A Comparison of Partial Coherence and Singular Value Partial Field Decomposition in the Context of Nearfield Acoustical Holography

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ACOUSTIC HOLOGRAPHIC METHOD

- What’s the Acoustic Holography

A Useful Sound Visualization Technique
- Localization of Sources
- 3D Characterization of Radiated Sound Fields

- Enhancement techniques
  - Wavenumber filtering
  - Minimum Error Windowing
- Moving frame technique
- Partial field separation
- Residual field technique
INTRODUCTION

What's the partial field?

Acoustic Holography:
Coherent field procedure
Partial field separation

finite number of
uncorrelated or
partially correlated
sources
total sound field

separation of sound
field related to each
noise source

engine noise

exhaust noise

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BASIC IDEAS

• How can we separate partial fields?

source 1  source 2

reference 1  reference 2

field points

– Unable to directly measure source signals
– Use of references close to sources; strong relation between each source and corresponding reference
– Partial field separation methods
  • Singular value decomposition method
  • Partial coherence method

Residual field method; modified from conventional method

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FUNDAMENTALS

- **Source/signal relations**
  - Source 1, $S_{AA}$
  - Reference 1, $S_{11}$
  - Source 2, $S_{BB}$
  - Reference 2, $S_{22}$
  - Field point, $S_{yy}$

- **Decomposition of reference signals**

- **Decomposed references**
  - SVD method
  - Partial Coh. D
  - Residual FD

- Orthogonal
  - 1st partial reference
  - 2nd partial reference

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2 SOURCE MODELLING

- 2 incoherent sources / 2 references

- Incoherent sources; $S_{AA} = 0$
- Transfer functions: $g_{ij}$, geometry relation
  - simple point source case
    $$ g_{ij} = \frac{e^{jkr_{ij}}}{r_{ij}} $$
    $r_{ij}$: distance between $i$ source & $j$ receiver
  - strong source/corresponding reference relation
    $$ g_{11}, g_{22} \gg g_{12}, g_{21}, g_{1y}, g_{2y} $$
METHODOLOGIES

- Reference spectral matrix
  
  \[ r = G_{\text{ref}} a \]
  
  source amplitudes
  
  TF's from sources to references, geometric relations
  
  \[ S_{\text{ref}} = E\{r^* r^T\} = G^* S_S G \]

- Partial field separation
  
  - Singular value decomposition method;
    - depends on the levels of sources & geometrical TFs \((g_{ij}, S_{AA}, S_{BB})\)
    - reordering, singular value swapping
  
  - Partial coherence method;
    - depends on the levels of sources & geometrical TFs \((g_{ij}, S_{AA}, S_{BB})\)
  
  - Residual field method;
    - depends only on the geometrical TF's \((g_{ij})\)
    - easily applicable to general acoustic problems, including structural acoustic problems
INCOHERENT REFS.

• 1-to-1 relation between sources & references
  
  source 1
  \( S_{AA} \)
  
  g_{11}

  reference 1

  No cross-relations
  between sources & refs.
  diagonal \( S_{\text{ref}} \)

  source 2
  \( S_{BB} \)
  
  g_{22}

  reference 2

  field points

  \( P_{\text{Coh}} \)
  Level
  \[ S_{zz} = |g_{zz}|^2 S_{BB} \]
  \[ S_{nn} = |g_{nn}|^2 S_{AA} \]

  freq.

  1st partial field

  2nd partial field

  \( S_{AA} \)

  \( S_{BB} \)

  \( g_{11} \)

  \( g_{22} \)

  \( g_{1y} \)

  \( g_{2y} \)

  \( \lambda_1 = |g_{zz}|^2 S_{BB} \)

  \( \lambda_2 = |g_{nn}|^2 S_{AA} \)

  \( \lambda_3 = |g_{zz}|^2 S_{BB} \)

  freq.

  1st partial field

  2nd partial field

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Simulation parameters

- N = 512
- Δf = 8 Hz
- $N_{avg} = 250$

Power spectra of Source 1 & 2

Coherence between Source 1 & 2

Power spectra of Reference 1 & 2

Coherence between Reference 1 & 2

19 field points

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SIMULATION

- Estimation of source strengths

- Comparison of separated partial fields
CONCLUSION

- Partial field separation methods
  - Use of as many references as sources, and place them close to incoherent sources
  - SVD, Partial coherence techniques
    • depend on source levels as well as geometries of sources & refs.
    • reordering of references & partial fields
  - A useful alternative: *Residual Field Technique*
    • modified from partial coherence technique
    • depends only on the geometries of source & refs.
    • simple extraction of partial field related to each reference

- Applicable to the structural acoustic problems
APPENDIX

• In case of 1-to-1 measurement

\[
S_{\text{ref}} = \begin{bmatrix}
|g_{11}|^2 S_{AA} & 0 \\
0 & |g_{22}|^2 S_{BB}
\end{bmatrix}
\]

\[
S_{11} = |g_{11}|^2 S_{AA}, S_{22} = |g_{22}|^2 S_{BB}
\]

\[
H_{1y} = \frac{g_{1y}}{g_{11}}, H_{2y} = \frac{g_{2y}}{g_{22}}
\]

\[
\lambda_1 = \max\{|g_{11}|^2 S_{AA}, |g_{22}|^2 S_{BB}\}, \quad H_{1y} = \frac{g_{1y}}{g_{11}}, H_{2y} = \frac{g_{2y}}{g_{22}} \quad \text{if} \quad |g_{11}|^2 S_{AA} > |g_{22}|^2 S_{BB}
\]

\[
\lambda_2 = \min\{|g_{11}|^2 S_{AA}, |g_{22}|^2 S_{BB}\}, \quad H_{1y} = \frac{g_{2y}}{g_{22}}, H_{2y} = \frac{g_{1y}}{g_{11}} \quad \text{if} \quad |g_{11}|^2 S_{AA} < |g_{22}|^2 S_{BB}
\]

• Cross-relations between sources/references

\[
|g_{11}|^2 S_{AA}, |g_{22}|^2 S_{BB} \gg |g_{12}|^2 S_{AA}, |g_{21}|^2 S_{BB} \quad S_{AA} \approx S_{BB} \quad \text{source level relation}
\]

\[
g_{11}, g_{22} \gg g_{12}, g_{21}, g_{1y}, g_{2y} \quad \text{geometric relation}
\]