Plane Wave Reflection Coefficient Estimation by Use of Spatial Parametric Signal Modeling

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Plane Wave Reflection Coefficient Estimation
By Use Of Spatial Parametric Signal Modeling

Hyu-Sang Kwon and J. Stuart Bolton
(Herrick Labs., Purdue Univ.)
INTRODUCTION

- How can we measure the reflection coefficient?
  - Plane wave
  - Axisymmetric source
  - Incoming and outgoing wave decomposition by two microphones
  - Plane wave decomposition by linear microphone array
  - Two plane linear microphone array
  - Reflection coefficient measurement w.r.t incident angle by wave decomposition theory
WAVE NUMBER SPECTRUM

- Plane wave decomposition theory

Spatial Fourier transform

P(x,y,z)

wave number domain

wave number domain

radiation circle

P(x,y,z)

space domain

P(x,y,z)

P(x,y,z)

P(x,y,z)

k_x

k_y

k_z

k_x

k_y

k_z

P(x,y,z)

k_x

k_y

k_z

k_x

k_y

k_z

P(x,y,z)

k_x

k_y

k_z

wave number domain

Radiation circle

P(x,y,z)

k_x

k_y

k_z

wave number domain

P(x,y,z)

k_x

k_y

k_z

wave number domain

P(x,y,z)

k_x

k_y

k_z

wave number domain

P(x,y,z)

k_x

k_y

k_z

wave number domain

P(x,y,z)

k_x

k_y

k_z

wave number domain

P(x,y,z)

k_x

k_y

k_z

wave number domain

P(x,y,z)
ZEROTH ORDER HANKEL TRANSFORM

- An alternative of Fourier transform
  - Axisymmetric field
  - Corresponds to 2dim. spatial Fourier transform

Non-axisymmetric field

2 dimensional spatial Fourier transform

Axisymmetric field

Space domain: \( r \) dependent

Wave number domain: \( k_r \) dependent

Non-axisymmetric wave number spectrum
**Reflection Coefficient**

**Procedure**
- $P_1(r) \rightarrow \text{Hankel TF} \rightarrow \tilde{P}_1(k_r)$
- $P_2(r) \rightarrow \text{Hankel TF} \rightarrow \tilde{P}_2(k_r)$

- Measured pressures
- Wavenumber spectra

- Wave decomposition
  - $\frac{\tilde{P}_{\text{incident}}(k_r)}{\tilde{P}_{\text{reflect}}(k_r)} = R(\theta)$

**Zeroth order Hankel transform**
- Definition: $\tilde{X}(k_r) = \int_0^\infty rX(r)J_0(k_r r)dr$
- Wave number and angle relation
  $$k_r = \sqrt{k_x^2 + k_y^2} \quad \theta = \cos^{-1}\left(\frac{\sqrt{k_x^2 - k_r^2}}{k}\right)$$
**PRONY APPROACH**

- **Finite measurement aperture**
  - Finite size of anechoic room & material
  - Finite number of measuring microphones

- **DHT (Discrete Hankel Transform)**
  - Leakage & poor spectral resolution
  - Less information
  
  \[
  \frac{\Delta k}{k} = \frac{\lambda}{N\Delta r}
  \]

- **Parametric spectral estimation**
  - Prony model: damped harmonic signal
  - High resolution of reflection coefficient
  - Less measurement positions
  - Model order selection: by using input white noise variance
  \[
  X(z) = H(z)U(z), \quad \rho_w = P_{uu}(z)
  \]

Ref.: Akaike Information Criterion, etc.
**Simulation setup**

- Monopole or dipole

  - 0.15m
  - 0.05m
  - 0.07m
  - 0.05m
  - 0.005m
  - 0.05m

- Rigid wall, R=1 or No wall, R=0
  - Monopole, dipole simulation
  - No wall, rigid wall simulation

- Frequency=343Hz
- Wavelength=1m
- No. of data=50 per line

**Reflection coefficients calculation**
- Discrete Hankel Transform
- Prony spectral estimation
MONOPOLE CASE

Model order, N=5
No. of poles=6

Model order, N=5
No. of poles=6

Monopole without reflecting wall
Monopole with rigid reflecting wall

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DIPOLE CASE

Model order, N=6
No. of poles=7

Dipole without reflecting wall

Model order, N=6
No. of poles=7

Dipole with rigid reflecting wall

Prony approach
DHT approach

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**EXPERIMENT**

- **Experimental setup**

  Axisymmetric source
  
  Unbaffled Loudspeaker

  ![Diagram of experimental setup]

  21 frequencies (1953Hz-4004Hz)
  No. of data=120 per line
  In a semi-anechoic room

  Reflecting surface, carpet

  - Reflection coefficient calculation
    - Discrete Hankel transform approach
    - Prony spectral estimation approach

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EXPERIMENTAL RESULTS

Model order, N=18
No. of poles=19

Model order, N=11
No. of poles=12

Frequency=1953Hz, N=120

Frequency=1953Hz, N=50

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EXPERIMENTAL RESULTS

Model order, N=14
No. of poles=15

Model order, N=11
No. of poles=12

Frequency=2832Hz, N=120

Frequency=2832Hz, N=50
CONCLUSIONS

- Reflection coefficient measurement
  - Wave decomposition theory
  - 2 line microphone array

- Prony approach
  - Higher spectral resolution than discrete Hankel transform approach
  - Effective with less measurement points
  - Less leakage due to finite aperture

- Simulation & Experiment
**APPENDIX**

- **Prony model**

\[ P(r) = \sum_{n=1}^{N} A_n e^{-\alpha_n r} \quad P(i) = \sum_{n=1}^{N} B_n Z_n^i \quad r = \delta \Delta r, (1+\delta) \Delta r, (2+\delta) \Delta r, \ldots \]

\[ \alpha_n = -\frac{\log Z_n}{\Delta r}, \quad A_n = B_n e^{\alpha_n \delta \Delta r}, \quad |Z_n| < 1 \]

\[ \tilde{P}(k_r) = \int_0^\infty rP(r)J_0(k_r r)dr \]

\[ = \sum_{n=1}^{N} A_n \int_0^\infty re^{-\alpha_n r} J_0(k_r r)dr \]

\[ = \sum_{n=1}^{N} \frac{A_n \alpha_n}{(\alpha_n^2 + k_r^2)^{3/2}} \]

Prony method:
Calculate \( A_n, \alpha_n \) by use of 2N or more data

Reflection coefficient

\[ R(k_z) = -\frac{\tilde{P}_1 e^{-j k_z z_2} + \tilde{P}_2 e^{-j k_z z_1}}{\tilde{P}_1 e^{j k_z z_2} - \tilde{P}_2 e^{j k_z z_1}} \]