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J. Stuart Bolton  
*Purdue University*, bolton@purdue.edu

Hyu-Sang Kwon

J. K. Hammond  
*ISVR*

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A Comparison of Partial Coherence and Singular Value Partial Field Decomposition in the Context of Nearfield Acoustical Holography

Hyu-Sang Kwon and J. Stuart Bolton
(Herrick Labs., Purdue Univ.)
J. K. Hammond
(ISVR)
ACOUSTIC HOLOGRAPHIC METHOD

- What’s the Acoustic Holography

A Useful Sound Visualization Technique
- Localization of Sources
- 3D Characterization of Radiated Sound Fields

- Enhancement techniques
  Wavenumber filtering
  Minimum Error Windowing
- Moving frame technique
- Partial field separation
- Residual field technique

Reconstruction plane
(Source plane)
Measuring plane
(Hologram plane)
What's the partial field?

Acoustic Holography:
Coherent field procedure
Partial field separation

finite number of uncorrelated or partially correlated sources

Reconstruction plane
(Source plane)

Measuring plane
(Hologram plane)

separation of sound field related to each noise source

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BASIC IDEAS

• How can we separate partial fields?

source 1 ➔ reference 1 ➔ reference 2 ➔ source 2

field points

- Unable to directly measure source signals
- Use of references close to sources; strong relation between each source and corresponding reference
- Partial field separation methods
  • Singular value decomposition method
  • Partial coherence method

*Residual field method; modified from conventional method*

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FUNDAMENTALS

- **Source/signal relations**
  - Source 1, $S_{AA}$
  - Reference 1, $S_{11}$
  - Source 2, $S_{BB}$
  - Reference 2, $S_{22}$

- **Decomposition of reference signals**
- **Field point, $S_{yy}$**
- **Partial field related to decomposed reference signal**

- **Decomposed references**
  - SVD method
  - Partial Coh. D
  - Residual FD

- **Orthogonal**
  - 1st partial reference
  - 2nd partial reference

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- **2 incoherent sources/2 references**

   - Incoherent sources; $S_{AB} = 0$
   - Transfer functions: $g_{ij}$, geometry relation
     - **simple point source case**
       \[
       g_{ij} = \frac{e^{jkr_{ij}}}{r_{ij}}
       \]
       $r_{ij}$: distance between $i$ source & $j$ receiver
     - **strong source/corresponding reference relation**
       \[
       g_{11}, g_{22} \gg g_{12}, g_{21}, g_{1y}, g_{2y}
       \]
METHODOLOGIES

- **Reference spectral matrix**
  
  \[ \mathbf{r} = \mathbf{G} \mathbf{a} \]  
  
  reference signals \( \mathbf{r} \) source amplitudes \( \mathbf{a} \)

  TF's from sources to references, geometric relations

  \[ \mathbf{S}_{\text{ref}} = E\{\mathbf{r}^* \mathbf{r}^T\} = \mathbf{G}^* \mathbf{S}_S \mathbf{G} \]

- **Partial field separation**
  
  - Singular value decomposition method;
    - depends on the levels of sources & geometrical TFs (\( g_{ij}, \mathbf{S}_{AA}, \mathbf{S}_{BB} \))
    - reordering, singular value swapping
  
  - Partial coherence method;
    - depends on the levels of sources & geometrical TFs (\( g_{ij}, \mathbf{S}_{AA}, \mathbf{S}_{BB} \))
  
  - Residual field method;
    - depends only on the geometrical TF's (\( g_{ij} \))
    - easily applicable to general acoustic problems, including structural acoustic problems
INCOHERENT REFS.

- 1-to-1 relation between sources & references

source 1 $S_{AA}$

No cross-relations between sources & refs., diagonal $S_{ref}$

source 2 $S_{BB}$

reference 1

g_{11}

reference 2

g_{22}

g_{1y}

g_{2y}

field points

PCoh

SVD

$S_{xx} = |g_{xy}|^2 S_{yy}$

$S_{yy} = |g_{yy}|^2 S_{xx}$

$\lambda_1 = |g_{xy}|^2 S_{yy}$

$\lambda_2 = |g_{yy}|^2 S_{xx}$

1st partial field

2nd partial field

freq.

freq.

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SIMULATION

- Simulation parameters
  source1

  $N = 512$
  $\Delta f = 8\text{Hz}$
  $N_{\text{avg}} = 250$

  ref. 1
  X
  0.02m

  source2

  ref. 2
  Z
  0.1m

  19 field points

---

Power spectra of Source 1 & 2

Source Levels (dB)

Source 1, $S_{AA}$
Source 1, $S_{BB}$

Frequency (Hz)

Coherence between Source 1 & 2

Coherence

0.2
0.4
0.6
0.8
1.0

Frequency (Hz)

Power spectra of Reference 1 & 2

Sound Levels (dB)

Reference 1, $S_{11}$
Reference 2, $S_{22}$

Frequency (Hz)

Coherence between Reference 1 & 2

Coherence

0.0
0.2
0.4
0.6
0.8
1.0

Frequency (Hz)

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**SIMULATION**

- **Estimation of source strengths**
  - [Graph showing conditioned power spectra of Reference 1 & 2]
  - [Graph showing singular values of reference crossspectral matrix]

- **Comparison of separated partial fields**
  - [Graph showing comparison of partial field 1 at 200 Hz]
  - [Graph showing comparison of partial field 2 at 200 Hz]
CONCLUSION

- Partial field separation methods
  - Use of as many references as sources, and place them close to incoherent sources
  - SVD, Partial coherence techniques
    - depend on source levels as well as geometries of sources & refs.
    - reordering of references & partial fields
  - A useful alternative: Residual Field Technique
    - modified from partial coherence technique
    - depends only on the geometries of source & refs.
    - simple extraction of partial field related to each reference

- Applicable to the structural acoustic problems
In case of 1-to-1 measurement

\[ S_{\text{ref}} = \begin{bmatrix} |g_{11}|^2 S_{AA} & 0 \\ 0 & |g_{22}|^2 S_{BB} \end{bmatrix} \]

\[ S_{11} = |g_{11}|^2 S_{AA}, \quad S_{22} = |g_{22}|^2 S_{BB} \]

\[ H_{1y} = \frac{g_{1y}}{g_{11}}, \quad H_{2y} = \frac{g_{2y}}{g_{22}} \]

\[ \lambda_1 = \max\{|g_{11}|^2 S_{AA}, |g_{22}|^2 S_{BB}\}, \quad H_{1y} = \frac{g_{1y}}{g_{11}}, \quad H_{2y} = \frac{g_{2y}}{g_{22}} \quad \text{if} \ |g_{11}|^2 S_{AA} > |g_{22}|^2 S_{BB} \]

\[ \lambda_2 = \min\{|g_{11}|^2 S_{AA}, |g_{22}|^2 S_{BB}\}, \quad H_{1y} = \frac{g_{2y}}{g_{22}}, \quad H_{2y} = \frac{g_{1y}}{g_{11}} \quad \text{if} \ |g_{11}|^2 S_{AA} < |g_{22}|^2 S_{BB} \]

Cross-relations between sources/references

\[ |g_{11}|^2 S_{AA}, |g_{22}|^2 S_{BB} \gg |g_{12}|^2 S_{AA}, |g_{21}|^2 S_{BB} \quad S_{AA} \approx S_{BB} \quad \text{source level relation} \]

\[ |g_{11}, g_{22}| \gg |g_{12}, g_{21}, g_{1y}, g_{2y}| \quad \text{geometric relation} \]