Finite Element Models for Sound Transmission through Foam Wedges and Foam Layers Having Spatially Graded Properties

J. Stuart Bolton
*Purdue University*, bolton@purdue.edu

Yeon June Kang

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Finite Element Models for Sound Transmission through Foam Wedges and Foam Layers Having Spatially Graded Properties

J. Stuart Bolton and Yeon June Kang
1077 Ray W. Herrick Laboratories
School of Mechanical Engineering
Purdue University
W. Lafayette, IN 47907-1077

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ELASTIC POROUS MATERIALS

- Foam Material Properties
  - Flow resistivity
  - Tortuosity (Structure Factor)
  - Porosity
  - Bulk Modulus of Elasticity
  - Poisson's Ratio
  - Loss Factor

- Biot Theory allows wave propagation to be expressed in terms of these macroscopically measurable properties.

- Multiple Wave Types → sensitive to boundary conditions
INTRODUCTION

- Analytical Capabilities — available by J.S. Bolton and J.F. Allard et al.

- Practical Treatments

plane infinite layers (air + panel + porous layers)
FOAM FINITE ELEMENTS

Elastic Porous Material Theory based on Biot

Dynamic Relations

Stress-Strain Relations

Wave Equations

Weak Forms

Galerkin's Approximations

Foam Finite Elements
Finite Element Models/Foam Properties

- FEM Models
  (i) Foam
    3-noded linear triangular elements
  (ii) Structure
    2-noded hermite cubic elements
  (iii) Airspace
    3-noded linear triangular elements

- Foam Properties
  bulk density of solid phase: 30 kg/m$^3$
  bulk Young's modulus: $8 \times 10^5(1 + 0.265) \text{ Pa}$
  bulk Poisson's ratio: 0.4
  flow resistivity: 25,000 MKS Rayls/m
  tortuosity: 7.8
  porosity: 0.9
BOUNDARY CONDITIONS

- OPEN SURFACE

\[-hp = s\]
\[-(1-h)p = \sigma_x\]
\[\tau_{xy} = 0\]
\[v_x = j\omega(1-h)u_x + j\omega hU_x\]

- MEMBRANE-SEALED SURFACE

\[v_x = j\omega W\]
\[u_x = W\]
\[U_x = W\]
\[\pm p \pm (\sigma_x + s) = -\omega^2 m_s W\]
$h \rho n_a = s n_f$

$(1 - h) p n_{ax} = \sigma_x n_{fx} + \tau_{xy} n_{fy}$

$(1 - h) p n_{ay} = \tau_{xy} n_{fx} + \sigma_y n_{fy}$

$v = j \omega (1 - h) u + j \omega h U$
SHAPE OPTIMIZATION OF FOAM WEDGE

- **Objective** - maximize absorption offered by a wedge over a specified frequency range

- wedge defined by θ when volume and a is held constant

- given volume find optimum angle, θ
SHAPE OPTIMIZATION OF FOAM WEDGE

non-optimal wedge

\[ \theta = 132^\circ \]

| 1.2 cm | 4.8 cm |

optimal wedge

\[ \theta = 36^\circ \]

| 8.4 cm | 1.2 cm |

flat layer

(180°)

| 5.4 cm |

(a)

\[
\begin{align*}
\alpha & \quad \text{Frequency (Hz)} \\
0 & \quad 0 \\
0.2 & \quad 500 \\
0.4 & \quad 1000 \\
0.6 & \quad 1500 \\
0.8 & \quad 2000 \\
1.0 & \quad 2000 
\end{align*}
\]

- \[ \theta = 36^\circ \] (optimal wedge)
- \[ \theta = 132^\circ \]
- \[ \theta = 180^\circ \]

(averaged over 500-2000Hz)

\[
\begin{align*}
\alpha & \quad \text{angle (\theta)} \\
16 & \quad 0 \\
28 & \quad 0.2 \\
36 & \quad 0.4 \\
41 & \quad 0.6 \\
48 & \quad 0.8 \\
59 & \quad 1.0 \\
74 & \quad 1.0 \\
97 & \quad 1.0 \\
132 & \quad 1.0 \\
180 & \quad 1.0 
\end{align*}
\]
MOTIVATIONS AND OBJECTIVE

- Motivations
  - could optimize shape of a foam wedge by maximizing absorption offered by a wedge over a specified frequency range
  - found interesting and useful sound transmission characteristics of foam wedge in some high frequency bands
  - required larger treatment spaces if foam wedges are used

- Objective
  - to find a way of increasing sound transmission loss of a plane foam layer based on facts that were found from wedge studies
In system (b), tortuosity of a foam layer is varied spatially across the duct (in y-direction).
SOUND TRANSMISSION THROUGH A WEDGE

\[ d = 5.4 \text{ cm} \]

\[ Z_n = \rho_0 c \]

\[ u_0 e^{i\omega t} \]

\[ \theta = 180^\circ \]
\[ \theta = 132^\circ \]
\[ \theta = 97^\circ \]
\[ \theta = 74^\circ \]
\[ \theta = 59^\circ \]
\[ \theta = 48^\circ \]
\[ \theta = 41^\circ \]
\[ \theta = 36^\circ \]
\[ \theta = 28^\circ \]

Transmission Loss (dB)

Frequency (Hz)
PHASES OF THE X-DISPLACEMENTS
OF THE SOLID AND FLUID PHASES (f = 3800 Hz)

\[ \theta = 48^\circ \]

\[ \theta = 74^\circ \]
MAGNITUDE AND PHASE OF THE X-DISPLACEMENT OF THE FLUID PHASE AT THE REAR SURFACE OF THE WEDGE

$f = 2300$ Hz and $\theta = 48^\circ$
MAGNITUDE AND PHASE OF THE X-DISPLACEMENT OF THE FLUID PHASE AT THE REAR SURFACE OF THE WEDGE

\[ f = 3800 \text{ Hz and } \theta = 48^\circ \]
SOUND TRANSMISSION THROUGH A WEDGE

\[ d = 5.4 \text{ cm} \]

\[ Z_n = \rho_0 c \]

---

Transmission Loss (dB)

<table>
<thead>
<tr>
<th>Angle (°)</th>
<th>Transmission Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>180°</td>
<td></td>
</tr>
<tr>
<td>132°</td>
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<td>97°</td>
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<td>36°</td>
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<tr>
<td>28°</td>
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</tr>
</tbody>
</table>

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Frequency (Hz)

---

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HERRICK LABORATORIES
SOUND TRANSMISSION THROUGH A FOAM LAYER HAVING SPATIALLY GRADED TORTUOSITY

\[ Z_n = \rho_0 c \]

\[ d = 5.4 \text{ cm} \]

\[ u_0 e^{i \omega t} \]

\[ d \]

\[ \text{Transmission Loss (dB)} \]

<table>
<thead>
<tr>
<th>Case</th>
<th>Initial Thickness (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 0</td>
<td>7.8 - 7.8</td>
</tr>
<tr>
<td>Case 1</td>
<td>7.8 - 7.0</td>
</tr>
<tr>
<td>Case 2</td>
<td>7.8 - 6.0</td>
</tr>
<tr>
<td>Case 3</td>
<td>7.8 - 5.0</td>
</tr>
<tr>
<td>Case 4</td>
<td>7.8 - 4.0</td>
</tr>
<tr>
<td>Case 5</td>
<td>7.8 - 3.0</td>
</tr>
<tr>
<td>Case 6</td>
<td>7.8 - 2.0</td>
</tr>
<tr>
<td>Case 7</td>
<td>7.8 - 1.0</td>
</tr>
</tbody>
</table>

\[ 10 \text{^2} \quad 10^3 \text{ (Hz)} \]
CONCLUSIONS

- It has been found that the transmission loss of the wedge is significantly higher than that of a plane foam layer of the same volume in some high frequency bands.

- The TL appears to be enhanced by "converting" the incident plane wave into a non-radiating higher-order symmetric mode.

- The same increase in TL can be produced using a plane, constant depth foam layer if tortuosity is varied across the width of the foam layer.