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Finite Element Models for Sound Transmission through Foam Wedges and Foam Layers Having Spatially Graded Properties

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Finite Element Models for Sound Transmission through Foam Wedges and Foam Layers Having Spatially Graded Properties

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ELASTIC POROUS MATERIALS

- **Foam Material Properties**
  - Flow resistivity
  - Tortuosity (Structure Factor)
  - Porosity
  - Bulk Modulus of Elasticity
  - Poisson's Ratio
  - Loss Factor

- Biot Theory allows wave propagation to be expressed in terms of these macroscopically measurable properties.

- Multiple Wave Types → sensitive to boundary conditions
INTRODUCTION

- Analytical Capabilities — available by J.S. Bolton and J.F. Allard et al.

- Practical Treatments

plane infinite layers
(air + panel + porous layers)
FOAM FINITE ELEMENTS

Elastic Porous Material Theory based on Biot

Dynamic Relations

Stress-Strain Relations

Wave Equations

Weak Forms

Galerkin's Approximations

Foam Finite Elements
Finite Element Models/Foam Properties

- FEM Models
  (i) Foam
    3-noded linear triangular elements
  (ii) Structure
    2-noded hermite cubic elements
  (iii) Airspace
    3-noded linear triangular elements

- Foam Properties
  bulk density of solid phase: 30 kg/m$^3$
  bulk Young's modulus: $8 \times 10^5(1 + 0.265)$ Pa
  bulk Poisson's ratio: 0.4
  flow resistivity: 25,000 MKS Rayls/m
  tortuosity: 7.8
  porosity: 0.9
**BOUNDARY CONDITIONS**

- **OPEN SURFACE**
  
  \[-hp = s\]
  \[-(1-h)p = \sigma_x\]
  \[\tau_{xy} = 0\]
  \[v_x = j\omega(1-h)u_x + j\omega h U_x\]

- **MEMBRANE-SEALED SURFACE**
  
  \[v_x = j\omega W\]
  \[u_x = W\]
  \[U_x = W\]
  \[\pm p \pm (\sigma_x + s) = -\omega^2 m_s W\]
BOUNDARY CONDITIONS — OPEN SURFACE

\[ h p n_a = s n_f \]

\[ (1 - h) p n_{ax} = \sigma_x n_f + \tau_{xy} n_{fy} \]

\[ (1 - h) p n_{ay} = \tau_{xy} n_f + \sigma_y n_{fy} \]

\[ \mathbf{v} = j \omega (1 - h) \mathbf{u} + j \omega h \mathbf{U} \]
SHAPE OPTIMIZATION OF FOAM WEDGE

- Objective – maximize absorption offered by a wedge over a specified frequency range

- wedge defined by $\theta$ when volume and $a$ is held constant
- given volume find optimum angle, $\theta$
SHAPE OPTIMIZATION OF FOAM WEDGE

non-optimal wedge

optimal wedge

flat layer (180°)

1.2 cm  4.8 cm

5.4 cm

1.2 cm  8.4 cm

36°

132°

5.4 cm

(a)

\[ a \]

\[ \theta = 36° \ (\text{optimal wedge}) \]

\[ \theta = 132° \]

\[ \theta = 180° \]

Frequency (Hz)

angle (\( \theta \))

(averaged over 500-2000Hz)
MOTIVATIONS AND OBJECTIVE

- Motivations
  - could optimize shape of a foam wedge by maximizing absorption offered by a wedge over a specified frequency range
  - found interesting and useful sound transmission characteristics of foam wedge in some high frequency bands
  - required larger treatment spaces if foam wedges are used

- Objective
  - to find a way of increasing sound transmission loss of a plane foam layer based on facts that were found from wedge studies
In system (b), tortuosity of a foam layer is varied spatially across the duct (in y-direction).
SOUND TRANSMISSION THROUGH A WEDGE

\[ u_o e^{j\omega t} \]

\[ y \]
\[ x \]

\[ \text{constrained edges} \]

\[ \text{air} \]
\[ \theta \]
\[ \text{foam} \]
\[ \text{air} \]

\[ d \]
\[ d = 5.4 \text{ cm} \]

\[ Z_n = \rho_o c \]

\[ \theta = 180^\circ \]
\[ \theta = 132^\circ \]
\[ \theta = 97^\circ \]
\[ \theta = 74^\circ \]
\[ \theta = 59^\circ \]
\[ \theta = 48^\circ \]
\[ \theta = 41^\circ \]
\[ \theta = 36^\circ \]
\[ \theta = 28^\circ \]

Transmission Loss (dB)

\[ 0 \]
\[ 10 \]
\[ 20 \]
\[ 30 \]
\[ 40 \]

Frequency (Hz)

\[ 10^2 \]
\[ 10^3 \]
SOUND TRANSMISSION THROUGH A WEDGE

\[ u_0 e^{j\omega t} \]

\[ y \]

\[ x \]

\[ \text{constrained edges} \]

\[ \text{air} \]

\[ \text{foam} \]

\[ \text{air} \]

\[ Z_n = \rho_0 c \]

\[ d = 5.4 \text{ cm} \]

\[ \theta = 74^\circ \]

\[ \theta = 48^\circ \]

Transmission Loss (dB)

\[ 40 \]

\[ 30 \]

\[ 20 \]

\[ 10 \]

\[ 0 \]

\[ 10^2 \]

\[ 10^3 \]

Frequency (Hz)

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PHASES OF THE X-DISPLACEMENTS OF THE SOLID AND FLUID PHASES (f = 3800 Hz)

\[ \theta = 48^\circ \] \hspace{2cm} \[ \theta = 74^\circ \]
MAGNITUDE AND PHASE OF THE X-DISPLACEMENT OF THE FLUID PHASE AT THE REAR SURFACE OF THE WEDGE

\[ f = 2300 \text{ Hz} \text{ and } \theta = 48^\circ \]

\[
\begin{array}{c}
\text{magnitude (x } 10^{-5} \text{ m)}} \\
\text{y-coordinate (cm)} \\
\end{array}
\]

\[
\begin{array}{c}
\text{phase (degree)} \\
y-coordinate (cm) \\
\end{array}
\]
MAGNITUDE AND PHASE OF THE X-DISPLACEMENT OF THE FLUID PHASE AT THE REAR SURFACE OF THE WEDGE

\( f = 3800 \text{ Hz and } \theta = 48^\circ \)

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**Graphs:**

1. **Magnitude vs. Phase:**
   - Y-coordinate (cm) vs. Magnitude (x 10^{-5} m)
   - Phase (degree) vs. Y-coordinate (cm)

   - **Plot 1:** Magnitude vs. Phase
     - Magnitude increases with phase.
     - Peak magnitude at approximately 5.4 x 10^{-5} m.
   - **Plot 2:** Y-coordinate vs. Phase
     - Y-coordinate varies with phase, showing a non-linear relationship.

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**References:**

- **Institution:** Purdue University
- **Laboratory:** Herrick Laboratories
SOUND TRANSMISSION THROUGH A WEDGE

\[ d = 5.4 \text{ cm} \]

\[ Z_n = \rho_0 c \]

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[Graph showing transmission loss vs. frequency for different angles.]
SOUND TRANSMISSION THROUGH A FOAM LAYER HAVING SPATIALLY GRADED TORTUOSITY

\[ Z_n = \rho_0 c \]

\[ d = 5.4 \text{ cm} \]

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transmission loss graph

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CONCLUSIONS

- It has been found that the transmission loss of the wedge is significantly higher than that of a plane foam layer of the same volume in some high frequency bands.

- The TL appears to be enhanced by "converting" the incident plane wave into a non-radiating higher-order symmetric mode.

- The same increase in TL can be produced using a plane, constant depth foam layer if tortuosity is varied across the width of the foam layer.