5-2011

Multi-Reference Methods for Nearfield Acoustical Holography

J. Stuart Bolton
Purdue University, bolton@purdue.edu

Yong-Joe Kim
Texas A&M University

Yaying Niu
Texas A&M University

Moohyung Lee
Purdue University

Follow this and additional works at: http://docs.lib.purdue.edu/herrick

http://docs.lib.purdue.edu/herrick/13

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact epubs@purdue.edu for additional information.
Multi-Reference Methods for Nearfield Acoustical Holography

Yong-Joe Kim and Yaying Niu
Department of Mechanical Engineering, Texas A&M University

Moohyung Lee
Samsung Electronic Co. Ltd.

J. Stuart Bolton
Ray W. Herrick Labs, Purdue University

161st Meeting of the Acoustical Society of America
May 24th 2011
Seattle, WA
NAH Measurements

**Snap-shot measurement**
- Non-stationary
- Large number of microphones

**Scanning measurement**
- Small number of scanning microphones
- Use of reference microphones to provide phase information
- Assumption: *stationary* sound field

Source non-stationarity compensation
Partial field decomposition

Uncorrelated or partially correlated sources

Decomposition

engine noise

exhaust noise
Partial field decomposition methods


**SVD method:** Orthogonal partial references.

**Partial Coherence (PC) method:** Incoherent, but not orthogonal partial references.

- **source 1**
- **source 2**
- **reference 1**
- **reference 2**

**field signal**

**decomposition of reference signals**

**decomposition of field signal into the set of partial fields**

**SVD method**

**Partial Coherence (PC) method**

1st partial reference

2nd partial reference
Partial field decomposition methods

\[ p = H_{pr} r \]

\( p \): hologram sound pressures
\( r \): reference microphone measured sound pressures

\[ S_{pr} = p r^H = H_{pr} S_{rr} \]

Cross-spectra between hologram and reference signals

Decompose reference cross-spectra \( S_{rr} \)

**SVD-based**

\[ S_{rr} = V \Sigma V^H \]

\( V \) is a unitary matrix

**PC-based**

\[ S_{rr} = LDL^H \]

\( L \) is the lower triangular matrix with diagonal entries equal to unity

\[ H_{pr} = S_{rp} V \Sigma^{-1} V^H \]

\[ H_{pr} = S_{rp} L D^{-1} L^{-1} \]

\[ S_{pp} = H_{pr} S_{rp} = H_{pr} S_{rr} H_{pr}^H \]

\[ S_{pp} = H_{pr} LD L^H H_{pr}^H \]

\[ S_{pp} = pp^H \]

Partial field:

\[ p = H_{pr} V \Sigma^{1/2} = S_{rp} V \Sigma^{-1/2} \]

\[ p = H_{pr} LD^{1/2} = S_{rp} L^H \Sigma^{-1/2} \]

Partial fields are different but the total fields are same if references sense all contributing sources.
Experimental design and selection of references

2 incoherent sources & 5 reference candidates

18 x 16 measurement points

Decomposed holograms (1200 Hz)

SVD method

PC method
Experimental setup of tire noise measurements

Tire driven by a roller
8 reference microphones
8 by 8 scanning mic array (10 cm spacing)
Hologram height: 6 cm

Aperture

16 Point Measurement with 5 cm Sampling Space

Tire driven by a roller
8 reference microphones
8 by 8 scanning mic array (10 cm spacing)
Hologram height: 6 cm

Aperture

16 Point Measurement with 5 cm Sampling Space

Smooth Roller Surface

32 Point Measurement with 5 cm Sampling Space
Virtual reference signals

Virtual reference procedure allows to decompose physically meaningful partial fields during post processing.

\[ Y = H_{yr} V \Lambda^{1/2} = H_{y1} \Lambda^{1/2} \]

**Y**: Partial field matrix on hologram surface

**\Lambda^{1/2}**: Principal reference signal matrix

**H_{y1}**: Transfer matrix

\[ Y' = H_{yy} Y = H_{yy} H_{yr} V \Lambda^{1/2} = H_{yy} H_{y1} \Lambda^{1/2} = H_{y1} \Lambda^{1/2} \]

**Y'**: Partial field matrix on reconstruction surface

**H_{yy}**: Transfer matrix (NAH projection procedure)

\[
X = \begin{bmatrix}
    c_1^T Y'_1 \\
    c_2^T Y'_2 \\
    \vdots \\
    c_K^T Y'_K
\end{bmatrix} = \begin{bmatrix}
    c_1^T H_{y1} \\
    c_2^T H_{y1} \\
    \vdots \\
    c_K^T H_{y1}
\end{bmatrix} \Lambda^{1/2} = H_{x1} \Lambda^{1/2}
\]

**X**: Virtual reference signal matrix

**c_m**: virtual reference selection vector on reconstruction surface

**Y'_m**: Partial field matrix for reconstruction surface

---

**References**

Virtual reference locations

- Location of a virtual reference in a 3-D space is determined by the position of the reconstruction surface in combination with the reference selection vector.
- MUSIC power is maximized at the optimal reference microphone location.

\[
P_{\text{MUSIC}} = \frac{1}{\mathbf{u}^H \mathbf{R}_{\text{noise}} \mathbf{u}}
\]

- \( P_{\text{MUSIC}} \): MUSIC power
- \( \mathbf{u} \): trial vector
- \( \mathbf{R}_{\text{noise}} = \sum_{n=K+1}^{N} \mathbf{w}_n \mathbf{w}_n^H \): Noise subspace
- \( \mathbf{w}_n \): Noise-related eigenvectors of \( \mathbf{S}_{yy'} (\mathbf{S}_{yy'})^H \)

\( K \) is the number of incoherent sources (i.e., rank of system).
MUSIC power for selection of virtual references (216 Hz)

maxima at $n = 15$ and $19$
Partial fields based on real and virtual references (216 Hz & 21 mph)

Based on real references

1st Partial Pressure Field

2nd Partial Pressure Field

Total Pressure Field

Based on virtual references

Allows successful separation of radiation from leading and trailing edges of contact patch.
Source non-stationarity compensation

- No realistic source is perfectly stationary.
- Assumptions for compensation
  - Slight source level changes do not affect sound radiation pattern
  - Acoustical transfer functions between reference and field microphones are only determined by their locations.
- Acoustical transfer function are invariant regardless of source level changes.
- Use of acoustic transfer functions between reference and field signals

\[
H_{pr} = S_{rp}^H S_{rr}^{-1}
\]

\[
S_{yy} = YY^H
\]

\[
Y = H_{yr,(step)} S_{rr,(avg)}^{1/2} = S_{ry,(step)}^H S_{rr,(step)}^{-1} S_{rr,(avg)}^{1/2}
\]

- Source non-stationarity causes scanned hologram to appear “patchy” and typically add spatial noise to the hologram image.

---

SVD-based partial field decomposition is applied.

**Without source non-stationarity compensation**

**With source non-stationarity compensation**

(a) First partial pressure field
(b) Second partial pressure field
Effect of the number of reference microphones (1)

- Cylindrical scanning surface around subsonic jet.
- 48 fixed reference microphones in 6 linear arrays.

Three array configurations
- Case 1: use 48 references
- Case 2: use 18 references (3 references from each array)
- Case 3: use 16 references (array #1 and #2)

Effect of the number of reference microphones (2)

The use of a large number of spatially-well-distributed references allows clean separation of signal and noise related singular values – The number of partial fields is minimized.
Effect of the number of reference microphones (3)

< Sum of the virtual coherence functions at 1 kHz >

- The 48-reference array used in this test was nearly optimal
Conclusions

- In scan-based, multi-reference NAH measurements, reference signals can be used to decompose a composite sound source into coherent partial fields. The partial fields can be physically meaningful or not depending on the decomposition methods.

- SVD-based and PC-based partial field decomposition methods were presented along with their applications to decompose the multi-component sound fields.

- The use of a source non-stationarity compensation procedure is implemented when measuring realistic, multi-component noise sources.

- The Virtual Reference Method can be used to decompose a total sound field into physically meaningful partial fields regardless of the locations of actual references during a measurement.

- The number of reference transducers should be much larger than the number of incoherent sound sources to model measurement noise components and to identify a minimum set of noise-suppressed partial fields.
References