV`, `EV_ALT`, and `WEV` are the same. This verifies that the new system avoids the gross errors of the old system in calculating equivalent variation with non-standard `INCPAR` (section 3.2).

- All variables except utility (u) are invariant with respect to changes in the initial level of the sum of the upper-level distribution parameters (B or `DPARSUM`).

This can be verified by comparing results between simulations `new`, with standard settings for `DPARSUM`, and `ndparsum`, with rescaled `DPARSUM`.

- In quantity homogeneity tests—that is, simulations in which uniform shocks are applied to population, factor endowments, and any other exogenous quantity variables—all components of the EV decomposition except the population component are zero. The simulation `nqhom` verifies that the new system has this property, while `oqhom` confirms that the old system doesn't.

The first property is not obvious, since rescalings of the expansion parameters do affect the utility elasticity of private consumption expenditure, and, in the linearised equation system, changes in the utility elasticity of private consumption expenditure affect the upper-level allocation of income. Nevertheless, as the following proposition shows, it does apply:

Proposition 7 *With an upper-level Cobb-Douglas demand system and a bottom-level CDE system, with distribution parameters calibrated to a given initial equilibrium, rescaling the CDE expansion parameters has no effect on quantities demanded, or on the equivalent variation.*

Proof. Suppose that the CDE expansion parameters R_i maximizing U^K with the old expansion parameters is equivalent to maximizing U with the new expansion parameters. So rescaling the CDE expansion parameters does not affect the private consumption demand system. It does affect the upper level of the final demand system, since the elasticity of private consumption expenditure with respect to utility from private consumption is linearly homogeneous in the CDE expansion parameters. To calibrate to the observed income allocation, however, when we rescale the expansion parameters by a factor K, we need also to multiply by K the upper-level distribution parameter for private consumption. With that adjustment, the new system yields the same upper-level budget shares as the old system. So lower-level quantities demanded are the same with the new system as with the old (though utility U_P from private consumption, and overall utility U, are different). Since the demand functions are unaffected by the parameter rescaling, the equivalent variation is also unaffected.■

In the private consumption demand system, as income increases, the budget share of commodities with higher expansion parameters increases. Then because of the expansion-parameter weighting of `XWCONSHR`, the utility elasticity `uepriv` also increases. This leads to a shift away from private consumption toward government consumption and saving. In addition, reductions in relative prices of commodities with low expansion parameters (with sufficiently low price elasticities) typically decrease their budget share, again leading to increases in `uepriv` and reallocation of income away from private consumption toward government consumption and saving.

In standard GTAP data bases, the greatest differences in expansion parameters are typically between food and non-food commodities, the expansion parameters of food commodities typically being much lower than those of non-food. Accordingly, the share of private consumption in regional income typically varies directly with food prices.

Table 1: EV under old and new treatments (US$ billion)

	Old	New	Percentage difference
North America	-2.756	-2.625	-4.8
Japan	47.473	47.230	-0.5
Australia and New Zealand	0.629	0.641	1.9
China incl. Hong Kong	6.064	6.146	1.4
Taiwan	4.517	4.458	-1.3
South Korea	9.242	9.055	-2.0
Malaysia and Singapore	1.914	1.926	0.6
Thailand and Philippines	-1.355	-1.329	-1.9
Indonesia	0.577	0.583	1.0
Rest of world	-15.272	-14.731	-3.5

Source: Simulations `old`, `new`

Experience to date with the old and new models with aggregated standard data bases suggest some tentative generalizations:

- In moving from the old to the new treatment, corrections to the welfare variables are typically small. For example, in our illustrative trade liberalisation experiment, the differences in regional EV between the old and new treatments are all less than 5 per cent (table 1).

- While under the new treatment, the upper-level allocation of income depends on the income level, it is rather insensitive to it. Table 2 shows changes in money income and in major income disposition aggregates in an experiment in which factor productivity doubles in all industries in all regions. It reports just two income disposition aggregates, private consumption expenditure and "other", where "other" includes both government consumption expenditure and saving; since in the absence of preference shifts, the percentage changes in government consumption expenditure and saving are equal. As it shows, in each region the ratio of "other" to private consumption expenditure rises, but in no region by very much; the maximum increase being 12.2 per cent in China and Hong Kong, and the minimum 2.7 per cent in Japan.

- Under the new treatment, the upper-level allocation of income may be affected appreciably by changes in relative prices of commodities with different expansion parameters. In particular, in low- and middle-income countries, the upper-level allocation of income is liable to be affected by changes in the price of food relative to other commodities. The upper-level allocation is typically less sensitive to food prices in high income countries, since there the share of food in private consumption expenditure is typically low.

  Table 3 shows results from an experiment in which subsidies are placed on food

Table 2: Effects on income and major disposition aggregates of a doubling of factor productivity (with new treatment; percentage changes)

	Income	Private consumption expenditure	Other	Ratio other: private
North America	113.6	111.8	119.3	3.5
Japan	114.3	112.4	118.2	2.7
Australia and New Zealand	112.7	109.0	121.8	6.1
China incl. Hong Kong	107.4	99.8	124.2	12.2
Taiwan	104.8	100.6	109.8	4.6
South Korea	108.6	103.0	116.5	6.6
Malaysia and Singapore	112.0	107.3	117.9	5.1
Thailand and Philippines	107.5	103.7	115.2	5.6
Indonesia	104.7	99.1	112.0	6.5
Rest of world	114.2	110.6	123.1	5.9

Note: "Other" includes government consumption expenditure and saving.
Source: Simulation `income`

production so that the ratio of the market value to producer earnings from food sales falls by one half. In each case there is an increase in the ratio of government consumption expenditure and saving to private consumption expenditure. This increase is due to, and equal to, the increase in the elasticity of private consumption expenditure to utility, Φ_P (equations (2.27)–(2.29) show why they are equal). The elasticity decreases because there is a decrease in the budget share of food, and food has an unusually low CDE expansion parameter R_{food}, ranging between 0.13 for Australian and New Zealand and 0.63 for Indonesia (as in standard GTAP databases, the expansion parameters are normalized so that across commodities they average to 1; for the relation between the expansion parameters and the utility elasticity, see equation (2.33)). The food budget share decreases because the price of food decreases and demand for food is price-inelastic. The effect on the macro expenditure ratio is greatest in two of the poorer regions, China and Indonesia, and least in two of the richer regions, North America and Japan.

6 Future work

Given the problems with the old system, there are some alternative approaches to that taken in this paper, some sketched briefly above (subsection 2.5). An obvious possibility for future work is to explore some of these approaches more fully.

In our judgement, however, a more fruitful approach may be to explore the empirics of the top-level demand system. The new system generates more complex behavior than the old; in particular, the share of private consumption in national income tends

Table 3: Effects of a food subsidy (with new treatment; percentage changes)

	Food price	Ratio other: private [a]	`uepriv`	Food budget share [b]	R_{food} (level)
North America	-60.3	4.0	4.0	-54.6	0.20
Japan	-60.9	4.2	4.2	-54.6	0.45
Australia and New Zealand	-59.4	5.8	5.8	-49.2	0.13
China incl. Hong Kong	-55.3	14.5	14.5	-32.9	0.52
Taiwan	-64.9	8.1	8.1	-54.2	0.57
South Korea	-57.3	8.5	8.5	-45.4	0.48
Malaysia and Singapore	-59.9	7.2	7.2	-49.5	0.49
Thailand and Philippine	-59.9	8.9	8.9	-43.5	0.58
Indonesia	-57.5	10.5	10.5	-40.3	0.63
Rest of world	-60.8	7.0	7.0	-51.9	0.32

[a] "Other" includes government consumption expenditure and saving.
[b] Share of food in private consumption expenditure
Source: Simulation `food`

to vary directly with national income. However the magnitude of the variation is not based on empirical estimates, but emerges as a side effect from other features of the demand system. A good next step would be to research the empirics of this relationship, and review the functional form and parameterization of the new demand system in the light of that research.

References

[1] A. Deaton and J. Muellbauer. *Economics and Consumer Behavior*. Cambridge, 1980.

[2] W.M. Gorman. Separable utility and aggregation. *Econometrica*, 27:469–81, 1959.

[3] G. Hanoch. Production and demand models in direct or indirect implicit additivity. *Econometrica*, 43:395–419, 1975.

[4] K Hanslow. A general welfare decomposition for CGE models. Paper presented to the Third Annual Conference on Global Economic Analysis, Melbourne, 2000.

[5] T.W. Hertel and M.E. Tsigas. *Global Trade Analysis: Modeling and Applications*, chapter 3, Structure of GTAP. Cambridge University Press, 1997.

[6] K. Huff and T.W. Hertel. Decomposing welfare changes in GTAP. Technical Paper 5, Center for Global Trade Analysis, Purdue University, West Lafayette, IN., 1996.

[7] Steve Talbott and Andy Oram. *Managing Projects with `make`*. O'Reilly, second edition, 1991.

[8] H.R. Varian. *Microeconomic Analysis*. Norton, second edition, 1984.

[9] L.M. Young and K.M. Huff. *Global Trade Analysis: Modeling and Applications*, chapter 9, Free trade in the Pacific Rim: On what basis? Cambridge University Press, 1997.

Appendix A The demand system and the Gorman conditions

As discussed in the main text (subsection 2.2), the old upper-level demand theory is in error. One way to view the error is that it mistakenly imposes a two-stage budgeting scheme on a preference system that does not support it. It is natural then to inquire how this error relates to the well-known Gorman conditions [2] for the feasibility of two-stage budgeting.

The Gorman conditions apply in the context of *weak separability*. A system is said to be *weakly separable* if the utility function can be represented in the form

$$U(\mathbf{Q}) = U_\bullet(U_1(\mathbf{Q_1}), \ldots, U_G(\mathbf{Q_G})),$$

where $\mathbf{Q_1}, \ldots, \mathbf{Q_G}$ is a partition of the quantity vector $\mathbf{Q}$ into subvectors representing groups of commodities. The function $U_\bullet$ is the upper-level utility function, and the $U_1, \ldots, U_G$ are *lower-level utilities* or *subutilities*. In a weakly separable system, the Gorman conditions are the necessary and sufficient conditions for the existence of an upper-level demand system

$$\max U_\bullet^*(U_1^*, \ldots, U_G^*) \text{ subject to } \sum_{i=1}^{G} P_i^* U_i^* = X, \tag{A.1}$$

where P_i^* and U_i^* are price and quantity indices for the i'th lower-level subsystem, and $U_\bullet^*$ a *utility index*, such that the solution for the upper-level system is consistent with the solution for the overall system. Note that the quantity indices U_i^* may or may not be the subutilities U_i. Also, the utility index $U_\bullet^*$ may or may not be similar in form to the upper-level utility function $U_\bullet$.

By construction, the GTAP regional household demand system is weakly separable. The error in the old derivation is the assumption that utility from private consumption can serve as a quantity index for private consumption in the upper-level demand system (A.1). As shown above (subsection 2.2), if we try to use utility from private consumption as the quantity index, we find there is no corresponding price index. This does not mean that the regional household demand system does not meet the Gorman conditions; price and quantity indices for private consumption might yet be found; it means only that the quantity index cannot be the subutility.

On the other hand, even if suitable price and quantity indices did exist, that would not necessarily validate the old treatment. It would show that we could specify an upper-level demand system that treated private consumption as an ordinary good, but it would not guarantee that the utility index $U_\bullet^*$ in that system was of the Cobb-Douglas form, nor that the demands had the Cobb-Douglas fixed budget shares property. In short, the requirements for the validity of the old treatment are more stringent than the Gorman conditions.

Gorman [2] shows that an upper-level system of the desired form can be constructed under either of two alternative conditions. One alternative is that the lower-level sys-

tems are homothetic. Under this alternative, the quantity indices are just the lower-level utilities, and the utility index is just the upper-level utility function. The other alternative is that the upper-level utility function is additive, $U_\bullet(U_1, \ldots, U_G) = \sum_{i=1}^{G} U_i$, and the lower-level systems admit indirect utility functions of the *Gorman generalized polar form*,

$$\Psi_i(\mathbf{P_i}, X_i) = F_i\left(\frac{X_i}{M_i(\mathbf{P_i})}\right) + A_i(\mathbf{P_i}). \tag{A.2}$$

Under this alternative, the quantity indices $U_i^* = X_i/M_i(\mathbf{P_i})$, the price indices $P_i^* = M_i(\mathbf{P_i})$, and the utility index $U_\bullet^*(U_1^*, \ldots, U_G^*) = \sum_{i=1}^{G} F_i(U_i^*)$.

As we have seen already, the GTAP final regional household demand system does not meet the first condition, that the lower-level demand systems be homothetic. It seems obvious, but is not easily proved, that except in degenerate cases, the CDE and Gorman generalized polar forms are incompatible.

Conjecture 1 *If a demand system is both a CDE system and a Gorman generalized polar form, then it is a CES system.*

As noted above, it is possible to satisfy the Gorman conditions without validating the old treatment of the upper-level demand system. More specifically, solutions involving homothetic lower-level demand systems validate the old treatment, but solutions involving the Gorman generalized polar form do not. In particular, the old treatment specifies a utility function of the Cobb-Douglas form, but the solutions involving the Gorman generalized polar form require a utility function of the additive form. Note that additivity is a much stronger requirement than additive separability; the Cobb-Douglas utility function can be written as additively separable ($U = \sum B_i log U_i$), but not as additive ($U = \sum U_i$).

On the one hand, it is true to say that the old treatment is erroneous because the CDE system does not satisfy the Gorman conditions. It is true because, if the old treatment were valid, the Gorman conditions would necessarily be satisfied. On the other hand, the Gorman conditions are a something of a distraction in this context. To show that the old derivation is invalid, we do not need to refer to the Gorman conditions; it is sufficient, and simpler, to show that the private consumption demand system is non-homothetic. Nevertheless, as we show below (appendix B), although the Gorman result is not useful in refuting the old treatment, it is potentially useful in remedying its defects.

Appendix B Alternative private consumption demand systems

As shown above (appendix A), unless we accept a homothetic private consumption demand system, we must accept some changes to the upper-level system. On the other hand, while not concerned to retain all aspects of the current upper-level system, we would like to retain at least the fixed shares property. In this section we investigate whether we can find a new form for the private consumption demand system such as to preserve the fixed shares property while perhaps affecting other aspects of the upper-level system.

Recalling equation (2.18),

$$\frac{X_i}{X} = \frac{\Phi_i^{-1} B_i}{\sum_j \Phi_j^{-1} B_j},$$

we see that even when the elasticities Φ are not all equal to one, the budget shares are constant provided that the elasticities are constant. This seems a hopeful notion: with constant elasticities, we change some aspects of the upper-level system but retain the fixed budget shares. As it turns out however, this approach imposes unacceptable restrictions on the form of the lower-level systems.

Proposition 8 *In any demand system, if the utility elasticity of expenditure is constant, the system is homothetic.*

Proof. Let $\bar{U}$ denote some arbitrary utility level, and Φ the constant utility elasticity. If the utility elasticity of expenditure is constant, then for any utility level U, $E(\mathbf{P}, U) = (U/\bar{U})^{\Phi} E(\mathbf{P}, \bar{U})$. But then we can write, for all $\mathbf{P}$, U, $E(\mathbf{P}, U) = \Pi(\mathbf{P})(U/\bar{U})^{\Phi}$, where $\Pi(\mathbf{P}) = E(\mathbf{P}, \bar{U})$. So, by proposition 1, the system is homothetic.■

Since homotheticity is empirically unacceptable, this idea does not help us find an acceptable form for the private consumption demand system.

We may also attempt to use the Gorman [2] conditions for two stage budgeting to find a functional form for the private consumption demand system that lets us preserve the upper-level demand system. This is a somewhat subtle strategy. We have seen above (appendix A) that there is no non-homothetic private consumption demand system that, in conjunction with a Cobb-Douglas upper-level utility function $U_\bullet$, leads to fixed upper-level budget shares. There might yet however be a non-homothetic private consumption demand system that, in conjunction with an upper-level utility function not of the Cobb-Douglas form, leads to a Cobb-Douglas upper-level utility index $U_\bullet^*$. This is in fact feasible.

Of the two alternative conditions in [2], one entails homothetic lower-level demand systems, which is unacceptable. The other condition however does allow an acceptable solution.

Proposition 9 *In a two-level demand system with an upper-level additive utility func-*

tion $U_\bullet(U_1, \ldots, U_G) = \sum_{i=1}^{G} U_i$ *and lower-level indirect utility functions*

$$\Psi_i(\mathbf{P_i}, X_i) = B_i \log \frac{X_i}{M_i(\mathbf{P_i})} + A_i(\mathbf{P_i}), \tag{B.1}$$

the upper-level expenditure shares are fixed.

Proof. In consumer equilibrium, the group expenditure levels X_i solve the problem

Find X_i to maximize $\sum_i \Psi_i(\mathbf{P_i}, X_i)$ such that $\sum_i X_i = X$;

that is, with the specified form for the lower-level indirect utility functions Ψ_i,

Find X_i to maximize

$$\sum_i B_i \log\left(\frac{X_i}{M_i(\mathbf{P_i})}\right) + \sum_i A_i(\mathbf{P_i})$$

such that $\sum_i X_i = X$.

Since the functions A_i do not involve group expenditure X_i, this is equivalent to

Find X_i to maximize $\sum_i B_i \log(X_i/M_i(\mathbf{P_i}))$ such that $\sum_i X_i = X$;

or, putting $U_i^* = X_i/M_i(\mathbf{P_i})$, $P_i = M_i(\mathbf{P_i})$,

Find U_i^* to maximize $\sum_i B_i \log U_i^*$ such that $\sum_i P_i U_i^* = X$.

This has the standard Cobb-Douglas solution

$$U_i^* = \frac{B_i}{\sum_j B_j} \frac{X}{P_i},$$

so the expenditure shares

$$\frac{X_i}{X} = \frac{P_i U_i^*}{X} = \frac{B_i}{\sum_j B_j},$$

so the expenditure shares are fixed, as was to be shown.■

The functional form (B.1) covers both (with zero A_G) the Cobb-Douglas demand system used in GTAP for government consumption, and (with non-zero A_P) a reasonably extensive class of non-homothetic private consumption demand systems. So with the demand system of proposition 9, we can preserve the Cobb-Douglas government consumption system and the upper-level fixed shares, by changing the upper-level utility function and the private consumption demand system.

Appendix C Accompanying Software

Accompanying this paper is a software package, including old and new versions of the GTAP model, the illustrative simulations discussed in section 5, and a small aggregated GTAP database.

The data files conform to the older Lahey binary structure rather than the new Lahey/Fujitsu structure.

The software is designed to work with Microsoft Windows operating systems (`http://www.microsoft.com`), the Cygwin working environment (`http://cygwin.com`), Lahey Fortran compilers (`http://www.lahey.com`), and GEMPACK (`http://www.monash.edu.au/policy/gempack.htm`). Windows, Lahey, and GEMPACK are likely familiar to most GTAP model users; Cygwin is a free software (`http://www.gnu.org/philosophy/free-sw.html`) package that operates under Microsoft Windows but provides a Unix-like modeler-friendly environment.

The build description file `Makefile` describes the relations between the source files included in the package and the model and simulation files that can be generated from them. It is written to work with the GNU implementation (`http://www.gnu.org/software/make/make.html`) of the `make` build management program ([7]) included in Cygwin. The shell script `ltg.sh`, adapted from the GEMPACK batch file `ltg.bat`, is used to compile and link the TABLO-generated Fortran programs.

The file `old.tab` contains the GTAP model theoretical structure before the revisions proposed in this paper. The file `old.fts` is the TABLO stored input file, containing condensation information; `ohom.cmf` is the command file for a price homogeneity test, containing closure information. The corresponding files for the revised model proposed in this paper are `gtap.tab`, `gtap.fts`, and `ghom.cmf`.

Most of the illustrative simulations are based on a trade liberalisation scenario involving removal of import barriers within the APEC group of countries (experiment 1 in [9]). The command file `old.cmf` defines this scenario for the old structure, `new.cmf` for the new. They all use a 10-region 3-commodity data base. The sets file is `gset.har` and the parameters file `gpar.har`; the flows data file is `odat.har` for the old theoretical structure, and `gdat.har` for the new. Tariff shocks are read from a file `tms.shk`.

The command files `oincpar.cmf` and `nincpar.cmf` define the same scenario for the old and new models, but with non-standard values for the expansion parameter (R_i or `INCPAR`). The non-standard values are contained in an alternative parameters file, `gpincpar.har`, created from the standard parameters file `gpar.har` using the GEMPACK utility program `modhar` and the stored input file `gpincpar.sti`.

The command file `ndparsum.cmf` defines the same scenario for the new theoretical structure, but with a non-standard value for the sum of the distribution parameters. This non-standard value is read from an alternative data file `gddparsum.har`, created from the standard data file `gdat.har` using the GEMPACK utility program `modhar` and the stored input file `gddparsum.sti`.

We also include command files for a few other scenarios. The files `oqhom.cmf` and `nqhom.cmf` define a quantity homogeneity test for the old and new theoretical structures. In the file `income.cmf` we increase real income by doubling factor productivity

in all regions; in `food.cmf`, we lower the price of food using a production subsidy. As discussed in section 5, these scenarios are designed to illustrate how the upper-level income allocation is affected by price and income changes. Finally, the file `dps.cmf` implements a 20 per cent increase in the propensity to save; this demonstrates the use of a non-standard closure for the upper level of the regional demand system, as discussed in subsection 2.14.

The software has been tested with GNU `make` under the Cygwin `bash` shell under MS Windows 4.10.2222 (that is, some flavor of MS Windows 98), with Lahey Fortran 90 version 4.5, and GEMPACK 7.0. The program files will likely also work under other Win32 operating systems (MS Windows 95, ME, 2K, XP, . . .) or (with a few simplifying adjustments) under Unix or Linux. The data files however are MS Windows-specific.

To compile the solution programs and run all simulations, just type

```
make
```

from the Cygwin bash prompt. To create a specific file type "make" with the target file name, say

```
make foo.sl4
```

There are a few dummy targets defined for special purposes: to make just the executable program files, make the dummy target `exes`:

```
make exes
```

to remove all derived files and return to the original distribution,

```
make clean
```

to remove just the simulation-related files but not the (slow to remake) executables,

```
make simclean
```

The advantage of using `make` is that it does as much or as little work as necessary to create or update the target files. So for example if you have an up-to-date executable for a simulation that you want to run, `make` knows that it need not recompile the executable before running the simulation; but if you have changed the source code for the model since last you compiled it, then `make` knows that it does need to recompile.

Appendix D Revision History

2003–09	Fix error in welfare decomposition.
2002–03	Initial version.