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Plane Wave Reflection Coefficient Estimation
By Use Of Spatial Parametric Signal Modeling

Hyu-Sang Kwon and J. Stuart Bolton
(Herrick Labs., Purdue Univ.)
INTRODUCTION

How can we measure the reflection coefficient?

- Plane wave
  - Incoming and outgoing wave decomposition by two microphones
  - Plane wave decomposition by linear microphone array
- Axisymmetric source
  - Reflection coefficient measurement w.r.t incident angle by wave decomposition theory
WAVE NUMBER SPECTRUM

- Plane wave decomposition theory

Spatial Fourier transform

P(x,y,z)

P(x,y,z)

P(x,y,z)

P(x,y,z)

P(x,y,z)

space domain

wave number domain

wave number domain

radiation circle

k_x

k_y

k_z

k_x

k_x

k_x

k_x

k_x

k_x
ZEROTH ORDER HANKEL TRANSFORM

- An alternative of Fourier transform
  - Axisymmetric field
  - Corresponds to 2dim. spatial Fourier transform

Non-axisymmetric field

Non-axisymmetric wave number spectrum

2 dimensional spatial Fourier transform

Axisymmetric field

Axisymmetric wave number spectrum

space domain

wave number domain

r dependent

k_r dependent

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**Reflection Coefficient**

**Procedure**

- \( P_1(r) \) \( \rightarrow \) Hankel TF \( \rightarrow \tilde{P}_1(k_r) \)
- \( P_2(r) \) \( \rightarrow \) Hankel TF \( \rightarrow \tilde{P}_2(k_r) \)

**Wave decomposition**

- \( \tilde{P}_{\text{incident}}(k_r) \)
- \( \tilde{P}_{\text{reflect}}(k_r) \)
- \( R(\theta) \)

**Zeroth order Hankel transform**

- **Definition:**
  \[
  \tilde{X}(k_r) = \int_0^\infty rX(r)J_0(k_r r)dr
  \]
- **Wave number and angle relation**
  \[
  k_r = \sqrt{k_x^2 + k_y^2} \\
  \theta = \cos^{-1}\left(\frac{\sqrt{k^2 - k_r^2}}{k}\right)
  \]
**PRONY APPROACH**

- **Finite measurement aperture**
  - Finite size of anechoic room & material
  - Finite number of measuring microphones

- **DHT (Discrete Hankel Transform)**
  - Leakage & poor spectral resolution
  - Less information

- **Parametric spectral estimation**
  - Prony model: damped harmonic signal
  - High resolution of reflection coefficient
  - Less measurement positions
  - Model order selection: by using input white noise variance

\[
\frac{\Delta k_r}{k} = \frac{\lambda}{N \Delta r}
\]

\[
P(r) = \sum_{n=1}^{N} A_n e^{-\alpha_n r}
\]

\[
X(z) = H(z)U(z), \quad \rho_w = P_{uu}(z)
\]

*constant*

Ref.: Akaike Information Criterion, etc.

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**Simulation**

**Simulation setup**

Monopole or dipole

- Frequency = 343 Hz
- Wavelength = 1 m
- No. of data = 50 per line

Rigid wall, $R=1$ or
No wall, $R=0$

- Monopole, dipole simulation
- No wall, rigid wall simulation

Reflection coefficients calculation
- Discrete Hankel Transform
- Prony spectral estimation
MONOPOLE CASE

Model order, N=5
No. of poles=6

Model order, N=5
No. of poles=6

Monopole without reflecting wall
Monopole with rigid reflecting wall
DIPOLE CASE

Model order, N=6
No. of poles=7

Model order, N=6
No. of poles=7

Dipole without reflecting wall

Dipole with rigid reflecting wall

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**EXPERIMENT**

- **Experimental setup**

  Axisymmetric source
  ![Unbaffled Loudspeaker](image)

  0.1m 0.012m 0.025m 0.002m 0.01m

  Reflecting surface, carpet

  21 frequencies (1953Hz-4004Hz)
  No. of data=120 per line
  In a semi-anechoic room

- Reflection coefficient calculation
  - Discrete Hankel transform approach
  - Prony spectral estimation approach
EXPERIMENTAL RESULTS

Model order, N=18
No. of poles=19

Model order, N=11
No. of poles=12

Frequency=1953Hz, N=120

Frequency=1953Hz, N=50

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EXPERIMENTAL RESULTS

Model order, N=14
No. of poles=15

Model order, N=11.3
No. of poles=12.14

Frequency=2832Hz, N=120
Frequency=2832Hz, N=50

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CONCLUSIONS

- Reflection coefficient measurement
  - Wave decomposition theory
  - 2 line microphone array

- Prony approach
  - Higher spectral resolution than discrete Hankel transform approach
  - Effective with less measurement points
  - Less leakage due to finite aperture

- Simulation & Experiment
**APPENDIX**

- **Prony model**

\[
P(r) = \sum_{n=1}^{N} A_n e^{-\alpha_n r} \quad P(i) = \sum_{n=1}^{N} B_n Z_n^i \quad r = \delta \Delta r, (1+\delta) \Delta r, (2+\delta) \Delta r, \ldots
\]

\[
\alpha_n = -\frac{\log Z_n}{\Delta r}, \ A_n = B_n e^{\alpha_n \delta \Delta r} \quad |Z_n| < 1
\]

\[
\tilde{P}(k_r) = \int_0^\infty r P(r) J_0(k_r r) dr
\]

\[
= \sum_{n=1}^{N} A_n \int_0^\infty r e^{-\alpha_n r} J_0(k_r r) dr
\]

\[
= \sum_{n=1}^{N} \frac{A_n \alpha_n}{(\alpha_n^2 + k_r^2)^{3/2}}
\]

**Prony method:**

Calculate \( A_n, \alpha_n \) by use of 2N or more data

**Reflection coefficient**

\[
R(k_z) = \frac{-\tilde{P}_1 e^{-jk_z z_2} + \tilde{P}_2 e^{-jk_z z_1}}{\tilde{P}_1 e^{jk_z z_2} - \tilde{P}_2 e^{jk_z z_1}}
\]