New Techniques for the Design of Diffractive Optical Devices for Optical Communications and Networking

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NEW TECHNIQUES FOR THE DESIGN OF DIFFRACTIVE OPTICAL DEVICES FOR OPTICAL COMMUNICATIONS AND NETWORKING

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New Techniques for the Design of Diffractive Optical Devices for Optical Communications and Networking

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Three new techniques are proposed for the design of optical devices for various applications in optical communications and networking. The first algorithm shows that the nonperiodic grating-assisted directional coupler design can achieve not only the complete power transfer but also superior performances over the conventional periodic grating-assisted directional coupler in nonsynchronous waveguides. The second method is the newly proposed angular spectrum and beam propagation methods using the continuous and discrete sine transforms. This alternative approach is useful especially when the boundary conditions need to be zeroes, for example at the dielectric-metal interfaces. Lastly, the ranked phased-array method (RPAM) helps achieve an optical device called ranked phased-array (RPA) that, along with the existing phased-array (PHASAR) based devices, can efficiently utilize most of the optical low-loss bandwidths in a single system, and has the potential to greatly increase the number of transmitted channels in dense wavelength division multiplexing (DWDM).
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CHAPTER 1

INTRODUCTION

Lightwave communications and networking have become major area of interest, especially with the advent of multimedia systems. The current trend is to develop all-optical devices in order to fully exploit the speed of optics. The objective of this thesis is to make contributions to this area by providing new algorithms for better diffractive optical devices, especially in the design of directional couplers, in the use of angular spectrum and beam propagation methods, and, in designing phased-array type of dense wavelength division multiplexing systems.

A directional coupler is a device with two waveguides closely placed in parallel so that an optical wavefield launched in one waveguide can be completely or partially transferred into the other waveguide depending on the coupling length allowed by the directional coupler. These characteristics allow a directional coupler to be used for optical switching, filtering, and so on [1].

In a practical case with the two waveguides having mismatched refractive indices, the grating-assisted structure in which the separation gap of the two waveguides is varied along the propagation direction can help achieve the complete power transfer. Generally, the nonsynchronous waveguide alone cannot accomplish the task. Most studies have paid attention to improve the grating-assisted directional coupler in periodic structures. In this study, a nonperiodic grating-assisted structure is introduced. Not only that the nonperiodic structure can succeed in complete power transferring but also can considerably reduce the coupling lengths. Such reductions can provide smaller dimension that can be significant in applications of integrated optics. The principle of nonperiodic grating-assisted directional coupler and its comparison to the conventional periodic type
are detailed in chapter 2. Because of the complexity of the nonperiodic structure, the experimental results are considered by the beam propagation method (BPM).

In Chapter 3, an alternative derivation of the BPM with the sine transform and the real Fourier transform in place of the complex Fourier transform is proposed for the cases of applications with zero boundary conditions. This would be the case when there are dielectric-metal interfaces, for example, with waveguides that are shielded to prevent electromagnetic interference. In such a structure, the restriction can hardly be applied to the conventional BPM. Because of its nature, the new algorithms can automatically take care of these boundary conditions and lead to more accurate numerical results.

In the course of increasing the amount of information transmitted in lightwave communications and networking, recent interest has been shifted to the technique called dense wavelength division multiplexing (DWDM). When each transmission waveguide carries several optical channels simultaneously, the high capacity system demands optical devices that can effectively perform some basic tasks in optical signal processing, such as switching, filtering, multiplexing, and demultiplexing, etc.

Currently, optical devices widely used as multiplexers or demultiplexers in an optical communication system can be categorized as grating-based and phased-array based devices. The grating-based type has a structure of vertically etched medium so that, after passing through the device at a specific location, an input wavefield is manipulated in amplitude and/or phase to yield the required focusing and dispersing characteristics for multiplexing or demultiplexing. In phased-array (PHASAR) based components, the phase shifting of the input field is caused by the arrangement of waveguides with different lengths into an array such that the anticipated output field can be realized.

In this study, the main concentration is on the PHASAR based devices, which are also called arrayed waveguide gratings (AWG). Because this structure can be implemented with conventional waveguide technology and does not require vertical etching as in the grating-based devices, it is more robust and fabrication tolerant[10].
The recent studies on PHASAR based devices [11]-[15] have shown that an AWG is very competent in separating wavelengths in the optical-fiber low-loss bandwidths that are in the vicinity of wavelengths 1.3 and 1.55 \( \mu m \). The wavelengths separations can be made as small as 0.08 nm or equivalent to 10 GHz [13] or the number of operating channels can be as many as 128 channels [20]. Even so, the usage of the optical bandwidth is small compared to each of the available low-loss regions because of the limitation by free space range (FSR). For example, the low-loss band around wavelength of 1.55 \( \mu m \) is approximately 14 THz or 120 nm wide, while the 10 GHz-AWG utilizes 122.8 GHz or 0.98 nm, and the 128 channel-demultiplexer occupies only 3.2 THz or 25.6 nm.

In order to accomplish better employment of the low-loss bandwidth available, a new method called the ranked phased-array method (RPAM) is proposed in chapters 4 and 5. The RPAM is based on a statistical ranking algorithm in order to design a phased-array device, which can separate subbands of optical channels. In Chapter 4, the background theory and general applications of the RPAM are described. Chapter 5 involves the combined use of the ranked phased-array (RPA) and AWG for enhancing the operating bandwidth and increasing the number of transmitting channels in comparison to the existing systems that is only composed of AWG.

In chapter 6, the conclusions are discussed and further researches are recommended.
CHAPTER 2
AN ITERATIVE METHOD FOR THE DESIGN OF NONPERIODIC GRATING-ASSISTED DIRECTIONAL COUPLER USING THE BEAM PROPAGATION METHOD

In this study, an iterative method using the beam propagation method (BPM) for the optimal design of the nonperiodic grating-assisted directional coupler has been proposed and investigated. The numerical technique is also compared to the analytical method and found to yield accurate results. By applying the iterative method to the design of nonperiodic grating-assisted directional coupler, the complete power coupling-length can be reduced by one-third of those of the periodic grating-assisted types with the same sets of waveguide parameters.

2.1 Introduction

It is sometimes difficult to determine the exact solution of the scalar wave equations by a conventional method, especially in a nonhomogeneous medium whose index of refraction is not constant at all locations. The BPM is an alternative method for approximately simulating optical wave propagation in such a medium of arbitrary shape with the constraints that reflected waves can be neglected and all refractive index differences are small [1].

The BPM is derived from the Helmholtz equation and the angular spectrum method as described in Section 2.2. The comparison of the exact solutions and the computer simulations for general cases of directional coupler by the BPM is discussed in Section 2.3. Using the BPM, the design of nonperiodic grating-assisted directional
coupler system are realized and the experimental results are compared with their periodic counterparts in Section 2.4.

2.2 Theory of The Beam Propagation Method

2.2.1 The Helmholtz equation

Let a scalar function \( u(x,y,z,t) \) represent the electric and/or magnetic components at position \( (x,y,z) \) and at time \( t \) of a wavefield propagating in a medium with the refractive index distribution \( n(x,y,z) \) with the wavelength \( \lambda \) or angular frequency \( \omega \). The monochromatic wavefield can be written in the form

\[
u(x, y, z, t) = U(x, y, z) \cos(\omega t + \Theta(x, y, z))\tag{2.1}
\]

where \( U(x,y,z) \) is the amplitude and \( \Theta(x,y,z) \) is the phase at position \( (x,y,z) \).

In complex notation, the wavefield can also be expressed as

\[
u(x, y, z, t) = \Re\{U(x, y, z) \exp(-j\omega t)\}\tag{2.2}
\]

where \( U(x,y,z) \) is the complex amplitude equal to \( U(x,y,z) \exp(-j\Theta(x,y,z)) \).

If the real expression in Eq. (2.2) is to represent an optical wavefield, it must satisfy the scalar wave equation

\[
\nabla^2 u(x, y, z, t) + n^2(x, y, z)k^2 u(x, y, z, t) = 0
\]

(2.3)

where the wave number \( k = \frac{\omega}{c} = \frac{2\pi}{\lambda} \).

Because the time dependence is known a priori, the complex function \( U(x, y, z) \) can as well provide adequate information about the wavefield. Substituting \( u(x,y,z,t) \) from Eq. (2.2) into the scalar wave equation (2.3), the complex notation of the wavefield \( U(x,y,z)=U(x,y,z)\exp(-j\Theta(x,y,z)) \) must also obey the wave equation and yields the Helmholtz equation:

\[
\left(\nabla^2 + n^2(x,y,z)k^2\right)U(x,y,z) = 0
\]

(2.4)
2.2.2 The angular spectrum of plane waves

The 2-D Fourier representation of \( U(x, y, z) \) in the space domain is given in terms of its Fourier transform \( A(\alpha, \nu, z) \) in the spatial frequency domain by

\[
U(x, y, z) = \mathcal{F}^{-1}\{A(\alpha, \nu, z)\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(\alpha, \nu, z) \exp(j2\pi(\alpha x + \nu y)) d\alpha d\nu \tag{2.5}
\]

where \( A(\alpha, \nu, z) \) is called the angular spectrum of \( U(x, y, z) \)

\[
A(\alpha, \nu, z) = \mathcal{F}\{U(x, y, z)\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(x, y, z) \exp(-j2\pi(\alpha x + \nu y)) dx dy \tag{2.6}
\]

With the assumption that the refractive index values \( n(x, y, z) \) are nearly the same everywhere in the waveguide, substituting \( U(x, y, z) \) from Eq. (2.5) into Eq. (2.4) with \( n(x, y, z) = n_0 \) and rearranging yields

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \frac{d^2}{dz^2} A(\alpha, \nu, z) + \left[ n_0^2 k^2 - 4\pi^2 (\alpha^2 + \nu^2) \right] A(\alpha, \nu, z) \right] \exp(j2\pi(\alpha x + \nu y)) d\alpha d\nu = 0 \tag{2.7}
\]

Eq. (2.7) is true for all waves only if the integrand is zero, or

\[
\frac{d^2}{dz^2} A(\alpha, \nu, z) + \left[ n_0^2 k^2 - 4\pi^2 (\alpha^2 + \nu^2) \right] A(\alpha, \nu, z) = 0 \tag{2.8}
\]

The solution depends only on the variable \( z \) in the form

\[
A(\alpha, \nu, z) = A(\alpha, \nu, 0) \exp(j\mu z) \tag{2.9}
\]

with the parameter \( \mu = \sqrt{n_0^2 k^2 - 4\pi^2 (\alpha^2 + \nu^2)} \).

If \( 4\pi^2 (\alpha^2 + \nu^2) < n_0^2 k^2 \), \( \mu \) is real. Each angular spectrum component \( A(\alpha, \nu, z) \) at any distance \( z \) is its response at \( z = 0 \), \( A(\alpha, \nu, 0) \), multiplied by a phase factor \( \exp(j\mu z) \).

In this case, the propagating wave is called the heterogeneous wave.
If $4n^2(a^2 + v^2) > n^2k^2$, $p$ is in the form $\mu = j\sqrt{4n^2(a^2 + v^2) - n^2k^2}$ and Eq. (2.9) becomes $A(\alpha, v, z) = A(\alpha, v, 0) \exp(-j\mu z)$. This result shows that the amplitudes of the angular spectrum components are exponentially attenuated while propagating along the $z$-direction. This is the case of evanescent waves.

If $4n^2(a^2 + v^2) = n^2k^2$, $\mu = 0$ which makes $A(\alpha, v, z) = A(\alpha, v, 0)$ at any distance $z$. This is the case of plane wave traveling perpendicular to the $z$-direction.

2.2.3 The beam propagation method (BPM)

The principal of the BPM is based on the understanding that, in a nonhomogeneous medium, the optical wave undergoes the same effect of diffraction as in a homogeneous medium with average index of refraction $n$, though, with different amounts of phase shift because of the nonhomogeneity. The additional phase shift can be considered to be due to the change $\{n(x, y, z) - n\}$ in the index of refraction. Whereas the alterations occur continuously while the optical wave travels along the guide with distance $\Delta z$, they can be determined separately in two serial processes: (1) wave propagating in a homogeneous medium of refractive index $n$, and, (2) the virtual lens effect. Thus, the BPM can be conceived as the system of homogeneous media with the periodic array of lenses as shown in Fig. 2.1.

Fig. 2.1. The equivalent optical system for simulating the BPM.
The effectiveness of this method depends on the choice of $\Delta z$ at each step to be small enough to achieve the degree of accuracy with a reasonable amount of computation time.

2.2.3.1 Wave propagation in a homogeneous medium $\tilde{n}$

The angular spectrum responses at any distance $z$, $\mathbf{A}(\alpha, \nu, z)$, which is the 2-D Fourier transform of the spatial field $\mathbf{U}(x, y, z)$, can be determined from $\mathbf{A}(\alpha, \nu, 0)$ as described in Eq. (2.9). $\mathbf{U}(x, y, z)$, can also be obtained from $\mathbf{U}(x, y, 0)$ with the help of 2-D Fourier transform and its inverse as follows:

From Eq. (2.5) and (2.9), the homogeneous waves for which $\tilde{n}^2 k^2 \geq 4\pi^2 (\alpha^2 + \nu^2)$ can be expressed as

$$
\mathbf{U}(x, y, z) = \mathcal{F}^{-1} \{ \mathbf{A}(\alpha, \nu, 0) \exp \left( j \sqrt{\tilde{n}^2 k^2 - 4\pi^2 (\alpha^2 + \nu^2)} z \right) \} 
$$

$$
= \int \int \mathbf{A}(\alpha, \nu, 0) \exp \left( j \sqrt{\tilde{n}^2 k^2 - 4\pi^2 (\alpha^2 + \nu^2)} z \right) \exp (j 2\pi (\alpha x + \nu y)) d\alpha d\nu 
$$

(2.10)

According to Eq. (2.6), $\mathbf{A}(\alpha, \nu, 0)$ can be obtained beforehand if $\mathbf{U}(x, y, 0)$ is known:

$$
\mathbf{A}(\alpha, \nu, 0) = \mathcal{F} \{ \mathbf{U}(x, y, 0) \} = \int \int \mathbf{U}(x, y, 0) \exp (- j 2\pi (\alpha x + \nu y)) dx dy 
$$

(2.11)

Therefore,

$$
\mathbf{U}(x, y, z) = \mathcal{F}^{-1} \{ \mathcal{F} \{ \mathbf{U}(x, y, 0) \} \exp (+ j \beta z) \} 
$$

(2.12)

where $\beta = \sqrt{\tilde{n}^2 k^2 - 4\pi^2 (\alpha^2 + \nu^2)}$.

According to [1], the transformation is confined to the region

$$
4\pi^2 (\alpha^2 + \nu^2) \leq \tilde{n}^2 k^2 \text{ or } (\alpha^2 + \nu^2) \leq \frac{\tilde{n}^2}{\lambda^2} 
$$

(2.13)
which exclude the spatial frequency region of evanescent waves. However, without the contribution from evanescent waves, the expansion would not be complete. In our numerical investigation reported in Section 2.4, inclusion of evanescent waves gave similar results. The numerical integration can be done according to the theory discussed in Numerical Computational Of the Fourier Transform in [2].

To separate the rapid $z$-variations caused by large $\beta$ inside the exponent terms in (2.12), $\beta$ is rewritten in the equivalent form as follows:

$$\beta = \sqrt{\tilde{n}^2 k^2 - 4\pi^2 (\alpha^2 + \nu^2)} = \tilde{n}k - \frac{4\pi^2 (\alpha^2 + \nu^2)}{\tilde{n}k + \sqrt{\tilde{n}^2 k^2 - 4\pi^2 (\alpha^2 + \nu^2)}}$$  \hspace{1cm} (2.14)

Hence, (2.12) becomes

$$U(x, y, z) = \mathcal{F}^{-1} \left[ \mathcal{F} \{U(x, y, 0)\exp \left(-jz \frac{4\pi^2 (\alpha^2 + \nu^2)}{\tilde{n}k + \sqrt{\tilde{n}^2 k^2 - 4\pi^2 (\alpha^2 + \nu^2)}} \right) \} \exp (j\tilde{n}k) \right]$$  \hspace{1cm} (2.15)

where $F(x, y, z)$ is the complex amplitude according to the propagation part.

The operation in Eq. (2.15) can be solved much faster by using the effective refractive index method [3] to modify the problem of 2-D into 1-D structure so it can be obtained by using only a single Fourier transform.

2.2.3.2 The virtual lens effect

The effect of the nonhomogeneity of the medium can be accounted by correcting the phases of the complex amplitude $U(x, y, z)$ from the first step. Using the same notation as specified in Eq. (2.2),

$$u(x, y, z, t) = \text{Re} \{U(x, y, z)\exp (-j\omega t)\},$$

the corresponding phase factors for multiplying the prior results can be approximately obtained from the wave equation in the form

$$\left(\nabla^2 + \chi k^2\right)G(z) = 0$$  \hspace{1cm} (2.16)
where the general solution can be written as \( G(z) = \exp(+ j\chi z) \), provided that \( G \) is a function of only the variable \( z \). In the nonhomogeneous case with \( \chi = n(x, y, z)k \), though the condition of \( x, y \) independence is violated, the solution yields a remarkably accurate characteristic of the phase change for plane wave traveling nearly parallel to the z-axis.

In the first step, the contribution of the homogeneous medium, \( \exp(+ j\bar{n}k) \), has already been included in phase as in Eq. (2.15). The phase correcting factors to \( U(x, y, z) \) must then be

\[
G(z) = \exp(+ j[n(x, y, z) - \bar{n}]kz) \tag{2.17}
\]

Finally, the mathematical model for the solution of the scalar wave equation by the BPM can be expressed as \( U(x, y, z) = F(x, y, z)\exp(- j\bar{n}k)G(x, y, z) \). If only the magnitude response of \( U(x, y, z) \) is considered, \( \exp(+ j\bar{n}k) \) term can be disregarded. The wave intensity is given by

\[
|U(x, y, z)| = |F(x, y, z)G(x, y, z)| = \left| \mathcal{F}^{-1}\left\{ \mathcal{F}[F(x, y, z)] \times \exp\left( jz \frac{4\pi^2(\alpha^2 + \beta^2)}{\bar{n}k + \sqrt{\bar{n}^2k^2 - 4\pi^2(\alpha^2 + \beta^2)}} \right) \right\} \times \exp(- j[n(x, y, z) - \bar{n}]kz) \right| \tag{2.18}
\]

2.3 Simulations of Wave Propagation in a Directional Coupler by the BPM

2.3.1 One-dimensional implementation of the beam propagation method (BPM) with the Fast Fourier Transforms (FFT)

Although the general theory is described with the utilization of 2-D Fourier transform and its inverse above, the example applications of a dielectric coupler allow the program to be written in 1-D with the discretization and truncation of variables, and parameters defined as follows:

\[
x = dx \times i \quad \text{for} \quad -\frac{N}{2} \leq i < \frac{N}{2} \tag{2.19}
\]
\[ f_s = df_x \times l \quad \text{for} \quad -\frac{N}{2} \leq l < \frac{N}{2} \]  

(2.20)

where \( dx \) and \( df_x \) are the sampling intervals, corresponding to index \( i \) and \( l \) on space and frequency planes, respectively.

In terms of the discretized space and spatial frequency variables, Eq. (2.5) and Eq. (2.6) become

\[ U(i, z) = df_x \sum_{l=-\frac{N}{2}}^{\frac{N}{2}-1} A(l, z) \exp \left( j2\pi \left[ \frac{i \times l}{N} \right] \right) \]  

(2.21)

\[ A(l, z) = dx \sum_{i=-\frac{N}{2}}^{\frac{N}{2}-1} U(i, z) \exp \left( -j2\pi \left[ \frac{i \times l}{N} \right] \right) \]  

(2.22)

where \((df_x)(dx)\) must equal \( \frac{1}{N}\), according to FFT restriction [2].

Hence, Eq. (2.18) for wave propagation with a small distance from \( z_i \) to \( z_o \) can be programmed with two operations as follows:

\[ A(l, z_o) = \left[ dx \sum_{i=-\frac{N}{2}}^{\frac{N}{2}-1} U(i, z_i) \exp \left( -j2\pi \left[ \frac{i \times l}{N} \right] \right) \exp \left( j \frac{4\pi^2 l^2(z_o - z_i)}{\eta k + \sqrt{\eta^2 k^2 - 4\pi^2 l^2}} \right) \right] \]  

(2.23)

\[ U(i, z_o) = \left[ df_x \sum_{l=-\frac{N}{2}}^{\frac{N}{2}-1} A(l, z_o) \exp \left( j2\pi \left[ \frac{i \times l}{N} \right] \right) \exp(-j[\sigma(i, z_o) - \eta k(z_o - z_i)]) \right] \]  

(2.24)

where \( k = \frac{2\pi}{\lambda_0} \).
2.3.2 Directional coupler and the coupled mode equations

A directional coupler is an optical device consisting of two parallel dielectric waveguides placed nearby each other so that an optical wave launched into one guide can be coupled into the other. The structure and notation used is as shown in Fig. 2.2.

![Diagram of a directional coupler](image)

Fig. 2.2. The directional coupler structure

To describe waveforms that travel in a directional coupler, the coupled mode equations are derived from the theory of directional coupler as follows [4]:

Assume that the expression for the wavefield in the absence of the other guide is $E_i(x, z) = u_i(x) \exp(-j\beta_i z)$ for $i = 1, 2$. The coupling effect modifies the amplitudes along $z$ of these modes with no interference to the transverse spatial distributions or the propagation constants. Define the peak amplitude variation of $E_i(x, z)$ along $z$ due to such effect as $a_i(z)$ for $i = 1, 2$. Hence, the coupled mode equations as given in [4] are

\[
\frac{\partial a_1}{\partial z} = -jC_{21} \exp(j\Delta\beta z)a_2(z) \\
\frac{\partial a_2}{\partial z} = -jC_{12} \exp(-j\Delta\beta z)a_1(z)
\]

(2.25)
The coefficients $\beta_1$ and $\beta_2$ are the uncoupled propagation constants depending on the refractive index in each guide and $\Delta \beta = \beta_1 - \beta_2$. $C_{12}$ and $C_{21}$ are the coupling coefficients defined as follows:

$$
C_{12} = \frac{1}{2} \left( n_1^2 - n_2^2 \right) \frac{k_0^2}{\beta_2} \int u_2(x) \mu_1(x) \, dx
$$

$$
C_{21} = \frac{1}{2} \left( n_2^2 - n_1^2 \right) \frac{k_0^2}{\beta_1} \int u_1(x) \mu_2(x) \, dx
$$

According to [4], the general solutions can be derived as:

$$
a_1(z) = \left\{ A_1 \left[ \cos(yz) - j \frac{\Delta \beta}{2} \sin(yz) \right] - A_2 \left[ \frac{C_{21}}{jy} \sin(yz) \right] \right\} \exp \left( j \frac{\Delta \beta}{2} z \right)
$$

$$
a_2(z) = \left\{ A_1 \left[ \frac{C_{21}}{jy} \sin(yz) \right] + A_2 \left[ \cos(yz) + j \frac{\Delta \beta}{2} \sin(yz) \right] \right\} \exp \left( - j \frac{\Delta \beta}{2} z \right)
$$

where $A_i$'s are initial peak amplitude in waveguide-i, $\gamma^2 = \left( \frac{\Delta \beta}{2} \right)^2 + C^2$ and $C = \sqrt{C_{12} C_{21}}$.

### 2.3.3 TM modes in slab dielectric waveguides as input wavefields

The input wavefield launched into one of the dielectric waveguides of a directional coupler in this study are considered in Transverse Magnetic (TM) modes, with the field components $H_y, E_x, E_z$. The $H_y(x,z)$ component of wavefield with wavelength $\lambda$ and angular frequency $\omega$ satisfying the scalar wave equation can be written as

$$
H_y(x,z,t) = H_y(x) \exp\left( -j [\omega t - \beta z] \right)
$$

The magnitude profile on the x-axis can be written as [5]
\[ H_y(x) = \begin{cases} -\frac{h}{q} \exp(-qx) & \text{for } 0 < x < \infty \\ C \left[ \frac{h}{q} \cos(hx) + \sin(hx) \right] & \text{for } -d < x < 0 \\ -C \left[ \frac{h}{q} \cos(hd) + \sin(hd) \right] \exp(q(x + d)) & \text{for } -\infty < x < -d \end{cases} \] (2.29)

where \( d \) is the width of the guide, and \( C \) is a constant. The refractive index in the structure are \( n_g \) and \( n_s \) for the guided and substrate regions, respectively. The parameters \( h \) and \( q \) are defined as

\[
\begin{align*}
h &= \sqrt{n_g^2 k^2 - \beta^2} \\
q &= \sqrt{\beta^2 - n_s^2 k^2} \\
k &= \frac{\omega}{c} = \frac{2\pi}{\lambda}
\end{align*}
\]

(2.30)

The boundary conditions that \( E_y(x,z,t) \) and \( H_y(x,z,t) = -\left( \frac{j}{\omega \mu} \right) \frac{dE_x}{dx} \) are continuous at \( x = 0 \) and \( x = -d \) yield the following relationship:

\[
\tan(hd) = \frac{2h\bar{q}}{\left( h^2 - \bar{q}^2 \right)}
\] (2.31)

\[ 2.3.4 \text{ Comparison of analytical results vs. computational simulations with three typical cases} \]

To show that the BPM is sufficiently accurate for propagation simulation, wave propagation in three different directional coupler structures are considered, and compared to the analytical results obtained from the coupled mode theory. The physical property of interest is the intensity of the propagating wave. According to Eq. (2.27), the analytical results for the case that input wavefield given as \( A_2 = 0 \) can be expressed as:
|a_i(z)|^2 = |A_i|^2 \left[ \cos^2(\gamma z) - \left( \frac{\Delta \beta}{2\gamma} \right)^2 \sin^2(\gamma z) \right]

|a_2(z)|^2 = |A_i|^2 \left[ \left( \frac{|C_{21}|}{\gamma} \right)^2 \sin^2(\gamma z) \right]

(2.32)

where

\[ \Delta \beta = \beta_1 - \beta_2, \quad \gamma^2 = \left( \frac{\Delta \beta}{2} \right)^2 + C^2 \quad \text{and} \quad C = \sqrt{C_{12}C_{21}} \]

(2.33)

### 2.3.4.1 Case of synchronous waveguides

The directional coupler in this case has the same refractive index values in both guides, \( n_1 = n_2 \). Because \( \beta_1 \) and \( \beta_2 \) depend on the refractive index in the guides, this case yields \( \beta_1 = \beta_2 \) or \( \Delta \beta = 0 \). The intensities of wavefields moving in each guide in this structure is

\[ |a_i(z)|^2 = |A_i|^2 \cos^2(\gamma z) \]

\[ |a_2(z)|^2 = |A_i|^2 \left( \frac{|C_{21}|}{\gamma} \right)^2 \sin^2(\gamma z) \]

(2.34)

Using \( A_1 = 1, n_1 = n_2 = 1.1, d_1 = d_2 = s = 1 \ \mu m \), the power exchange determined from the analytical expressions is illustrated in Fig. 2.3. According to Eq. (2.34), complete power exchange can be achieved at \( z = \frac{\pi}{2\gamma} \). In the BPM simulations, power at any \( z \) in each waveguide is calculated from \[ \int |U(x, z)|^2 \, dx \]. As shown in Fig. 2.4, the result from the BPM simulation with the same parameters yields the same characteristics as in Fig. 2.3.
Fig. 2.3. Analytical results for power exchange between synchronous waveguides.

Fig. 2.4. The BPM simulation of power exchange in directional coupler with synchronous waveguides.
2.3.4.2 Case of nonsynchronous waveguides

In the nonsynchronous case, the mismatch \( n_1 \neq n_2 \) makes \( \Delta \beta = \beta_1 - \beta_2 \neq 0 \). The expression for power of propagating wave in waveguide 2 yields

\[
|a_2(z)|^2 = |A_1|^2 \left[ \left( \frac{|C_{21}|}{\gamma} \right)^2 \sin^2(\gamma z) \right]
\]  

(2.35)

The term \( \gamma = \sqrt{\left( \frac{\Delta \beta}{2} \right)^2 + C^2} \) does not allow the quantity in the bracket to be equal to one. Complete power exchange between the two guides cannot be accomplished at any \( z \) as shown in Fig. 2.5, which illustrates the system with \( n_1 = 1.12 \), \( n_1 = 1.08 \), and \( \Delta \beta = 0.1887 \mu m^{-1} \).

For the same structure, the BPM simulation illustrates the similar characteristic, with power in each waveguide determined as \( \int_{\text{guide}} |U(x,z)|^2 \, dx \), shown in Fig. 2.6. The differences in distances caused by the numerical computations for the power profile along the propagation direction are approximately 5-10\%.
Fig. 2.5. Analytical results for power exchange between nonsynchronous waveguides

Fig. 2.6. Computer simulation of power exchange between nonsynchronous waveguides
2.3.4.3 Case of a periodic grating-assisted directional coupler

When the two waveguides have different refractive indices, the complete power exchange cannot be achieved because the superposition fields in the two guides travel with different propagating constant $\beta$. The contribution from one guide can be added to the existing field in the other guide either in-phase or out-of-phase, when power flows in the same or opposite directions, respectively. While the fields are out-of-phase, the power in one guide is reduced because the coupling power has been transferred back to the other guide.

To achieve complete power exchange, the coupling coefficient has to be modulated so that it allows strong coupling while the two fields are in-phase, and reduces the coupling effect while the two fields are out-of-phase. The idea of assigning proper coupling coefficients at specific $z$ can be realized by adjusting the separation distances $s$ between the two waveguides. The closer the separation distance is, the stronger the coupling effect it yields. Consequently, the resulting structure is called "grating-assisted" directional coupler, in which one or both guides have periodic width variations along $z$ as shown in Fig. 2.7.

![Fig. 2.7. Schematic of a grating-assisted directional coupler](image-url)
The grating period \( \Lambda \) can be determined as follows [1]. If the two fields begin propagating in-phase at \( z = 0 \), they will be out-of-phase at \( z = \frac{\Lambda}{2} = \frac{\pi}{(\beta_1 - \beta_2)} \). Hence, the grating period must satisfy the condition

\[
\Lambda = \frac{2\pi}{(\beta_1 - \beta_2)}
\]

(2.36)

where \( \beta_1 \) and \( \beta_2 \) are propagation constants in waveguides 1 and 2, respectively.

According to [1], the coupling length for complete power exchange to be accomplished in grating-assisted case is

\[
L = \frac{\pi}{2\hat{\kappa}_g}
\]

(2.37)

where \( \hat{\kappa}_g \) is the coupling coefficient due to the presence of grating.

By considering only the slowly \textbf{varying} parts of the propagating fields that are significant to the characteristics of \textbf{power exchange}, the coupling coefficients for TM modes of a slab directional coupler in Fig. 2.2 can be \textbf{expressed} in terms of parameter \( \hat{\kappa}_g \) given by [1]

\[
\hat{\kappa}_g = \frac{b(n_2^2 - n_1^2)}{(e_0/\mu_0)\beta_1 n_1^2 n_2^2 \beta_e \beta_o} \left[ \frac{n_1^4 + n_2^4}{2 n_2^2 n_1^2} \beta_e \beta_o H_y \frac{\partial H_y}{\partial x} + \frac{\partial H_y}{\partial x} \right]_{x=x_2}
\]

(2.38)

where the subscript "e" and "o" indicate "even" and "odd" modes of the fields \((H_y)\) propagating in the structure. The "+" and "-" signs in front of the last \textbf{term} in the bracket are used in the forward and \textbf{backward grating-assisted} case, respectively.

In the case of grating-assisted directional coupler with the following parameters \( n_1 = 1.12, n_2 = 1.08, n_s = 1, \lambda = 1 \mu m, d_1 = d_2 = 1 \text{ ym} \) with \( 2b = 0.4 \text{ ym} \) and \( s = 0.5 \text{ ym} \), the coupling coefficient due to the \textbf{presence} of grating can be obtained from Fig. 2.8.
which, for the forward case, yields the value of $\hat{\kappa}_g = 0.01 \, \mu m^{-1}$. The analytical results give $L = \frac{n}{0.01} = 157 \, \mu m$ and $A = \frac{n}{(\beta_1 - \beta_2)} = 28.8 \, \mu m$

Fig. 2.8. Ratio of coupling coefficient to grating amplitude as a function of slab separation [1].

For the same structure, the BPM simulation results yield the important parameters approximately the same as those of analytical results. To obtain total power exchange, the grating period and the coupling length are determined as 29.0 $\mu m$ and 155 $\mu m$, respectively.

Note that the power in each waveguide is considered as the integral of the wavefield profile illustrated in Fig. 2.9. Hence, the initial power may not be equal to one in the input waveguide and be zero in the output waveguide. At a specific distance, the complete power transfer is determined when the ratio of power from the output waveguide over that from input waveguide reaches the same amount as the initial power ratio of the input over the output waveguides.
Fig. 2.9. Input power profile over the directional coupler cross-section at $z = 0$.

Fig. 2.10. The BPM simulation of power exchange in the grating-assisted waveguides with grating period of $29 \, \mu m$ and coupling length of $155 \, \mu m$.
2.4 Nonperiodic Grating-Assisted Directional Coupler

Although the periodic grating-assisted directional coupler can achieve total power exchange between nonsynchronous waveguides, further effort has been made to effectively reduce the coupling length. In the previous results, since the BPM has been shown competent for analyzing a complicated optical system, it will be used as a computational tool to characterize the power transfer inside the directional coupler system.

The basic concept of grating-assisted directional coupler with nonsynchronous waveguides is to amplify the coupling coefficient when the power is coupled from the input to the output guides by reducing the separation between the guides. On the other hand, when power is coupled back from the output guide into the input guide, the coupling coefficient is attenuated by increasing the gap between the guides.

For the sake of simplicity in the derivation, the prior grating-assisted case has been obtained analytically by using the periodic assumption. With the same parameters, the BPM also simulates the similar characteristics as shown in Fig. 2.10. Though the complete power exchange can be achieved by the periodic structure, such grating profile has been found inefficient in adjusting the coupling coefficients to match the actual characteristics of power exchange between the guides at some points. According to Fig. 2.10, the gap between the waveguides such as from $z = 40$ to $45$ $\mu$m should be large so that the coupling coefficients would be decreased because the power is coupled back from the output guide to the input guide.
2.4.1 The basic design of the nonperiodic grating-assisted directional coupler (Method I)

In case of 2-level gap, the procedure for determining the effective gap between the two waveguides at any specific distance \( z \) as illustrated in Fig. 2.11 can be described as follows:

i) Determine the input fields of the first segment from directional coupler's parameters. Then, the input power at \( z = 0 \) for both guides is considered.

ii) Use the BPM to simulate wavefields propagating in a segment of a short distance \( dz \)

iii) Calculate the power in each guide according to the derived wavefields.

iv) If the power in the output guide declined in comparison to the previous segment, the gap between the two waveguides for the next segment is increased. Contrarily, if the power in the output guide still climbs up as desired, the gap will be kept the same.

v) Repeat for the next segment from step ii)-iv) with new structure and input fields until the complete power transfer is achieved in the output guide.
Fig. 2.11. The flowchart of simulation with BPM for the nonperiodic gating-assisted directional coupler.
2.4.2 The nonperiodic grating-assisted design with implementational constraints (Method II)

Although the basic algorithm for grating-adjusting can generate a desirable result as in Fig. 2.11, the ideal design may not be realized due to the limitation in the current fabrication technology. With the practical issue in concern, another experiment is set up. For the grating of a minimum acceptable resolution, say, 1-μm, the procedure is modified so that the gap will not be adjusted at the distance less than 1 μm from the previous adjustment.

Since the infinitesimal distance \( dz \) between segment-boundaries is significant to the accuracy of the BPM simulation, the procedure cannot be altered by simply setting \( dz \) equal to 1 μm. Instead, the distance \( dz \) is kept as small as in the original procedure but the determination of grating adjustment will be withheld until a segment is at least 1-μm long.

The computer simulation shown in Fig. 2.13 also depicts the comparable result to that of the ideal-implementation case. The total power transfer length is also reduced to approximately 100 μm, which is one-third less than that of the periodic case.

2.4.3. The nonperiodic grating-assisted design with adaptive grating adjustment (Method III)

The idea of adjusting the grating level according to the direction of power transfer is similar to the gradient-descent optimization in the same sense of being a greedy algorithm. The grating level is switched as soon as the power transfer direction changes from the input guide to the output guide and vice versa. Consequently, the original technique can lead to a local minimum instead of the global minimum.

To solve this problem, an adjusting parameter \( p \) is introduced into the algorithm to avoid too sensitive adjustments. If the power in the input waveguide: \( P_i \) does not decrease appreciably as described by \( 0 < p \leq 1 \), the grating level is not changed from
wide to narrow. Similarly, if the input waveguide power does not increase appreciably as decided by $\rho$, the grating level is not changed from narrow to wide. In more detail, the original procedure is modified as follows:

(i)-(iii) The same as in the original procedure.

(iv) This step is to adjust the grating levels according to two conditions applied to power in the input guide as follows:

(a) If power at $z$ is transferring from the input guide to the output guide, the grating should be at the level of the narrow gap. This condition is applied more conservatively by testing whether $P(z) > \rho P(z - dz)$. If so, the grating level is changed to the narrow gap. Otherwise, the current grating level is maintained at the wide gap.

(b) If power at $z$ is transferring from the output guide to the input guide, the grating should be changed to the wide gap level. This condition is applied more rigorously by testing whether $\rho P(z) > P(z - dz)$. If so, the grating level is changed to a wide gap. Otherwise, the current grating level is maintained at the narrow gap.

(v) Repeat the steps (ii) through (iv) for the succeeding intervals until the complete power transfer is achieved into the output guide.

The flowchart of this modified procedure is illustrated in Fig. 2.12.
Fig. 2.12. The flowchart of the iterative method III for the design of the nonperiodic grating-assisted directional coupler with BPM.
2.5 Experimental Results

The computational results as shown in the following figures are simulated with the same parameters as in the periodic case, which are \(n_1=1.12\), \(n_2 = 1.08\), \(n_S = 1\), \(\lambda = 1 \, \mu m\), \(d_1 = d_2 = 1 \, \mu m\) and \(s = 0.5 \, \mu m\). The two levels of the adjusted grating are 0.3 \(\mu m\) for the narrow gap and 0.7 \(\mu m\) for the wide gap.

<table>
<thead>
<tr>
<th>Simulation Description</th>
<th>Coupling Length ((\mu m))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Periodic grating-assisted directional coupler</td>
<td>(\approx 155)</td>
</tr>
<tr>
<td>Method I: the basic nonperiodic grating-assisted design</td>
<td>(\approx 130)</td>
</tr>
<tr>
<td>Method II: the design with constraint of (l_{min}=1 , \mu m)</td>
<td>(\approx 100)</td>
</tr>
<tr>
<td>Method III: the adaptive design with (p = 0.995)</td>
<td>(\approx 130)</td>
</tr>
<tr>
<td>Method III: the adaptive design with (p = 0.999)</td>
<td>(\approx 100)</td>
</tr>
</tbody>
</table>

The results listed in Table 2.1 are from the simulations illustrated in Fig. 2.10, 2.13 - 2.16, respectively. The basic design or Method I, which can also be considered the same as Method III with \(p\) equals to 1, can reduce the coupling length from 155 \(\mu m\) with periodic design to 130 \(\mu m\). However, the best result is obtained when \(p\) is chosen equal to 0.999 and yields the coupling length 100 \(\mu m\) meaning the distance that complete power exchange is reduced by 23 % as compared to the coupling length of the periodic grating-assisted directional coupler. In this experimental setting, the design with
constraints or Method II also shows a satisfactory result in this setting. Though it may not always achieve the best possible outcome for the nonperiodic design, it is guaranteed to obtain complete power transfer in a shorter distance than the periodic case.

More computational simulations with parameter settings suitable for practical optical communications and networking technology can be found in [6], which is also included as Appendix I.
Fig. 2.13. The BPM simulation of power exchange in the nonperiodic grating-assisted directional coupler.

Fig. 2.14. The BPM simulation of the design using Method II with resolution of at least 1 \( \mu m \) for each segment as the implementational constraint.
Fig. 2.15. The BPM simulation of the design using Method III with adaptive parameter $p = 0.995$

Fig. 2.16. The BPM simulation of the design using Method III with adaptive parameter $p = 0.999$
CHAPTER 3

THE ANGULAR SPECTRUM AND BEAM PROPAGATION
METHODS USING SINE TRANSFORMS

3.1 Introduction

The beam propagation method (BPM) is a powerful method to determine how waves propagate inside a medium with arbitrary refractive index profile, such as a grating-assisted directional coupler, etc. However, the conventional BPM uses the complex Fourier transform (CFT), which is not well suited when zero boundary conditions are required. In this method with successive steps, the final consequence can be too much approximation with a finite number of expansion terms. When zero boundary conditions exist, it is better to use a transform with basis functions, which automatically generate zero values at given boundaries. Because the sine transform can generate zero values at specific boundaries with proper indices, the beam propagation method using sine transform (BPMS) is more suitable to handle such problems. For comparison purposes, the conventional BPM will be referred to as the beam propagation method by complex Fourier transform (BPMCF) in this study.

3.2 The Angular Spectrum and Beam Propagation Methods by Sine Transforms

3.2.1 Zero boundary condition at the origin

Let a scalar waveform propagating in the z-direction be expressed in the form

\[ u(x,z,t) = U(x,z) \cos(\omega t + \Theta(x,z)) \quad \text{for} \quad -\infty \leq x \leq \infty \quad (3.1) \]

where \( U(x,z) \) is the amplitude, and \( \Theta(x,z) \) is the phase at position \((x,z)\).
In complex notation, this can be written as

\[ u(x,z,t) = \text{Re}\{U(x,z)\exp(-j\omega t)\} \quad \text{for} \quad -\infty \leq x \leq \infty \quad (3.2) \]

where \( U(x,z) \) is the complex amplitude equal to \( U(x,z)\exp(-j\Theta(x,z)) \).

The 1-D Fourier transform \( A(\alpha,z) \) of the complex amplitude \( U(x,z) \) is defined as

\[ A(\alpha,z) = \mathcal{F}\{U(x,z)\} = \int_{-\infty}^{\infty} U(x,z)\exp(-j2\pi\alpha x)dx \quad \text{for} \quad -\infty \leq \alpha \leq \infty \quad (3.3) \]

The zero boundary condition at the origin is achieved by imposing odd-symmetry property that satisfies the following condition:

\[ U(x,z) = -U(-x,z) \quad \text{for} \quad 0 \leq x \leq \infty . \quad (3.4) \]

Then, Eq. (3.3) can be rewritten as

\[ A(\alpha,z) = \mathcal{F}\{U(x,z)\} = \int_{0}^{\infty} U(x,z)\exp(-j2\pi\alpha x)dx + \int_{-\infty}^{0} U(x,z)\exp(-j2\pi\alpha x)dx \]

\[ = \int_{0}^{\infty} U(x,z)[\exp(-j2\pi\alpha x) - \exp(j2\pi\alpha x)]dx \]

\[ = -2j\int_{0}^{\infty} U(x,z)\sin(2\pi\alpha x)dx \quad (3.5) \]

The integral above is known as the continuous-space sine transform.

The complex amplitude can be written as

\[ U(x,z) = U_r(x,z) + jU_i(x,z) \quad (3.6) \]

where \( U_r(x,z) \) and \( U_i(x,z) \) represent real and imaginary parts of the complex amplitude \( U(x,z) \), respectively. The odd-symmetry property is also preserved in both parts as follows:

\[ U_r(-x,z) = -U_r(x,z) \]
\[ U_i(-x,z) = -U_i(x,z) \quad (3.7) \]

According to Eqs. (3.5)-(3.7), the angular spectrum \( A(\alpha,z) \) by the sine transform is defined as
\[ A(\alpha,z) = A_R(\alpha,z) + jA_I(\alpha,z) \]
\[ = -2j \int_0^\infty [U_R(x,z) + jU_I(x,z)] \sin(2\pi \alpha x) dx \]
\[ = -j2 \int_0^\infty U_R(x,z) \sin(2\pi \alpha x) dx + 2 \int_0^\infty U_I(x,z) \sin(2\pi \alpha x) dx \] (3.8)

Hence,
\[ A_R(\alpha,z) = +2 \mathcal{S} \{U_I(x,z)\} = +2 \int_0^\infty U_I(x,z) \sin(2\pi \alpha x) dx \]
\[ A_I(\alpha,z) = -2 \mathcal{S} \{U_R(x,z)\} = -2 \int_0^\infty U_R(x,z) \sin(2\pi \alpha x) dx \] (3.9)

Likewise, \( A(\alpha,z) \), \( A_R(\alpha,z) \), and \( A_I(\alpha,z) \) have odd-symmetry property at \( \alpha = 0 \).

According to Eq. (3.3), the corresponding 1-D inverse Fourier transform can be written as
\[
U(x,z) = \mathcal{F}^{-1} \{A(\alpha,z)\} = \int \tilde{A}(\alpha,z) \exp(j2\pi(\alpha x)) d\alpha
\]
\[
= \int \tilde{A}(\alpha,z) \exp(j2\pi(\alpha x)) d\alpha
\]
\[
= 2j \int \tilde{A}(\alpha,z) \sin(2\pi \alpha x) d\alpha
\] (3.10)

Likewise, the 1-D inverse sine transform of an odd-symmetry angular spectrum corresponding to Eqs. (3.8) and (3.9) can be derived as follows:
\[
U(x,z) = U_R(x,z) + jU_I(x,z)
\]
\[
= 2j \int [A_R(\alpha,z) + jA_I(\alpha,z)] \sin(2\pi \alpha x) d\alpha
\]
\[
= j2 \int A_R(\alpha,z) \sin(2\pi \alpha x) d\alpha - 2 \int A_I(\alpha,z) \sin(2\pi \alpha x) d\alpha
\] (3.11)

Therefore,
\[ U_R(x,z) = -2\mathcal{S}^{-1}\{A, (\alpha, z)\} = -2\int_0^\infty A(\alpha, z)\sin(2\pi \alpha) d\alpha \]
\[ U_I(x,z) = +2\mathcal{S}^{-1}\{A, (\alpha, z)\} = +2\int_0^\infty A(\alpha, z)\sin(2\pi \alpha) d\alpha \] (3.12)

Note that this inverse sine transform also yields \( U(x,z), U_R(x,z), \) and \( U_I(x,z) \) with odd-symmetry.

### 3.2.2 Zero boundary conditions at \( x = a \) and \( x = b \)

In the discussion of Section 3.2.1, it appears that the zero condition is achieved only on one boundary considered as the origin of the x-axis. In this section, the more general cases of zero boundaries are to be investigated.

Consider the case of a wavefield defined in the domain from \( x = a \) to \( b \), where \( b > a \), and the zero boundary conditions occur at both ends. For this problem, it is more convenient to apply the generalized Fourier transform \([22]\), yielding the following spectrum:

\[ A(\alpha, z) = \mathcal{F}\{U(x,z)\} = \int_{-\infty}^{\infty} U(x, z)\exp(-j2\pi \alpha(x-a)) dx \] for \(-\infty \leq \alpha \leq \infty\) (3.13)

The zero boundary condition at \( x = b \) can be achieved by assuming the wavefield is periodic with period \( L = b - a \), Eq. (3.13) then becomes

\[ A(n, z) = \frac{1}{L} \int_a^{a+L} U(x, z)\exp\left(-\frac{j2\pi n(x-a)}{L}\right) dx, \] (3.14)

which will be referred to as the complex Fourier series (CFS) coefficients of \( U(x,z) \). The wavefield itself can be expanded as

\[ U(x, z) = \sum_{n=-\infty}^{\infty} A(n,z)\exp\left(\frac{j2\pi n(x-a)}{L}\right) \] for \( a \leq x \leq a + L \) (3.15)
The spatial frequencies of the angular spectrums are now discrete with a resolution of $\frac{1}{L}$

or $\alpha = \frac{n}{L}$.

By imposing the property of odd-symmetry that satisfies the following condition:

$$U(x,z) = -U(2a-x,z) \quad \text{for } a \leq x \leq a + L$$  \hspace{1cm} (3.16)

The wavefield is now defined in the range of $a - L \leq x \leq a + L$ with period $2L$,

Eq. (3.14) can be rewritten as

$$\mathcal{A}(n,z) = \frac{1}{2L} \int_{a-L}^{a+L} U(x,z) \exp\left(-j \frac{2\pi n(x-a)}{2L}\right) dx$$

$$= \frac{1}{2L} \int_{a-L}^{a+L} U(x,z) \left[ \exp\left(-j \frac{2\pi n(x-a)}{2L}\right) \right] - \exp\left(\frac{j 2\pi n(x-a)}{2L}\right)$$

$$= -\frac{j}{L} \int_{a-L}^{a+L} U(x,z) \sin\left(\frac{\pi n(x-a)}{L}\right) dx$$

for $-\infty \leq n \leq \infty$ \hspace{1cm} (3.17)

The corresponding generalized sine series can be expressed as

$$U(x,z) = j \sum_{n=-\infty}^{\infty} \mathcal{A}(n,z) \sin\left(\frac{\pi n(x-a)}{L}\right) \quad \text{for } a \leq x \leq a + L$$  \hspace{1cm} (3.18)

Similar to the previous derivation, we obtain

$$\mathcal{A}_h(n,z) = +\frac{1}{L} \int_{a-L}^{a+L} U_j(x,z) \sin\left(\frac{\pi n(x-a)}{L}\right) dx$$

$$\mathcal{A}_l(n,z) = -\frac{1}{L} \int_{a-L}^{a+L} U_l(x,z) \sin\left(\frac{\pi n(x-a)}{L}\right) dx$$  \hspace{1cm} (3.19)

And

$$U_h(x,z) = -\sum_{n=-\infty}^{\infty} A_h(n,z) \sin\left(\frac{\pi n(x-a)}{L}\right)$$

$$U_l(x,z) = +\sum_{n=-\infty}^{\infty} A_l(n,z) \sin\left(\frac{\pi n(x-a)}{L}\right)$$  \hspace{1cm} (3.20)
Note that by imposing the restriction of odd-symmetry to the wavefield, the period in consideration is doubled to $2L$, which causes the spatial frequency domain to gain the better resolution of $\frac{1}{2L}$. The odd-symmetry property as well as the zero boundary conditions are now valid both at $x = a$ and $x = b$.

### 3.2.3 The angular spectrum method by the sine transform (ASMS)

By applying the concept of the angular spectrum in the spatial frequency domain as discussed in chapter 2, the wavefield with wavelength $\lambda$ propagating from $z_0$ to $z_0 + \Delta z$ can be characterized in space domain with proper transformation (7) as [8]

$$U(x, z = z_0 + \Delta z) = \mathcal{F}^{-1}\{A(\alpha, z_0)\exp(j\mu \Delta z)\} = \mathcal{F}^{-1}\{\mathcal{F}\{U(x, z_0)\}\exp(j\mu \Delta z)\}$$

(3.21)

where the propagation constant $\mu$ is a function of the spatial frequency $\alpha$, refractive index $\overline{n}$, and, the wave number $k = \frac{2\pi}{\lambda}$ as follows:

$$\mu = \sqrt{\overline{n}^2k^2 - 4\pi^2\alpha^2} = \overline{n}k - \frac{\alpha^2}{\overline{n}k + \sqrt{\overline{n}^2k^2 - 4\pi^2\alpha^2}}$$

(3.22)

with the restriction that $4\pi^2(\alpha^2 + \nu^2) \leq \overline{n}^2k^2$ to exclude the effect of evanescent waves.

The wavefield is assumed to have odd-symmetry at $x = a$ and $x = b$ as discussed in the previous section. Substituting the complex amplitude and the exponential terms in Eq. (3.21) with their corresponding real and imaginary parts, the equivalent expressions for the ASM by the sine basis function can be derived as follows:

$$U(x, z_0 + \Delta z) = U_k(x, z_0 + \Delta z) + jU_i(x, z_0 + \Delta z)$$

$$= -2\mathcal{F}^{-1}\{A, (\alpha, z_0 + \Delta z)\} + j2\mathcal{F}^{-1}\{A, (\alpha, z_0 + \Delta z)\}$$

(3.23)

where

$$A_k(\alpha, z_0 + \Delta z) = A_k(a, z_0)\cos(\mu \Delta z) - A_i(a, z_0)\sin(\mu \Delta z)$$

$$A_i(\alpha, z_0 + \Delta z) = A_i(a, z_0)\cos(\mu \Delta z) + A_i(a, z_0)\sin(\mu \Delta z)$$

(3.24)

Note that $A, (\alpha, z_0 + \Delta z)$ and $A, (\alpha, z_0 + \Delta z)$ still have odd-symmetry.
3.2.4 The beam propagation method by the sine transform (BPMS)

The BPMS is obtained when odd-symmetry is imposed with zero boundary conditions. Similar to the BPMCF as discussed in chapter 2, the BPMS first determines the propagation effect along an infinitesimal distance in a medium of constant refractive index \( \bar{n} \) by using the ASMS. Due to the refractive index profile \( n(x,z) \) of the nonhomogeneous medium, the wavefield is then multiplied with the phase correcting factors,

\[
\Phi = \exp(j\Delta z[n(x,z) \hat{k} - \bar{n} \hat{k}])
\]  

(3.25)

The flowchart of the BPMS is illustrated in Fig. 3.1.

For applications in 3-D requiring 2-D transform, the effective index method [1] can be used to reduce the dimension of calculation to 1-D before either the BPMCF or BPMS can be applied. Equivalently, the y-dimension can be included by generalizing the results above.
Input $U_r(x, z_0) + jU_i(x, z_0)$
for $z_0 = 0$ and $a \leq x \leq a + L$

Determine $A_r(a, z_0) + jA_i(a, z_0)$ from Eq.(3.19)

Apply ASMS as in Eqs. (3.22) - (3.24) with effective index $\bar{n}$

Multiply Eq. (3.25) by the lens effect
$\Phi = \exp(j\Delta z[n(x, z)k - \bar{n}k])$

The final distance $z$ has been reached

No

STOP

Yes

Fig. 3.1. The flowchart illustrating the BPMS.
3.3. Numerical Computations

The input wavefields for the two methods used in the computer simulations are illustrated in Fig. 3.2. The domains of the input fields are set at arbitrary positions and both tails decay to zeroes at the boundaries. The simulations are done in 1-D with the discretization and truncation of variables, and parameters defined below.

3.3.1 Computational programming for the BPMCF

The discretizations of the variables for the numerical computation of the complex Fourier-transform for the BPMCF are set as follows:

\[ x = \Delta x \times i \quad \text{for} \quad 0 \leq i < N \quad (3.26) \]
\[ \alpha = \Delta \alpha \times n \quad \text{for} \quad 0 \leq n < N \quad (3.27) \]

where \( A_x = \frac{L}{N} \) and \( A \alpha \) are the sampling intervals with corresponding index \( i \) and \( n \) on space and frequency planes, respectively. Note that \( (\Delta \alpha \times A_x) \) must be equal to \( \frac{1}{N} \), according to FFT restriction [2].

In terms of the discretized space and spatial frequency variables, Eq. (3.14) and Eq. (3.15) become

\[ A(n, z) = \sum_{i=0}^{N-1} U(i, z) \exp \left( -j2\pi \left[ \frac{i \times n}{N} \right] \right) \quad (3.28) \]
\[ U(i, z) = \frac{1}{N} \sum_{n=0}^{N-1} A(n, z) \exp \left( j2\pi \left[ \frac{i \times n}{N} \right] \right) \quad (3.29) \]

These equations are essentially the same as the discrete Fourier transform (DFT) and inverse DFT, respectively.

Because of the numerical calculation, the wavefield is represented by a discrete function defined within \((a, a+L)\). This discretization on the space domain causes the angular spectrum on the spatial domain to be periodic as well. Hence, the angular
spectrum as illustrated in Fig. 3.3 (b) and (c) are equivalent for the same periodic wavefield in Fig. 3.3 (a).

Fig. 3.2. Input wavefields used in the experimental simulations for L = 10μm and -2.5 ≤ x < 7.5μm (a) U(x,0) = \sin \left( \frac{\pi x}{L} \right) (b) U(x,0) = \sin \left( \frac{\pi x}{L} \right) + \sin \left( \frac{2\pi x}{L} \right) + \sin \left( \frac{3\pi x}{L} \right).
Fig. 3.3. The periodic angular spectrum as a result of discretization space domain of

(a) \( U(i) = 0.4256 \times \left[ \sin\left( \frac{2\pi(20)}{200} i \right) + \sin\left( \frac{2\pi(30)}{200} i \right) + \sin\left( \frac{2\pi(60)}{200} i \right) \right] \)

(b) The angular spectrum by \texttt{fft{.}}

(c) The angular spectrum by \texttt{fftshift{fft{.}}}. 
To exclude the effect of evanescent waves, the frequencies in the computation by FFT must be in the range such that \(4\pi^2\left((\Delta\alpha \times n)^2 + v^2\right) \leq \pi^2k^2\). The numerical solutions require shifting of frequencies so that \(N\) terms of spatial frequencies expand on both negative and positive as \(-\frac{N}{2} \leq n < \frac{N}{2}\) instead. Hence, (3.28) becomes

\[
A(n, z) = \sum_{i=-\frac{N}{2}}^{\frac{N}{2}} U(i, z) \exp\left(-j2\pi \left[\frac{i \times n}{N}\right]\right)
\]

(3.30)

Next, the propagation part can be obtained by multiplying the angular spectrum with phase shifts by the corresponding propagation constants as in Eq. (3.22). Finally, the wavefield can be achieved by inverse transforming the angular spectrum at \(z = z + \Delta z\) using Eq. (3.29) and including the virtual lens effect as in Eq. (3.25).

### 3.3.2 Computational programming for the BPMS

The BPMS always has zero amplitudes at both boundaries so its routine requires only initial \(N-1\) input points, while the BPMCF uses \(N\).

The sine transform in the BPMS has the following notations:

\[
x = \Delta x \times i \quad \text{for} \quad 0 < i < N
\]

(3.31)

\[
\alpha = \Delta \alpha \times n \quad \text{for} \quad 0 < n < N
\]

(3.32)

where \(Ax = \frac{L}{N}\) and \(A\alpha\) are the sampling intervals with corresponding index \(i\) and \(n\) on space and frequency planes, respectively. According to the odd-symmetry restriction, the spatial frequency resolution \(A\alpha = \frac{1}{2L}\). Therefore, \((\Delta\alpha \times Ax)\) must be equal to \(\frac{1}{2N}\).

In terms of the discretized space and spatial frequency variables, Eqs. (3.19) with the domain shifting and (3.20) become
Equations (3.33) and (3.34) are essentially the same as the discrete sine transform (DST) and inverse-DST, respectively. Note that $N-1$ points are used since $i$ or $n$ equal $0$ or $N$ give zero output values. Additionally, the domain shift in the spatial frequency domain need not be applied in the transformation by sine basis function because both positive and negative frequencies are already included in one $\sin(.)$ term. The output wavefields by BPMS can be obtained by following the step as discussed in $i)$ with proper transformation as defined in Eqs. (3.33) and (3.34), and the corresponding phase factors as in Eqs. (3.22) and (3.25).

3.4 Experimental Results

3.4.1 Simulations of single-mode wavefield propagating in a shielded waveguide

In this study, we consider wave propagation in a homogeneous waveguide in which refractive index is constant along x- and z-axis. A single-mode wavefield is assumed to be launched into the shielded waveguide with zero boundary conditions as shown in Fig. 3.3. The computer simulations are demonstrated in Figs. 3.4 - 3.9. The comparison of the results from the BPMCF and the BPMS are summarized in Table 3.1, where the error is determined as

$$
err = \frac{\sum_{x \in T} [U_{\text{analytical}}(x,z) - U_{\text{BPM}}(x,z)]^2}{N}
$$

(3.35)
Table 3.1
The comparison of the computer simulations from the BPMCF and the BPMS

<table>
<thead>
<tr>
<th>Simulation Description</th>
<th>No. of sampling points</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BPMCF</td>
</tr>
<tr>
<td>Input wavefield as shown in Fig. 3.2 (a)</td>
<td>N=128</td>
<td>1.44698</td>
</tr>
<tr>
<td></td>
<td>N=256</td>
<td>1.44702</td>
</tr>
<tr>
<td></td>
<td>N=512</td>
<td>1.44703</td>
</tr>
<tr>
<td>Input wavefield as shown in Fig. 3.2 (b)</td>
<td>N=128</td>
<td>4.39190</td>
</tr>
<tr>
<td></td>
<td>N=256</td>
<td>4.39239</td>
</tr>
<tr>
<td></td>
<td>N=512</td>
<td>4.39252</td>
</tr>
</tbody>
</table>

The analytical solutions for wavefields of period $L = 10 \mu m$ propagating to distance $z = 10 \mu m$ are as follows:

$$U(x,z) = \sin \left( \frac{\pi x}{L} \right) \times \exp \left( j2\pi \sqrt{\left( \frac{n}{\lambda} \right)^2 - \left( \frac{1/2}{L} \right)^2} \right)$$

(3.36)

$$U(x,z) = \sum_{i=1}^{3} \sin \left( \frac{\pi x i}{L} \right) \times \exp \left( j2\pi \sqrt{\left( \frac{n}{\lambda} \right)^2 - \left( \frac{1/2 \times i}{L} \right)^2} \right)$$

(3.37)

The numerical simulations BPMS has shown to produce more accurate results than those from the BPMCF with the same number of sampling points because the consideration by the conventional BPMCF cannot sustain the zero boundary conditions along the propagation direction. The BPMS also yields better results as the numbers of sampling points increase.
Fig. 3.4. The output amplitudes of wavefield as in Fig. 3.2 (a) propagating with distance $z = 10 \mu m$ by (a) BPMCF (b) BPMS
Fig. 3.5. The output phases of wavefield as in Fig. 3.2 (a) propagating with distance $z = 10 \mu m$ by (a) BPMCF (b) BPMS
Fig. 3.6. The amplitude profiles of wavefield as in Fig. 3.2 (a) propagating along z-direction by (a) BPMCF (b) BPMS
Fig. 3.7. The output amplitudes of wavefield as in Fig. 3.2 (b) propagating with distance $z = 10 \mu m$ by (a) BPMCF (b) BPMS
Fig. 3.8. The output phases of wavefield as in Fig. 3.2 (b) propagating with distance $z=10\mu m$ by (a) BPMCF (b) BPMS
Fig. 3.9. The amplitude profiles of wavefield as in Fig. 3.2 (b) propagating along $z$-direction by (a) BPMCF (b) BPMS
3.4.2 Simulations of single-mode wavefield propagating in a shielded waveguide with tapering contour

The following simulation is an example of waveguides with arbitrary shapes that also require zero boundary conditions as shown in Fig. 3.10 (a). The input wavefield is similar to the profile illustrated in Fig. 3.2 (a) and can be expressed as

\[ U(x,0) = \sin \left( \frac{\pi x}{L} \right) \quad \text{for} \quad L = 10 \mu m \text{ and } -5 \leq x < 5 \mu m \] (3.38)

The operation begins by using BPMS to obtain the wavefield profile within the constraint zero boundaries of \(-5 \leq x < 5 \mu m\) similar to the procedure previously described. Then, the period of consideration is increased or decreased by one sampling point on each side with the distance of propagation \(\Delta z\) corresponding to the slope of the taper contour. The simulation results are as illustrated in Figs 3.10 (b) and 3.11.

3.5 Conclusion

The BPMS has been found effective and helpful for simulating wave propagation in a waveguide of arbitrary shape that demands zero boundary conditions, while it can be very difficult, if not impossible, to consider such problem by the conventional BPMCF.
Fig. 3.10. The simulation by BPMS in an optical system with arbitrary shapes
(a) the geometry of a shielded waveguide with taper contour
(b) The output amplitude along z-direction
Fig. 3.11. (a) The amplitude profiles on the output plane. (b) The amplitude profiles on the output plane in 3-D.
CHAPTER 4

THE RANKED PHASED-ARRAY METHOD

4.1 Introduction

Dense Wavelength Division Multiplexing (DWDM) has obtained prevalent interest in the field of optical communications because of its capability to accommodate very large number of channels in the low transmission-loss regions of optical fiber bandwidth. Additionally, such systems can simultaneously transmit diversified types of signals, such as synchronous, asynchronous, and analog, via the same routes of optical fibers.

To effectively realize the idea of the wavelength-selective DWDM systems, efforts have been made to achieve essential optical elements that can combine/distinguish different wavelengths into/from multi-wavelength signals, which are called optical multiplexers/demultiplexers, respectively. Such elements can be divided into two categories: optical filters, and, angularly dispersive devices [9]. Examples of optical filters are interference filters, Fabry-Perot filters, tuned semiconductor amplifiers, heterodyne receivers, etc. phased-array (PHASAR) based devices [10]-[15] are the popular implementation of the angularly dispersive type, which relies on wavelength-dependent angles through media, such as gratings, to separate one channel from the others.

The ranked phased-array method is a new design technique for the device used in dense wavelength division multiplexing (DWDM). It involves ranking the variances of phases of all wavelengths in order to pick the better locations for implementing a number of waveguides that can accomplish separation of wavelengths as discussed in this chapter, and separation of subbands of wavelengths that will be discussed in chapter 5.
4.2 Principle Of The Ranked Phased-Array Method

Based on the algorithm developed for constructing point images in digital holography [16]-[18], the idea of focusing monochromatic optical fields from multiple point-sources on the input plane onto one specific position on the output plane can be realized if the output fields according to all input sources are in-phase at the focal point.

This approach works as well with nonmonochromatic or independent multiwavelength optical sources. Moreover, wavefields with different wavelengths propagate through the same media with different velocities. It is possible to manipulate phases for each wavelength on the input plane such that wavefields from different waveguides happen in-phase at a particular output point only at one wavelength, while the other wavelength components are out-of-phase there. This method is useful in separating different wavelengths to be focused on different points. It will be shown later that each wavelength and its corresponding focal point can be set arbitrarily.

4.2.1 An algorithm for construction of point images in digital holography

Let an optical field at \((x, z)\) be represented by \(U(x, z)\) with subscript "i" and “o” specify the coordinates on the input and output plane as shown in Fig. 4.11, respectively.

With number of input points as \(N_i\), the output field can be expressed as [16]

\[
U(x_o, z_o) = \sum_{i=1}^{N_i} U(x_i, z_i) G(x_i, z_i; x_o, z_o; \lambda) \Delta x_i
\]  

(4.1)

where \(G(x_i, z_i; x_o, z_o; \lambda)\) defines the transformation function for the optical system. For propagation in free space, the Huygens-Fresnel approximation can be used to describe the mechanism as follows [8]:

where $\lambda$ is the wavelength of propagating field in medium with index of refraction $n$. $k = \frac{2\pi n}{\lambda}$ and $r_{oi} = \sqrt{(x_o - x_i)^2 + (z_o - z_i)^2}$. $\delta$ is the angle between the vector $r_{oi}$ and the z-axis. Because $(z_o - z_i) \gg (x_o - x_i)$, $r_{oi} \equiv (z_o - z_i)$ and $\delta \equiv 1$ are constant for all coordinates $(x_i, z_i; x_o, z_o)$.

When launching into the system with uniform-amplitude input field $U \exp(\varphi_i)$, Equation (4.1) can be rewritten as

$$U(x_o, z_o) = \frac{U \Delta x_i}{j \lambda r_{oi}} \left( \sum_{i=1}^{N} \exp(\varphi_i) \exp(jkr_{oi}) \right)$$ (4.3)
According to the principle of superposition, the amplitude of the output field at $(x_o, z_o)$ will be maximized if the output phases generating from all input fields are in-phase at the specified point. In other words, the following condition must be satisfied:

$$k r_{oi}(x_i, z_i; x_o, z_o) + \varphi_i(x_i, z_i) = 2\pi M + \varphi_o(x_o, z_o)$$

(4.4)

where $M$ is an integer. $\varphi_i(x_i, z_i)$ is the initial phase at input position $(x_i, z_i)$, and $\varphi_o(x_o, z_o)$ is a chosen constant output phase at the output coordinate $(x_o, z_o)$.

Hence, there are two possible means to accomplish the intensity focusing at an arbitrary point $(x_o, z_o)$. One is to shift the positions $x_i$ of the input sources, which will lead to adjusting $r_{oi}$ to satisfy Equation (4.4). Alternatively, if $x_i$ or $r_{oi}$ are fixed, the other variable that can be altered is $\varphi_i(x_i, z_i)$. Such phase compensation can be achieved by using phase mask or grating.

4.2.2 Application to a phased-array for optical communications and networking

The same principle from digital holography can be applied in the design of a phased-array. However, the signals or wavefields in optical communications and networking are transmitted through waveguides as shown in Fig. 4.2, instead of being generated from point sources or plane waves as in digital holography. A different type of wavefield has to be considered in the derivation of phase modulation conditions. The exact solution for modal wavefields in such structures can be found from the same approach as described in Chapter 2. However, it is sufficiently accurate to approximate the modal solution from a channel waveguide as a gaussian beam [12]. In a homogeneous medium, a gaussian beam propagating with distance $z$, $U(x, z)$, can be related to the initial gaussian beam at $z = z_i$ as follows [13]:

$$U(x_o, z_o) = U(x_i, z_i) \frac{\omega_0}{\omega_z} \exp\left\{-\frac{r_{oi}^2}{\omega_z^2}\right\} \exp\left\{-j \left[kz - \eta_z + \frac{kr_{oi}^2}{2R_z}\right]\right\}$$

(4.5)
where \( r_{n} = \sqrt{(x_{o} - x_{i})^{2} + (z_{o} - z_{i})^{2}} \) and \( z = z_{o} - z_{i} \). 
\( \omega_{0} = \omega_{n}\sqrt{\frac{2}{\pi}} \) is the width of the equivalent gaussian far field. Details about the waveguide mode effective width and its relationship with \( \omega_{0} \) can be found in Appendix A in [12]. The other parameters can be determined from \( \omega_{0} \) as follows:

\[
\begin{align*}
    z_{o} &= \frac{\pi \omega_{0} n_{s}}{\lambda} \quad (4.6) \\
    \omega_{z} &= \omega_{0}\sqrt{1 + \frac{z^{2}}{z_{o}^{2}}} \quad (4.7) \\
    R_{z} &= z\left(1 + \frac{z_{o}^{2}}{z^{2}}\right) \quad (4.8) \\
    \eta_{z} &= \tan^{-1}\left(\frac{z}{z_{o}}\right) \quad (4.9)
\end{align*}
\]

Regarding the means of phase modulation, this study emphasizes generation of input phases by adjusting the input waveguide lengths rather than shifting locations of the input guides to satisfy the conditions in Equation (4.4). Yet, if a simulation begins with a very fine resolution \( \Delta x_{i} \), and selectively implements waveguides only at some chosen locations, the outcome will appear in the same manners as shifting waveguide positions in the coarser resolution.

When the input optical signals are not monochromatic, each wavelength, say \( \lambda_{j} \), can be separated from the others by focusing it to a different location \( x_{o-j} \). According to Eq. (4.5), the amount of phase shift for a gaussian beam with wavelength \( \lambda_{j} \) after propagating with distance \( z \) is

\[
    k_{j}z - \eta_{z} + \frac{k_{j}l^{2}}{2R_{z}}
\]

where \( k_{j} = \frac{2\pi n_{s}}{\lambda_{j}} \). Hence, the new phase condition is given by
By adjusting the length of each input waveguide, the corresponding input phase \( \phi_i(x_i, z_i) \) can be modified to satisfy the required conditions. Although the condition in Eq. (4.11) shows that different \( \lambda_j \)'s demand different sets of initial phases, there can only be one common set of adjusted waveguide lengths \( \hat{\delta} \) to work for all different wavelengths in one single structure. Therefore, the actual implementation must optimally represent the characteristics of all wavelengths.

To achieve the optimal \( \hat{\delta}_i \), the relationship between the phase \( \phi_i(x_i, z_i; \lambda_j) \) and the corresponding adjusted waveguide length \( \delta_i(\lambda_j) \) is established as

\[
\phi_i(x_i, z_i; \lambda_j) = \frac{2\pi n_g}{\lambda_j} \delta_i(\lambda_j)
\]

(4.13)

where \( n_g \) is the refractive index of input waveguides. The index "i" denotes the i-th position of the input waveguides, and, index "j" refers to the j-th input wavelength.
Fig. 4.2. Schematic of an optical system in communications and networking.
Note that if the optimal quantities are determined directly in terms of mean of adjusted length $\bar{\delta}_i$, the actual effect of phase adjustment will be weighted by $\hat{\lambda}/\lambda_j$, $\hat{\lambda}$ being the representing wavelength yielding the common $\bar{\delta}_i$ by Eq. (4.13). Therefore, the optimal set must be considered firstly as mean of the adjusted phases $\bar{\theta}_i(x_i, z_i)$ and later translated into a common adjusted length $\delta_i$ by Eq. (4.12) with optimal wavelength $\hat{\lambda}$, such as mean of all the input wavelengths, as follows:

$$\delta_i = \frac{\hat{\lambda}}{2\pi n} \bar{\theta}_i(x_i, z_i),$$  \hspace{1cm} (4.14)

### 4.2.3 The ranked phased-array method

The concept of finding the best representatives for all wavelengths does not work well in practice. The experimental simulations confirm that the phase modulation in this fashion with initially chosen constant waveguide positions does not yield satisfactory results.

An algorithm is proposed to select only a few good $\bar{\delta}_i$ that help achieve best possible characteristics for the whole structure. In the ranked phased-array method (RPAM) proposed below, these difficulties are overcome by selecting waveguide positions by ranking waveguides and choosing a number of them out of a pool of possible waveguides for best performance.

#### 4.2.3.1 The procedure of the RPAM

Before the RPAM can be executed, the wavelengths $\lambda_j$ and their corresponding output focal points $(x_o, z_o)$ need to be specified. To simplify the algorithm, the phase output $\bar{\theta}_o(x_o, z_o; \lambda_j)$ for each $\lambda_j$ can be presumed to be randomly selected, or assigned as zeros. Regarding the flowchart illustrated in Fig. 4.3, the procedure for the RPAM can be described as follows:
i) For each specific wavelength, determine the set of input phases by using Eq. (4.12).

ii) For each input waveguide, determine the mean of the input phase $\bar{\phi}_i$ and its variance $\sigma_i^2 = \frac{1}{N} \sum_{j=1}^{N} \left| \phi_{i,j} - \bar{\phi}_i \right|^2$.

iii) Ranking the input phase variance $\sigma_i^2$ from the smallest to the largest, select only the first $N_{\text{pick}}$ positions with the least $\sigma_i^2$ as the set of locations that can produce optimal input phases for all wavelengths used.

iv) Using Eq. (4.14), translate the mean compensated phase $\bar{\phi}_i$ into the adjusted length $\delta_i$ for the waveguide at each selected location.

With the locations and the corresponding lengths of the chosen waveguides known, the design gives a device called ranked phased-array (RPA) that consists of an array of adjusted waveguides on the input plane, and receiving waveguides at the specified coordinates on the output plane as illustrated in Fig. 4.2.

The computer simulations using gaussian beams as input fields through the designed RPAs exhibit that the effectiveness of wavelength separation is independent of focal locations and sequences of wavelengths, provided that the differences of the adjacent wavelengths are considered large compared to structure dimensions. Nonetheless, the intensities at the focal points of various wavelengths sometimes vary from the others considerably. Consequently, the iterative-RPAM is introduced to help normalize the peak intensities, as discussed in the next section.
Nonmonochromatic wavefields with number of wavelengths $= N_j$

Given arbitrary $\phi_o(x_o, y_o, z_o)$ for each $\lambda$

Use Eq. (4.12) to find phase compensation for each $\lambda_j$

$$\phi_{i,j} = \varphi_i(x_i, z_i; \lambda_j)$$
$$= \text{remainder} \left\{ \frac{2 \pi r}{\lambda_j} \left( z + \frac{r_{el}^2}{2R_z} \right) + \eta_z + \phi_o(x_{o,j}, z_o) \right\}$$

Determine: mean $\bar{\varphi}_i = \frac{1}{N_j} \sum_{j=1}^{N_j} \phi_{i,j}$, and variances $\sigma_i^2 = \frac{1}{N_j} \sum_{j=1}^{N_j} |\phi_{i,j} - \bar{\varphi}_i|^2$

Find best $N_{\text{pick}}$ of positions $x_i$ and their corresponding compensated phase $\bar{\varphi}_i$ by ranking the variance $\sigma_i^2$ from smallest to largest.

Compensated length for the $i$-th waveguide $\delta_i \propto \lambda \bar{\varphi}_i$

Fig. 4.3. The flowchart of the ranked phased-array method (RPAM).
4.2.3.2 The iterative ranked phased-array method

At the beginning of the RPAM, the output phases are randomly selected or assigned as zeros. In the iterative RPAM, the output phases are adjusted at the end of each iteration so that the RPAM gives better correspondence between the chosen output phases and the actual output phases obtained after the application of the RPAM.

The flowchart of the iterative RPAM is shown in Fig. 4.4 and can be described as follows:

i) Design an RPA by using the RPAM as described in 4.2.3.1

ii) Simulate the output waveforms by launching gaussian beams as input fields through the designed RPA.

iii) Check whether the variations of peak intensities are within the specified tolerances. If so, terminate. If not, use the computed output phases for the current iteration as the assigned output phases for the next iteration.

iv) Continue the next iteration by redoing i)-iii)
Nonmonochromatic wavefields with number of wavelengths $N$

Given arbitrary $\tilde{\varphi}_o(x_{o,j}, z_o)$ for each $\lambda_j$

**RPAM**

$$U(x_i, z_i) = \begin{cases} 
U \exp \left( j \frac{2\pi n_x}{\lambda} \delta_i \right) & \text{where } i \text{ - th } \lambda \in \text{best } N_{\text{pick}} \\
0 & \text{otherwise}
\end{cases}$$

Simulate the wave-propagation from the phase-compensated input waveguides and obtain the actual $\varphi_o(x_o, z_o)$

peak amplitude normalized?

**NO**

$$\tilde{\varphi}_o(x_{o,j}, z_o) = \varphi_o(x_{o,j}, z_o)$$

**YES**

TERMINAT

Fig. 4.4. The flowchart of the iterative RPAM.
4.3 Experimental Results

All computer simulations are considered for implementing 256 input waveguides out of possible 600 locations on the input plane with waveguide width of 4 μm and minimum separation of adjacent waveguides is 1 μm. Optical signals are launched through the input waveguides, whose refractive index \( n_e \) at the average wavelength equals 1.5, and the substrate with the refractive index \( n_s \) of 1.45.

In the simulations, the RPAM has been found to be an effective method to design an RPA for separating multiple wavelengths from each other without any regards to the order of wavelengths, and/or the output locations, etc. In the following results, the wavelengths of interest will be within two of the optical-fiber low-loss bandwidths, which are approximately 200 nm in the vicinity of 1300 nm and 1550 nm, respectively. The simulation results are illustrated in Fig. 4.5-4.16 and summarized in Table 4.1.

In simulation # 1, which is designed for wavelengths of 1300 and 1550 nm, the results have shown that the design works very well for the signal from different optical-fiber low-loss bandwidths. The signal-to-noise ration (SNR), defined as

\[
\text{SNR} = \frac{\text{Power of the desired output}}{\sum \text{power from the other wavelengths}},
\]

is low because the two wavelengths are very much different from each other. When the wavelengths in consideration are in the same bandwidth as in simulation # 2, the RPAM can still concentrate most of the power from each wavelength to its corresponding location. Therefore, the RPAs by the RPAM can be operated with wavelengths within or beyond the same low-loss bandwidth.

The RPAM can also be used to design a RPA that not only separate wavelengths but also get rid of the unwanted channel from a multi-wavelength signal. Both simulation # 3 and # 4 are for operating with input signals of 3 wavelengths at 1511.6 nm, 1550.0 nm, and 1588.0 nm. The RPAM design in simulation # 3 takes all 3 wavelengths into consideration, whereas only two wavelengths of 1511.6 and 1588.0 nm are used in simulation # 4 and thereby 1550.0 nm is filtered out.
In most cases, the basic RPAM can produce satisfactory results so that the iterative-RPAM may not be necessary. It can be observed from simulation # 5 and #6, with 4 wavelengths that the performance in terms of SNR is not much affected when the iterative-RPAM is used to equalize the intensities for all wavelengths.
Table 4.1
The summary of simulation results

<table>
<thead>
<tr>
<th>Simulation Description</th>
<th>Wavelength @ Output Location</th>
<th>Intensity</th>
<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Simulation #1: Figs. 4.5, 4.6</strong> from different 2 bands ($z_0 = 15 \text{ mm}$)</td>
<td>1300.0 nm @ -500 um</td>
<td>4.1619</td>
<td>4.2260</td>
</tr>
<tr>
<td></td>
<td>1550.0 nm @ 500 um</td>
<td>3.5700</td>
<td>3.6127</td>
</tr>
<tr>
<td><strong>Simulation #2: Figs. 4.7, 4.8</strong> 2 wavelengths from the 1.5-um band ($z_0 = 19 \text{ mm}$)</td>
<td>1537.2 nm @ -500 um</td>
<td>4.0455</td>
<td>4.0901</td>
</tr>
<tr>
<td></td>
<td>1588.0 nm @ 500 um</td>
<td>3.8720</td>
<td>3.9487</td>
</tr>
<tr>
<td><strong>Simulation #3: Figs. 4.9, 4.10</strong> from the 1.5-um band ($z_0 = 16 \text{ mm}$)</td>
<td>1511.6 nm @ -750 um</td>
<td>2.6672</td>
<td>2.7970</td>
</tr>
<tr>
<td></td>
<td>1550.0 nm @ 0 um</td>
<td>3.4114</td>
<td>3.0153</td>
</tr>
<tr>
<td></td>
<td>1588.0 nm @ 750 um</td>
<td>2.3217</td>
<td>2.5909</td>
</tr>
<tr>
<td><strong>Simulation #4: Figs. 4.11, 4.12</strong> 3 wavelengths with one filtered out ($z_0 = 14 \text{ mm}$)</td>
<td>1511.6 nm @ -750 um</td>
<td>4.3662</td>
<td>4.0146</td>
</tr>
<tr>
<td></td>
<td>1550.0 nm filtered out</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>1588.0 nm @ 750 um</td>
<td>4.2024</td>
<td>3.9303</td>
</tr>
<tr>
<td><strong>Simulation #5: Figs. 4.13, 4.14</strong> 4 wavelengths from the 1.5-um band ($z_0 = 15 \text{ mm}$)</td>
<td>1550.0 nm @ -300 um</td>
<td>2.5434</td>
<td>1.6979</td>
</tr>
<tr>
<td></td>
<td>1580.0 nm @ 300 um</td>
<td>2.0571</td>
<td>1.8847</td>
</tr>
<tr>
<td></td>
<td>1610.0 nm @ 900 um</td>
<td>1.4139</td>
<td>1.6461</td>
</tr>
<tr>
<td><strong>Simulation #6: Figs. 4.15, 4.16</strong> 4 wavelengths with arbitrary order of wavelengths and output locations ($z_0 = 18 \text{ mm}$)</td>
<td>1525.0 nm @ -900 um</td>
<td>1.1587</td>
<td>1.3728</td>
</tr>
<tr>
<td></td>
<td>1600.0 nm @ -300 um</td>
<td>1.6445</td>
<td>1.4636</td>
</tr>
<tr>
<td></td>
<td>1570.0 nm @ -100 um</td>
<td>1.4153</td>
<td>1.3882</td>
</tr>
<tr>
<td></td>
<td>1550.0 nm @ 400 um</td>
<td>1.6182</td>
<td>1.5610</td>
</tr>
</tbody>
</table>

*(a) by the basic RPAM, and (b) by the iterative-RPAM*
Fig. 4.5. The output intensities of simulations for separating optical signal with 2 wavelengths by (a) the basic RPAM (b) the iterative-RPAM.
Fig. 4.6. The adjusted waveguide lengths for separating optical signal with 2 wavelengths by (a) the basic RPAM (b) the iterative-RPAM.
Fig. 4.7. The output intensities of simulations for separating optical signal with 2 wavelengths by (a) the basic RPAM (b) the iterative-RPAM.
Fig. 4.8. The adjusted waveguide lengths for separating optical signal with 2 wavelengths by (a) the basic RPAM (b) the iterative-RPAM.
Fig. 4.9. The output intensities of simulations for separating optical signal with 3 wavelengths by (a) the basic RPAM (b) the iterative-RPAM.
Fig. 4.10. The adjusted waveguide lengths for separating optical signal with 3 wavelengths by (a) the basic RPAM (b) the iterative-RPAM.
Fig. 4.11. The output intensities of simulations for separating optical signal with 3 wavelengths by (a) the basic RPAM (b) the iterative-RPAM.
Fig. 4.12. The adjusted waveguide lengths for separating optical signal with 3 wavelengths by (a) the basic RPAM (b) the iterative-RPAM.
Fig. 4.13. The output intensities of simulations for separating optical signal with 4 wavelengths by (a) the basic RPAM (b) the iterative-RPAM.
Fig. 4.14. The adjusted waveguide lengths for separating optical signal with 4 wavelengths by (a) the basic RPAM (b) the iterative-RPAM.
Fig. 4.15. The output intensities of simulations for separating optical signal with 4 wavelengths by (a) the basic RPAM (b) the iterative-RPAM.
Fig. 4.16. The adjusted waveguide lengths for separating optical signal with 4 wavelengths by (a) the basic RPAM (b) the iterative-RPAM.
CHAPTER 5

COMBINED RANKED PHASED-ARRAY AND ARRAY
WAVEGUIDE GRATINGS AS DENSE WAVELENGTH DIVISION
DEMULTIPLEXERS

5.1 Introduction

A major objective for developing the RPAM in this research is to improve the overall performances of existing demultiplexers operating in optical dense wavelength division multiplexing. A properly designed RPA can assist an angularly dispersive device, such as an array waveguide grating (AWG), to expend most of the low-loss bandwidth as dense wavelength division demultiplexer in which AWG by itself cannot efficiently exploit.

Recent researches on AWG [3]-[7] have shown very impressive improvements that can accomplish implementation of multiplexers/demultiplexers operating in the low-loss bandwidth around 1,550 nm with wavelength-spacing down to 10 GHz (0.08 nm) for 11 channels [5]. In another case, the number of channels is maximized up to 64 for 50 GHz (0.4 nm) spacing [7]. Still, most of the low transmission-loss bandwidth is not employed due to the operating limitation of some devices. For single-mode waveguides, the two important low-loss windows locate near the operating wavelengths of 1,300 and 1,550 nm. The low-loss band in the vicinity of 1,300 nm is approximately 14,000 GHz, or equivalent to 78.9 nm wide, while there is about 15,000 GHz or equivalent to 120.1 nm wide for the low-loss wavelengths around 1,550 nm [9]. Therefore, an optical fiber can theoretically support up to 1,500 channels with 10-GHz spacing and 300 channels with 50-GHz spacing provided that the optical powers and transmission distances agree with the limitations imposed by optical-fiber nonlinearities[19].
The scheme to enhance demultiplexing by AWG has become a major contribution of FWAM as discussed in this chapter. The strategy is to initially divide the low-loss windows into a few predetermined subbands that the current AWG structure can operate with. Then, the FWAM will design an FWA that guides each subband to a AWG in the following stage. For example, to extract 300 channels from input multiple wavelength signals of 1490-1610 nm with 0.4 nm (or 50-GHz) spacing, the FWAM provides an RPA that can divide such optical signals into five different groups of wavelengths as follows: 1490-1515.2 nm, 1515.6-1540.8 nm, 1541.2-1566.4 nm, 1566.8-1592 nm, and 1592.4-1610 nm. The RPA separates each group or subband from the others and focus all wavelengths within each subband to the same designated location. At each location, there is an input waveguide linking that subband to the appropriate AWG that is correspondingly fabricated to finely isolate an individual wavelength in that particular subband to a specific AWG output waveguide. Hence, an RPA and five AWG elements are capable of demultiplexing \((4 \times 64) + 44 = 300\) optical channels.

![Graph showing optical fiber loss as a function of wavelength in conventional single-mode silica fiber][21].
5.2 Principle of Arrayed Waveguide Grating (AWG)

As illustrated in Fig. 5.2, an arrayed waveguide grating (AWG) consists of the following parts: the transmitter waveguides, the phased-array waveguides (PHASAR), the receiver waveguides, and two regions of homogeneous wave propagation for making use of diffraction. The transmitter and receiver waveguides have similar structures with input/output waveguides usually arranged in a confocal geometry with diameter \( R_a \). According to [8], this kind of configuration functions similarly to an optical lens with focal length of \( R_a \). Linking between the two sections, an array of waveguides with constant length differences is placed to create phase shifts from one waveguide to the other. These linear phase adjustments cause the same angularly dispersing effect as a sampled diffraction grating.

The mechanism of AWG as a demultiplexer can be described as follows. Launching via a transmitter waveguide in the middle of the input curvature, optical signals will expand in the slab, and, later appear at one end of each array waveguide with uniform amplitudes and phases for the same wavelength. This process is equivalent to wavefields propagating through a lens with a point source at its focal point. The array waveguides then create wavelength-dependent phase shifts at the other end according to the constant differences in lengths between adjacent waveguides \( \Delta L \). After travelling past another slab to the receiver waveguides, the optical signal peaks at the position where phases satisfy the following condition:

\[
\frac{2\pi}{\lambda} \left( n_s d_s \sin \theta_i + n_s \Delta L + n_s d_s \sin \theta_o \right) = 2m \pi
\]

or,

\[
n_s d_s \sin \theta_i + n_s \Delta L + n_s d_s \sin \theta_o = m \lambda
\]  (5.1)

where \( m \) is an integer. \( n_s \) and \( n_c \) are the effective refractive indices of the slab and array waveguides at the central wavelength, respectively. The angles \( \theta_i \) and \( \theta_o \) are defined as
the angles with which the corresponding i-th input and Z-th output waveguides deviate from the middle point of the curvatures, and are given by

\[ \delta_i = \frac{i \times d_\alpha}{R_a} \quad \text{and} \quad \delta_z = \frac{z \times d_\alpha}{R_a} \]  

(5.2)

The central wavelength \( \lambda_0 \) is supposed to occur at the center of the output plane and comply with

\[ n_c \Delta L = m \lambda_0 \]  

(5.3)

Therefore, \( m \) can also be considered as the diffraction order of grating.

AWG is categorized as an angularly dispersive device because of the way it manipulates the output diffraction angles, depends on the differences in frequencies or wavelengths of the optical signals. The angular dispersion is regarded as the measure of the angular deviation \( A\delta \) on the output plane between two channels different in frequencies by \( \Delta f \), which can be expressed as

\[ \frac{\Delta \theta}{\Delta f} \approx \frac{d \theta_\alpha}{df} = \frac{m \lambda^2}{cd} \frac{n_g}{n_e} \]  

(5.4)

where the wavelength and frequency of the same signal can be related by the velocity of light as

\[ f = \frac{c}{\lambda} \quad \text{and} \quad \Delta f = -\frac{c}{\lambda^2} \Delta \lambda \]  

(5.5)

The group refractive index of array waveguide \( n_g \) is defined as

\[ n_g = n_e - \lambda \frac{dn_e}{d\lambda} \]  

(5.5)

Given the frequency channel spacing \( \Delta f \), the output spacing between two adjacent channels \( \Delta x \) can be found as

\[ \Delta x = \Delta f \times R_a \left( \frac{d \theta_\alpha}{df} \right) \]  

(5.6)
Another important parameter in the AWG design is the free space range (FSR). Assume that the AWG can accommodate exactly N channels. FSR = \( N\Delta f \) must not be overlapped by channels from the next diffraction order \((m+1)\). The condition in Eq. (5.1) can be rewritten as

\[
(n + A_n)\lambda d\sin\theta + (n_c + A_n)\Delta L + (n + A_n)\lambda d\sin\theta = (m+1)\frac{c}{f + \text{FSR}} \tag{5.7}
\]

where \((n + A_n)\) and \((n_c + \Delta n_c)\) are the effective refractive indices of the slab and array waveguides at the center frequency \((f + \text{FSR})\), respectively. Each refractive index change is described as

\[
A_n = \frac{dn}{df} \text{FSR} = -\frac{\varepsilon}{f^2} \frac{dn}{d\lambda} \text{FSR} \tag{5.8}
\]

According to Eqs. (5.1), (5.7), and (5.8), FSR can be calculated as follows:

\[
\text{FSR} = \frac{c}{\left(n_c - \lambda \frac{dn_c}{d\lambda}\Delta L + \left(n_c - \lambda \frac{dn_c}{d\lambda}\right)(d\sin\theta_1 + d\sin\theta_o)\right)} \tag{5.9}
\]

Practically, the terms \(n_c - \lambda \frac{dn_c}{d\lambda}\) and \(n_c - \lambda \frac{dn_c}{d\lambda}\) yield approximately the same values, near 1.45 for silica-based waveguides, for example. With \(\Delta L \gg (d\sin\theta_1 + d\sin\theta_o)\), FSR in Hz can be estimated as

\[
\text{FSR} = \frac{c}{n_c \Delta L} \tag{5.10}
\]

Further details for the principles, design, and additional applications of AWG can be found in [10], [14], and [20].
Fig. 5.2. The AWG structures: (a) the layout of AWG as demultiplexer (b) geometry of the receiver side [10].
5.3 Combined RPA and AWG Design

After the proper FSR is predetermined from the arrangement of AWGs, all wavelengths of interest are suitably partitioned into several subbands such that each range does not extend beyond the FSR. Thereafter, one or several RPAs are designed to generate groups of specified wavelengths appearing at their corresponding locations as the dominant input optical signals, which each particular AWG can further demultiplex.

In the design by the RFAM, only one wavelength from each subband is taken into consideration. As illustrated in Fig. 5.3, an optical system operating with wavelengths $\lambda_1, \lambda_2, \ldots, \lambda_{N-1}, \lambda_N$, is divided into 2 subbands: $\lambda_1 - \lambda_N$ and $\lambda_{N+1} - \lambda_{2N}$ are assigned to subband # 1 and subband # 2, respectively. Next, the RPAM designs an RPA to focus $\lambda_{s,1}$ for $\lambda_1 \leq \lambda_{s,1} \leq \lambda_N$ to position $x_{\text{out},1}$, and to focus $\lambda_{s,2}$ for $\lambda_{N+1} \leq \lambda_{s,2} \leq \lambda_{2N}$ to position $x_{\text{out},2}$, accordingly. Although the other wavelengths in each subband are not directly included in the design by the RPAM, their peak intensities still appear at the specified location for the corresponding subband. These characteristics can be observed from the computer simulations discussed in Section 5.4. Afterwards, an AWG with the input waveguide at $x_{\text{out},1}$ processes the optical signals from that location at which wavelengths in subband # 1 dominate. Another AWG handles subband # 2 at the location $x_{\text{out},2}$ in the same manner.

The consideration of the RPAM with only one chosen wavelength in each subband, followed by an AWG processing, is crucial to the overall performance of the optical system. The RPA by itself may not be able to split beams with many different wavelengths to individually different locations. Consequently, not only the channels within its own subband but the channels from the other subbands may show up at the output waveguides. This undesirable interference is capable of significantly inducing crosstalks if, in the RPAM, the wavelength representing each subband is not chosen properly.

There are various approaches to determine the appropriate wavelength as a fitting candidate for each subband. Two interesting strategies to be discussed in the following
sections are the middle-wavelength representation and boundary-wavelength representation.

5.3.1 The RPAM with middle-wavelength representation

The general concept for the best representation of any quantity is to find the mean, median, or mode. This approach follows the similar idea and uses the median wavelength from each subband in the RPAM design.

As illustrated in Fig. 5.4, the middle-wavelength representation can concentrate high intensities of the dominating channels into the specified input AWG waveguide. The disadvantage of this representation is that the interference can be as large as the signals themselves for the channel around the output AWG boundaries.

5.3.2 The RPAM with boundary-wavelength representation

This representation is developed to reduce crosstalk between subbands. In the boundary-wavelength representation with two subbands, the representative wavelength for the first subband is chosen as the smallest wavelength, and the representative wavelength for the second subband is chosen as the largest wavelength as illustrated in Fig. 5.5. Unlike the previous RPAM with middle-wavelength representation, this arrangement results in most channels having better signal-to-noise ratio (SNR).

It can be noticed that this kind of representation can employ all channels in case of the system with only 2 subbands. To work with more subbands without dropping some channels in between subbands, at least one middle-representation must be included.
Fig. 5.3. Combined design with RPA and AWGs.
Fig. 5.4. The intensities vs. channel wavelengths by the RPAM with the middle-representation at the corresponding location of the input AWG waveguide designed for (a) subband #1 (b) subband #2.
Fig. 5.5. The intensities vs. channel wavelengths by the RPAM with the boundary-representation at the corresponding location of the input AWG waveguide designed for
(a) subband #1 (b) subband #2.
5.4 Experimental results

The experimental setups simulate 2 optical systems as follows:

**System #1**: The system consists of 1 RPA and 2 AWGs, functioning as a demultiplexer for signals of wavelength 1537.2-1588.0 nm. For the frequency-spacing of 50-GHz (0.4 nm), there are total of 128 channels with 64 channels in each subband. The input optical signals are divided into 2 subbands as:

- **Subband # 1** (channel number 1 to 64) will appear as dominating channels at the location for the designated AWG # 1.

- **Subband # 2** (channel number 65 to 128) will appear as dominating channels at the location for the designated AWG # 2.

The only RPA is designed by the RPAM with boundary-wavelength representation. The two wavelengths in the calculation are 1537.2 and 1588.0 nm as the representative channels for subband # 1 and # 2, respectively.

The 2 AWGs are implemented with following parameters:

- **AWG # 1** is designed to separate the wavelengths of 1537.2-1562.4 nm, with the following parameters: \( m = 58 \); \( R_a = 4390.0 \mu m \), \( \Delta L = 61.94 \mu m \).

- **AWG # 2** is designed to separate the wavelengths of 1562.8-1588.0 nm, with the following parameters: \( m = 58 \); \( R_a = 4388.8 \mu m \), \( \Delta L = 62.98 \mu m \).

**System #2**: The system consists of 1 RPA and 3 AWGs, functioning as a demultiplexer for signals of wavelength 1511.6-1588.0 nm. For the frequency-spacing of 50-GHz (0.4 nm), there are total of 192 channels with 64 channels in each subband. The input optical signals are divided into 3 subbands as:

- **Subband # 1** (channel number 1 to 64) will appear as dominating channels at the location for the designated AWG # 1.
Subband # 2 (channel number 65 to 128) will appear as dominating channels at the location for the designated AWG # 2.

Subband # 3 (channel number 129 to 192) will appear as dominating channels at the location for the designated AWG # 3.

The only RPA is designed by the RPAM with hybrid of boundary- and middle- wavelength representations. The boundary wavelengths in the calculation as representative channels for subband # 1 and # 3 are 1511.6 and 1588.0 nm, respectively. The representative channel for subband # 2 has to be the middle-wavelength representation, 1550.0 nm, so that there are no dropping channels in between the adjacent subbands.

The 3 AWGs are implemented with following parameters:

AWG # 1 is designed to separate the wavelengths of 1511.6-1536.8 nm, with the following parameters: \( m = 58 \); \( R_a = 4391.3 \) \( \mu m \), \( \Delta L = 60.90 \) \( \mu m \).

AWG # 2 is designed to separate the wavelengths of 1537.2-1562.4 nm, with the following parameters: \( m = 58 \); \( R = 4390.0 \) \( \mu m \), \( AL = 61.94 \) \( \mu m \).

AWG # 3 is designed to separate the wavelengths of 1562.8-1588.0 nm, with the following parameters: \( m = 58 \); \( R = 4388.8 \) \( \mu m \), \( AL = 62.98 \) \( \mu m \).

5.4.1 Computer simulations for system # 1

The results of computer simulations for system # 1 are presented in Figs. 5.6 to 5.13. Fig. 5.6 shows the representative wavelength intensities and the arrangement of the RPA designed by the RPAM with boundary-wavelength representation. After input optical signals with 128 channels past the RPA, the intensities for individual wavelength at 2 different locations are illustrated in Fig. 5.7. These intensities are the input signals for the corresponding AWGs at the specific locations. Although all interested channels arrive at each AWG location, the intensities of the wavelengths in the interfering subband are smaller than those of the dominating subband.
Then, each AWG tries to separate and focus input wavelengths at the location into its 64 receiving waveguides. Fig. 5.8 shows the output intensities for each wavelength after being processed by AWG # 1. Because of the limitation by FSR, the 128 channels appear on the receiving curvatures of AWG as illustrated in Fig. 5.9. Although the distances between the adjacent waveguides are 25 μm, the receiving waveguides placed in the middle of the pitches are only 4 μm wide. While the wavelengths of dominating subband can match the predetermined waveguide locations, the responses of the interfering channels mostly fall into the gap between waveguides. Hence, the power coupled into each receiving waveguide is mainly from the specified channel in the dominating subband. This effect can obviously be noticed at the receiving waveguides around the border of AWG, such as channels 1-16 in Fig. 5.9.

Due to the characteristics of propagating gaussian beams, the far-field envelopes, denoting intensity profiles on the receiving curvatures, indicates that the peak intensities of channels in the tail area are much less than those in the middle of the AWG. Therefore, the overlap of interfering channels with mismatched locations is beneficial to such channels and can reduce the effect of crosstalk, defined as

$$\text{crosstalk} = -10 \log \left( \frac{\text{Power of the interfering channels}}{\text{Power of the desired channel}} \right) \quad \text{in dB} \quad (5.11)$$

According to Fig. 5.10, although the crosstalks are small for channels in the middle of AWG, their peak output intensities are very large. The optical signals in these channels providing with appropriate thresholds can be detected directly, while those of channels in the tail area may be detected after being amplified to acceptable levels.

The similar results for the subband demultiplexed by AWG # 2 are also presented in Figs. 5.11 to 5.13. The crosstalks for all operating channels from both subbands are no less than 5 dB, which is equivalent to SNR of 3.13.
Fig. 5.6. The RPAM with boundary-wavelength representation for an optical system with 128 channels of wavelengths, 1537.2-1588.0 nm, as divided into 2 subbands: (a) the representative wavelength responses at the specified AWG locations (b) the adjusted waveguide lengths for the designed RPA.
Fig. 5.7. The input intensities vs. all wavelengths to AWGs at each location
(a) at location of dominating subband # 1 of wavelengths 1537.2-1562.4 nm.
(b) at location of dominating subband # 2 of wavelengths 1562.8-1588.0 nm.
Fig. 5.8. The output intensities vs. wavelengths from AWG #1
(a) the dominating subband #1 of wavelengths 1537.2-1562.4 nm.
(b) the interfering subband #2 of wavelengths 1562.8-1588.0 nm.
Fig. 5.9. The output intensities vs. channel waveguide numbers from AWG #1.
Fig. 5.10. The crosstalks vs. channel waveguide numbers from AWG # 1.
Fig. 5.11. The output intensities vs. wavelengths from AWG # 2
(a) the interfering subband # 1 of wavelengths 1537.2-1562.4 nm.
(b) the dominating subband # 2 of wavelengths 1562.8-1588.0 nm.
Fig. 5.12. The output intensities vs. channel waveguide numbers from AWG # 2.
Fig. 5.13: The crosstalks vs. channel waveguide numbers from AWG #2
5.4.2 Computer simulations for system # 2

The results of computer simulations for system # 2 are presented in Figs. 5.14 to 5.24. Fig. 5.14 shows the intensities of the 3 representative wavelengths and the arrangement of the RPA designed by the RPAM with hybrid of boundary- and middle-wavelength representation. For input optical signals with 192 channels, the RPA projects the transmitting wavelengths to 3 different locations as illustrated in Fig. 5.15. The intensities of those wavelengths are the input signals for the corresponding AWGs at the specific locations. Similar to the responses in system # 1, all transmitting channels arrive at each AWG location but the intensities of the wavelengths in the interfering subband are smaller than those of the dominating subband.

After each AWG beaming input wavelengths at the location to its 64 receiving waveguides, the output intensities for each wavelength after being processed by AWG # 1, 2, and 3 are plotted in Figs. 5.16, 5.19, and 5.22, respectively. The overlapping responses corresponding to the receiving waveguides are also shown in Fig. 5.17 for AWG # 1, Fig. 5.20 for AWG # 2, and Fig. 5.23 for AWG # 3. As discussed in the section of the RPAM with middle-wavelength representation, the characteristic of intensities in AWG # 2 around the tail area are more affected by the interfering channels than the other 2 AWGs working the boundary-wavelength representations. Still, the crosstalks for channels in subband # 2 dominating in this AWG, as shown in Fig. 5.21, do not fall below the 5-dB level.

Though input signals for the AWG # 3 are provided by the boundary-wavelength representation, the interfering channels from subband # 2 appear at the receiving waveguides with significant output intensities. This interference, especially in the middle waveguides, which the overlapping channels are less likely mismatched, causes the interfering power to be mostly coupled into the channel waveguides. As shown in Fig. 5.24, the crosstalks of channel 90 to 106 drop below 5-dB level to around 2 - 3 dB.

Because the experimental setup in this study refers to the actual implementation of an AWG operating with 64 channels around 1.55 μm as published in [15], the diffraction order m and other parameters are suitable for subband # 2. This experimental
setup may not be appropriate for AWG #1 and #3. The two AWGs are designed with similar parameters to AWG #2 so that all 3 AWGs become approximately the same in dimensions. Although they can effectively demultiplex the desired channels, the further research to investigate and determine the optimal parameters in the design for each AWG will help minimize the problem and allow AWG #3 to execute more efficiently.
Fig. 5.14. The RPAM with both boundary- and middle-wavelength representations for an optical system with 192 channels of wavelengths, 1511.6-1588.0 nm, as divided into 3 subbands: (a) the representative wavelength responses at the specified AWG locations (b) the adjusted waveguide lengths for the designed RPA.
Fig. 5.15. The input intensities vs. all wavelengths to AWGs at each location
(a) at location of dominating subband #1 of wavelengths 1511.6-1536.8 nm.
(b) at location of dominating subband #2 of wavelengths 1537.2-1562.4 nm.
(c) at location of dominating subband #3 of wavelengths 1562.8-1588.0 nm.
Fig. 5.16. The output intensities vs. wavelengths from AWG # 1
(a) the dominating subband # 1 of wavelengths 1511.6-1536.8 nm.
(b) the interfering subband # 2 of wavelengths 1537.2-1562.4 nm.
(c) the interfering subband # 3 of wavelengths 1562.8-1588.0 nm.
Fig. 5.17. The output intensities vs. channel waveguide numbers from AWG # 1.
Fig. 5.19. The output intensities vs. wavelengths from AWG #2 (a) the interfering subband #1 of wavelengths 1511.6-1536.8 nm. (b) the dominating subband #2 of wavelengths 1537.2-1562.4 nm. (c) the interfering subband #3 of wavelengths 1562.8-1588.0 nm.
Fig. 5.20. The output intensities vs. channel waveguide numbers from AWG # 2.
Fig. 5.22. The output intensities vs. wavelengths from AWG # 3
(a) the interfering subband # 1 of wavelengths 1511.6-1536.8 nm.
(b) the interfering subband # 2 of wavelengths 1537.2-1562.4 nm.
(c) the dominating subband # 3 of wavelengths 1562.8-1588.0 nm.
Fig. 5.23. The output intensity vs. channel waveguide numbers from AWG # 3.
Fig. 5.24. The crosstalks vs. channel waveguide numbers from AWG # 3.
CHAPTER 6

CONCLUSIONS AND FURTHER RESEARCH

In this chapter, conclusions and recommendations for further research are presented in three sections corresponding to the three algorithms proposed for the design of all-optical devices to be used in optical communications and networking.

6.1 An Iterative Method for the Design of Nonperiodic Grating-assisted Directional Coupler using the Beam Propagation Method

The iterative method for the design of nonperiodic grating-assisted directional coupler using the BPM has been introduced and discussed. This new algorithm has shown superior performance in reducing the coupling lengths for accomplishing complete power transfer. This feat benefits the implementation in optical system of very small dimension, such as in integrated optical circuits, etc.

To optimally realize this new design, possible further research can be done in investigating the effects of quantization and multiple grating levels to the overall characteristics of the nonperiodic grating structures.

6.2 The Angular Spectrum and Beam Propagation Methods Using Sine Transform

The ASM and the BPM have been alternative methods for efficiently considering how waves propagate in a complicated structure. However, it can be difficult for the conventional ASM and BPM using the complex Fourier transform to handle an optical system with zero boundary conditions, as in shielded optical waveguides, for example. The simulations by the new ASMS and BPMS help solve such difficulties due to the nature of the sine function and period of consideration. The computational results have
confirmed that the ASMS and BPMS provide more accurate wave profiles to the analytical solutions than those by the conventional methods in such cases.

The ASMS and BPMS algorithms are valid when one boundary condition is at \( x=0 \). In the more general case with the boundary condition at, say, \( x=a \), the real Fourier transform and its discrete counterpart, the real discrete Fourier transform (RDFT) become the more suitable transform for the same purpose. The restrictions of zero boundary conditions, symmetry/antisymmetry are the reasons dictating this type of transform to be used. Further research would be recommend to determine optimal transform for other similar conditions.

6.3 The Ranked Phased-Array Method and Its Applications

The main objective in developing the RPAM is to design a device that can assist a popularly used AWG as a multiplexer/demultiplexer in DWDM to better utilize the optical-fiber low-loss bandwidth around wavelength 1.3 and 1.55 \( \mu \text{m} \). The new device called ranked phased-array (RPA) has been proved to successfully perform the task of dividing multiple wavelengths into subbands that a single AWG can manage in the later stage. The combination of RPA and AWGs can multiply increase the total number of transmitting channels that AWG by itself cannot handle because of the limitation by free space range (FSR).

In this study, 64-channel AWGs along with an RPA designed for using with 2 subbands can operate up to 128 channels with crosstalk no less than 5 dB. The experimental setup of combined RPA and AWGS for 3 subbands with total of 192 channels has also achieved the demultiplexing task effectively, though the characteristics in terms of crosstalks fall below 5 dB at some wavelengths. To realize more efficient devices as demultiplexers, further researches are needed for better designs with optimal parameters, such as the suitable diffraction order for each individual AWG, the appropriate arrangement of subbands, etc.
Because an RPA by itself can also complete tasks similar to AWG, further study can be attempted to develop an RPA that can independently perform adding/dropping channels, filtering, and/or switching, etc.
REFERENCES


Iterative method for the design of a nonperiodic grating-assisted directional coupler

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We propose and investigate the optimal design of a nonperiodic grating-assisted directional coupler by iterative methods using the beam propagation method. Computer simulations were carried out at wavelengths of 0.8, 1.3, and 1.5 µm, which are often used in optical communications and networking. We found that the complete power coupling lengths can be reduced considerably in comparison with those in the case of the periodic grating-assisted waveguides with the same set of parameters. © 2001 Optical Society of America

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1. Introduction

A directional coupler is an optical device consisting of two parallel dielectric waveguides placed nearby each other so that an optical wave launched into one guide can be coupled into the other. The structure and notation used are shown in Fig. 1.

The directional coupler has become an important component in many systems used in integrated optics, optical communications, and networking. Because of its ability to couple power between two waveguides, it can be used as a power divider with a specific amount of power achieved by adjustment of the coupling distance of the two guides. Another interesting application of a directional coupler is in light switching and modulation. If any of the guides are made of electro-optics material, their refractive index can be adjusted by external fields. This feature allows modulation to be applied to the optical system. In addition, a directional coupler can be used as an optical filter. Because the coupling coefficients depend on the propagation constants, which relate to the wavelengths, only specified wavelengths can be detected at the output ends with a proper adjustment of the waveguides.

In a directional coupler, complete power exchange can be achieved when \( n_1 = n_2 \), which is called synchronous waveguides. In the case of nonsynchronous waveguides, in which the refractive index \( n_1 \neq n_2 \), although a complete power transfer can be obtained at a specific wavelength, such a wavelength may not be useful for a practical optical system. In general, only a small portion of power can be coupled into the mismatched guide no matter how long the coupling length is. The distance at which a complete power transfer occurs, called the coupling length in synchronous waveguides, and the maximum power that can be transferred in a nonsynchronous structure can be determined by coupled-mode theory, which is derived from the scalar wave equation.

Some applications may require a complete power transfer between nonsynchronous waveguides. The grating-assisted directional coupler has been introduced and shown to accomplish this task within an acceptable coupling length.

Because the exact solution can sometimes be difficult to achieve by conventional methods, especially when the refractive index varies in a complex way within a waveguide, many techniques have been proposed for investigating the characteristics of an optical wave in a complicated waveguide structure. The beam propagation method (BPM) is one such method that is used to approximately simulate the optical wave propagation in a nonhomogeneous medium of arbitrary shape with the constraints that reflected waves can be neglected and all refractive-index differences are small.

A review of the BPM and its derivation from the Helmholtz equation and the angular spectrum method can be found in Refs. 7 and 9. By use of this method, both the guided modes and the radiation...
modes are included in the calculation because the integration range covers all real positive values of the propagating constant \( \beta \). In many studies, the BPM has been proven an effective technique to simulate wave propagation in an arbitrary medium.\(^{10-13}\)

## 2 Periodic Grating-Assisted Directional Coupler

When the two waveguides have different refractive indices, complete power exchange is difficult to achieve because the superposition fields in the two guides travel with different propagation constant \( \beta \). The contribution from one guide can be added to the existing field in the other guide either in phase or out of phase when power flows in the same or opposite directions, respectively. While the fields are out of phase, the power in one guide will be reduced because the coupling power has been transferred back to the other guide.

To achieve complete power exchange, the coupling coefficient has to be modulated so that strong coupling occurs while the two fields are in phase and coupling is reduced while the two fields are out of phase. The idea of assigning the proper coupling coefficients at a specific \( z \) can be realized by adjustment of the separation distances \( z \) between the two waveguides. Closer separation distance yields a stronger coupling effect. The resulting structure is called the grating-assisted directional coupler, in which one or both guides have periodic width variations along \( z \) as shown in Fig. 2.

The basic concept of the grating-assisted directional coupler with nonsynchronous waveguides is to amplify the coupling coefficient when the power is coupled from the input guide to the output guide by a reduction of the separation between the guides and to attenuate the coupling coefficient by an increase in the gap between the guides when the power is coupled back from the output guide into the input guide.

By assuming a periodic variation with the grating period \( \Lambda \), we can derive the coupling coefficient that is due to the presence of a grating from its shape function \( f(z) \) by using the theory of the grating-assisted directional coupler.\(^7\) In the case of the step grating, the fundamental Fourier component of the structure can be found as

\[
f(z) = \left( \frac{4b}{\pi} \right) \cos \left( \frac{2\pi}{\Lambda} z \right),
\]

where \( b \) represents the amplitude of the grating variation and \( \Lambda \) is the grating period, as shown in Fig. 2.

If the two fields begin propagating in phase at \( z = 0 \), they will be out of phase at \( z = \Lambda/2 = \pi/(\beta_1 - \beta_2) \). Hence the grating period must satisfy the condition

\[
\Lambda = \frac{2\pi}{(\beta_1 - \beta_2)},
\]

where \( \beta_1 \) and \( \beta_2 \) are propagation constants in waveguides 1 and 2, respectively.

The coupling length for complete power exchange to be accomplished in the grating-assisted case is given by\(^7\)

\[
L = \frac{\pi}{2k_x},
\]

where \( k_x \) is the coupling coefficient that is due to the presence of the grating.

By considering only the slowly varying parts of the propagating fields that are significant to the characteristics of power exchange, we can express the coupling coefficients for TM modes of a slab directional coupler in Fig. 2 in terms of the parameter \( k_x \) given by\(^14\)

\[
k_x = \frac{b(n_2^2 - n_1^2)(\varepsilon_0/\mu_0)n_2^2n_1^2\beta_1\beta_2}{2n_2^4 + n_1^4 - \beta_1\beta_2H_{nm}H_{nm}^{*} - \frac{\partial H_{nm}}{\partial x} \frac{\partial H_{nm}^{*}}{\partial x}},
\]

where the subscripts \( n \) and \( m \) indicate even and odd modes of the fields \( (H_{nm}) \) propagating in the structure. The + and − signs in front of the last term in the brackets are used in the forward and backward grating-assisted case, respectively.

Many recent new techniques have been proposed to improve the design of the grating-assisted directional coupler, such as weighted coupling,\(^15\) photonic bandgap structures,\(^16\) and the appropriate geometric criterion for the ideal waveguides.\(^17\) Nonetheless, these techniques are all based on periodic grating structures.

## 3 Design of the Nonperiodic Grating-Assisted Directional Coupler

Although the periodic grating-assisted directional coupler can achieve total power exchange between nonsynchronous waveguides, further effort in the form of the design of a nonperiodic grating-assisted directional coupler has been made to effectively reduce the coupling length. In the previous results, the BPM has been shown as competent to analyze a complicated optical system. In this section it is used as a computational tool to characterize the power transfer inside the directional coupler system.
Fig. 3. Simulation of power exchange in the periodic grating-assisted directional coupler with the following parameters: \( n_1 = 1.47 \), \( n_2 = 1.48 \), \( n_3 = 1.45 \), \( d_1 = 1.3 \) pm, \( d_2 = 2 \) pm, \( b = 0.4 \) pm, \( s = 1 \) pm, and grating period \( \Lambda = 138 \) pm.

The prior grating-assisted case was obtained analytically by use of the periodic assumption. With the same parameters, the BPM also simulates similar characteristics as shown in Fig. 3. Although complete power exchange can be achieved by the periodic structure, such a grating profile has been found inefficient in adjusting the coupling coefficients to match the actual characteristics of the power exchange between the guides at some points. For example, according to Fig. 3, the gap between the waveguides, such as \( \Delta z = 550-600 \) pm, should be large so that the coupling coefficient would be decreased because the power is coupled back from the output guide to the input guide. A nonperiodic design would be sensitive to this issue, as discussed in the following subsection.

A. Algorithm for the Design of the Nonperiodic Grating-Assisted Directional Coupler (Method I)

When we use the BPM, propagation is considered in short segments of length \( dz \). In the case of a two-level gap, the procedure used to determine the effective gap between the two waveguides at any specific distance \( z \) can be described as follows:

(i) Determine the input fields of the first segment from the directional coupler’s parameters. Then the input power at \( z = 0 \) for both guides is considered.

(ii) Use the BPM to compute wave fields propagating in a segment of a short distance \( dz \).

(iii) Calculate the power in each guide according to the derived wave fields.

(iv) If the power in the output guide declined in comparison with the previous segment, the gap between the two waveguides for the next segment is increased. In contrast, if the power in the output guide still climbs up, the gap is kept at the narrow level.

(v) Repeat steps (ii)-(iv) for the next segment with a new structure and input fields until a complete power transfer is achieved in the output guide.

The flow chart of the iterative procedure developed for the design of the nonperiodic grating-assisted directional coupler is illustrated in Fig. 4. The computer simulations with this method are discussed in Section 4.

B. Incorporation of Implementational Constraints (Method II)

Although the original algorithm discussed above is highly effective in reducing the coupling length, as discussed in Section 4, the ideal design may be difficult to implement because of the limitation in current fabrication technology when the separation distances between the waveguides change too quickly, representing very high-frequency information.

To minimize this problem, we further modify the procedure by applying a constraint to the gap adjustment. Because the infinitesimal distance \( dz \) between segment boundaries is significant to the accuracy of the BPM simulation, we cannot simply alter the procedure by setting each \( dz \) to a specific resolution. Instead, the distance \( dz \) is kept as small as in the original procedure, but the determination of the grating adjustment in step (iv) is withheld until the minimum length \( \Lambda_{\text{min}} \) along the guides as shown in Fig. 5 has been reached.

C. Adaptive Algorithm (Method III)

The idea of adjusting the grating level according to the direction of the power transfer is similar to
gradient-descent optimization in the same sense of being a greedy algorithm. The grating level is switched as soon as the power transfer direction changes from the input guide to the output guide and vice versa. Consequently, the original technique can lead to a local minimum instead of the global minimum.

To solve this problem, a factor $\beta$ is introduced into the algorithm to avoid adjustments that are too sensitive. If the power in the input waveguide $(P_I)$ does not decrease appreciably as described by a parameter $\beta$, say, $0.99 < \beta \leq 1$, the grating level is not changed from wide to narrow and vice versa. The original procedure is modified in more detail as follows:

(i)-(iii) The same as in the original procedure.

(iv) This step is to adjust the grating levels according to two conditions that are applied to power in the input guide:

(a) If power at $z$ begins to transfer back from the output guide to the input guide, the coupling coefficient should be decreased by an adjustment of the gap to the wide level. One can apply this condition more rigorously by testing whether $P_I(z) > P_I(z - dz)$. If so, the grating level is changed to a wide gap. Otherwise the current grating level is maintained at the narrow gap.

(b) If power at $z$ begins to transfer from the input guide to the output guide, the coupling coefficient should be enhanced by a reduction of the gap to the narrow level. One can apply this condition more conservatively by testing whether $P_I(z) > P_I(z - dz)$. If so, the grating level is changed to the narrow gap. Otherwise the current grating level is maintained at the wide gap.

(v) Repeat steps (ii) through (iv) for the succeeding segments until a complete power transfer is achieved into the output guide.

The flow chart of this modified procedure is illustrated in Fig. 6.

4. Computer Simulation Results

In the computer simulations, the dimensions of the directional coupler in each experiment are chosen so
Fig. 9. Simulation of power exchange in the grating-assisted directional coupler with the following parameters: \( n_1 = 1.47, n_2 = 1.48, n_3 = 1.45, \lambda = 0.8 \text{ pm}, d_1 = d_2 = 1 \text{ pm}, b = 0.2 \mu\text{m}, \) and \( s = 0.5 \text{ pm}. \) (a) The periodic grating with \( \Lambda = 78 \mu\text{m}, \) (b) the nonperiodic grating with \( \ell_{\text{min}} = 0.2 \mu\text{m}, \) (c) the nonperiodic grating with \( \ell_{\text{min}} = 10 \mu\text{m}. \)

Fig. 10. Simulation of power exchange in the nonperiodic grating-assisted directional coupler with the following parameters: \( n_1 = 1.47, n_2 = 1.48, n_3 = 1.45, \lambda = 1.55 \mu\text{m}, d_1 = d_2 = 2 \mu\text{m}, b = 0.4 \mu\text{m}, s = 1 \text{ pm}. \) (a) The periodic grating with \( \Lambda = 150 \mu\text{m}, \) (b) the nonperiodic grating with \( \beta = 1, \) (c) the nonperiodic grating with \( \beta = 0.99. \)
that the study can be restricted to a single-mode system. The program starts with generating a proper input wave field to be launched into the system as illustrated in Fig. 7. Note that the input wave field is not totally guided into the input waveguide. When the two waveguides are placed adjacent to each other, the tail part of the input wave field is already coupled into the output waveguide at \( z = 0 \). The total input power in the system is considered from both power in the input waveguide \( (P_0) \) and power in the output waveguide \( (P_2) \). Hence a complete power transfer is determined when

\[
\frac{P_2(z)}{P_0(z = 0)} = 1. 
\]

With \( \lambda = 1.3 \) \( \mu \)m, the complete power transfer for the nonsynchronous directional coupler can be obtained by the computer simulation with \( \lambda = 138 \) \( \mu \)m as in Fig. 3. The computer simulation as shown in Fig. 8 indicates that the nonperiodic structures can achieve complete power exchange as well. Moreover, the design of a nonperiodic grating-assisted directional coupler yields superior performance to the periodic case in that it can reduce the coupling length of complete power exchange from 790 \( \mu \)m in the periodic structure to approximately 509 \( \mu \)m in the nonperiodic structure.

In the experiment with \( \lambda = 0.8 \) \( \mu \)m, the periodic coupler with \( \lambda = 1.6 \) \( \mu \)m results in a complete power transfer of \( \frac{P_2(z = 0)}{P_0(z = 0)} = 0.99 \) as shown in Fig. 9(a). The nonperiodic design method I reduces the coupling length to approximately 274 \( \mu \)m, as illustrated in Fig. 9(b). Furthermore, the design with implementation constraints (method II) with a chosen resolution \( (l_{\text{min}} = 10 \) \( \mu \)m) as in Fig. 9(c) achieves a coupling length reduction in the same fashion as in the design without any constraint \( (l_{\text{min}} = dz = 0.2 \) \( \mu \)m) as shown in Fig. 9(b).

In the case of \( \lambda = 1.55 \) \( \mu \)m, the periodic design with \( \lambda = 150 \) \( \mu \)m yields the coupling length of 987 \( \mu \)m as shown in Fig. 10(a). Although the nonperiodic design method I can also accomplish a coupling length reduction to 850 \( \mu \)m as shown in Fig. 10(b), the best result is obtained by method III when \( \beta \) is chosen equal to 0.99. According to Fig. 10(c), the coupling length is 733 \( \mu \)m, which is reduced further to three fourths of the complete power coupling length in the periodic grating-assisted directional coupler.

The numerical results are summarized in Table 1. It can be observed that, when nonperiodic grating-assisted waveguides are used, the coupling length reduction can be achieved in all three different wavelengths, 0.8, 1.3, and 1.55 \( \mu \)m, which are generally used in optical communications and networking systems.

5. Conclusions

An iterative method incorporating the BPM for the design of a nonperiodic grating-assisted directional coupler was discussed. The directional coupler with nonperiodic grating-assisted structure can achieve complete power exchange in the coupling length that is significantly less than those of the periodic grating-assisted design in all three important wavelengths used in optical communications and networking systems.

Further research can be attempted to analyze and to understand how to estimate the best \( \beta \) for method III. In addition, more research can be performed to utilize more effective iterative optimization algorithms for further reductions in the coupling length.

References


