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Increased Risk of Children For Subtle Closed Head Injury From Soccer Heading

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Key words: acceleration, brain, concussion, football, trauma

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ABSTRACT

Controversy surrounds the risk of heading a soccer ball. The research objectives were to determine the statistical distributions of head accelerations during soccer heading for adult versus youth players, to compare results with known safe levels, and to suggest steps to reduce risk for children. Monte Carlo simulations of large numbers of head-ball impacts were done using equations of motion based upon Newton’s second law. Results were compared with presumed safe accelerations. Calculated head accelerations experienced by youth and adult players vary greatly, depending on ball speed, player weight, and other factors. Both mean and maximal accelerations experienced by youth players under realistic game conditions are greater than those experienced by adults. A lighter weight, lower pressure youth soccer ball can reduce impact intensity experienced by youth players to the safer levels experienced by adults. Pediatricians can be helpful advocates for reform in this arena.

Key words: acceleration, brain, concussion, football, trauma

________________________________________________________________________

INTRODUCTION

The harmful effects of cumulative subtle injury to the body are increasingly recognized in medicine. Familiar examples include skin cancer caused by sun exposure, auditory nerve damage or deafness caused by loud noises, and brain damage caused by alcohol ingestion1. In the particular case of mechanical injury to the brain, repeated concussions or sub-concussive blows are now well documented to produce and lasting brain damage2,3.
Soccer is the only sport in which a ball is purposefully re-directed with the head—a play that is termed “heading” the ball. There has been recent expression of concern about the safety of head-ball contact in the sport of soccer. A series of studies reported from Norway has suggested that brain atrophy upon computed tomographic scanning, electroencephalographic changes, and subtle deficits in memory, concentration, planning, and alertness occur in long-time soccer players that can be related to the number of balls headed. Soccer players may also suffer repeated mild concussions from collisions with other players, from high kicks, or from falling to the ground, which might possibly explain brain abnormalities in the absence of injurious effects of heading the ball. Interestingly, both Barnes and Boden studied the incidence of concussions in players from contact with objects other than the ball, such as the ground, a goal post, or another player. Both studies found the incidence of one such non-ball-related concussions to be 1 per 20 years of active playing. This incidence is much too low to explain putative brain injury in typical soccer players, especially when two thirds of such concussions were mild, without amnesia. Hence one must seriously consider head-ball contact as a possible cause, either in heading accidents, in which balls strike players unawares, or in normal heading that occurs in the routine course of a playing career.

Matser and coworkers found that Dutch amateur and professional soccer players performed significantly worse in tests of memory and planning than did control runners and swimmers. A study of high level youth players in the United States found 40 percent of those who headed the ball frequently had impaired IQ scores. In contrast, a well-conducted study of U.S. National team players with a long history of heading found no evidence of brain injury compared with track athletes unexposed to heading. Neropsychological testing of elite college players immediately after a training session involving intense heading did not reveal significant effects. The issue thus remains controversial and of substantial concern to parents of younger players who take up the sport early and may quickly advance to higher competitive levels where heading is an encouraged and expected part of the game. These same players will be exposed to a lifetime of play and could be at risk of cumulative acceleration-related brain injury. The Committee on Sports Medicine and Fitness of The American Academy of Pediatrics has recommended further study before a conclusion can be made about the safety of heading by young soccer players.

One major determinant of head acceleration is the ratio of the mass of the soccer ball to the effective mass of the player. This fact suggests that children may be at increased risk of acceleration injury to the brain unless the mass of the soccer ball is properly adjusted to account for the reduced size of youth players. Although the mass of the current size 4 youth soccer ball is less than the adult size 5 ball, the reduction in mass may not be sufficient to balance differences in body mass between youth and adult players. Other

*Combining data for men and women, Boden found the probability of concussion to be 0.0005 per exposure (i.e. a game or practice). At 100 games or practices per year, this is a concussion rate of 0.5 concussions per 10 years or one concussion per 20 years. Barnes found that the odds are 50 percent that a male player will sustain a concussion within a 10-year period, which also gives an expected value of 1 concussion per 20 years.
factors that determine head acceleration during impact include ball speed and player technique. Both of these factors are statistically weighted against smaller, younger players. Measurements of ball speeds at head height in youth soccer games are paradoxically somewhat greater than ball speeds at head height in adult games (Figure 1). The differences are related to the greater effect of air resistance in slowing larger size 5 balls used by adults. Moreover, proper heading technique is less well developed in youth players, who tend to muster a lower fraction of total body mass behind the ball in heading than do adults. These secondary factors may also conspire to make heading relatively more dangerous for youth players than for adults.

The controversy regarding risk/safety is compounded by the statistical or stochastic nature of the problem. There is a very wide range of body sizes in children who play soccer, as well as a wide range of technical abilities of younger players. There is also a wide range of ball speeds in youth soccer games and practices. Accordingly a stochastic approach to research on this problem is needed. Even if only a small percentage of headers are dangerous (a likely situation), their cumulative effect could be clinically significant over a playing career lasting from age 6 to adulthood. Even if only a small percentage of children are affected, the public health impact in terms of subtle learning disabilities and behavioral problems could be substantial, given the estimated 200 million active soccer players worldwide, including 12 to 18 million players in the United States.

Accordingly, the present research was conducted (1) to estimate the statistical distributions of head acceleration in soccer heading for adult versus youth players under realistic game conditions, (2) to compare the results with reasonable safe levels, and (3) to explore improved designs of youth soccer balls that could make the sport of soccer more brain-friendly for younger players.

METHODS

Approach

The equations of motion in the horizontal dimension for both the head and the soccer ball during heading have been previously described. It is the horizontal dimension that is most relevant, because forces are predominantly in this dimension and because the neck, spine, body mass, and contact with the ground normally attenuate acceleration of the head in the vertical dimension. (For occasional “diving headers” the acceleration is also horizontal.) Hence to simplify the analysis, a one-dimensional approach may be used. Also for simplicity, consideration is limited to straight-on, frontal impacts. Twisting headers and backward headers are not formally considered.

Here Monte Carlo methods are used to calculate the statistical distribution of head accelerations in large populations of children and adults who head a variety of properly inflated, under inflated, and over inflated balls of varying speeds in a large number of games and practices, using a range of good to poor technique, which results in more or
less effective mass behind the ball. The idea of a Monte Carlo simulation is to use an
ordinary personal computer to compute the magnitude and duration of head acceleration
for many thousands of impacts. The relevant independent variables are selected at
random from realistic sampling distributions representing the expected variability in body
mass, ball inflation pressure, ball speed, etc. These are assumed to be independent for the
purpose of the simulation. The resulting distributions of head acceleration for youth and
adult players can be compared with each other and also with presumed safe accelerations
that occur during running, dancing, or head nodding, and impulses associated with non-
concussive falls and measured head accelerations in the sport of boxing (ref Adams and
boxing).

**Physics of head-ball impact**

Relevant variables of the problem are defined as shown in Table 1. The magnitude of the
mean value of head acceleration during the interval of head-ball contact is given to good
approximation (on the order of 1 percent) by the expression

\[
\bar{X} \approx \frac{2}{\pi} \cdot \frac{\Delta v_0}{m_1} \cdot \sqrt{CPm_2},
\]

as shown in Appendix 1.

**Table 1. Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Ball circumference</td>
<td>meters</td>
</tr>
<tr>
<td>(m_1)</td>
<td>Effective mass of player</td>
<td>kg</td>
</tr>
<tr>
<td>(m_2)</td>
<td>Mass of ball</td>
<td>kg</td>
</tr>
<tr>
<td>P</td>
<td>Ball inflation pressure</td>
<td>atmospheres</td>
</tr>
<tr>
<td>t</td>
<td>Time after the onset of head-ball contact</td>
<td>sec</td>
</tr>
<tr>
<td>(x)</td>
<td>Distance moved by the head</td>
<td>meters</td>
</tr>
<tr>
<td>(\dot{x})</td>
<td>Velocity of the head in the horizontal dimension</td>
<td>m/sec</td>
</tr>
<tr>
<td>(\ddot{x})</td>
<td>Acceleration of the head in the horizontal dimension</td>
<td>m/sec²</td>
</tr>
<tr>
<td>(\Delta v_0)</td>
<td>Horizontal ball speed with respect to the head at t=0</td>
<td>m/sec</td>
</tr>
</tbody>
</table>

The term \(\Delta v_0\) in Eq. (1) is the speed of the ball with respect to the player before impact.
For example, if the ball is traveling at 10 m/sec in one direction and the player is
traveling at 1 m/sec in the opposite direction, the effect is the same as if the player is
standing still and the ball is traveling at 11 m/sec. Statistical distributions of ball speeds
for actual competitive games of players aged 9-years-old to adult have been reported
previously\(^{22}\). From these distributions of measured ball speeds the term \(\Delta v_0\) were
estimated for an adult player is running forward at a speed of 1 m/sec and a child player is running forward at 0.5 m/sec.

In term $m_1$ in Eq. (1) is the “effective mass” of the player mass that opposes horizontal acceleration of the head in the x-direction. The effective can be defined as $m_1 = F_x/a_x$, the ratio of horizontal force to horizontal acceleration. For a player using ideal technique with neck strong and stiff, the effective mass is approximately one half the total body mass\(^{22}\). For players using extremely poor technique, in which the head is allowed to wobble, the effective mass is approximately that of the head. Thus a range of heading skill or technique from most safe to least safe can be characterized by a distribution of effective mass values ranging from one half the body mass to merely head mass (a minimum value of 3 kg in the present simulations).

The term $C$ in Eq. (1) $C$ is the circumference of the ball, and the term $P$ is the inflation pressure of the ball. The term $m_2$ is the mass of the ball. When computing the effects of modified ball designs having reduced size, mass, or inflation pressure, the effects of air resistance on the final speeds of larger, lighter weight balls were accounted for as described in Appendix 2. For the purposes of Monte Carlo simulations the small correction for subtle effects of air resistance can be described using the expression

$$\frac{v_{h2}}{v_{h1}} \approx \frac{C_1^2}{C_2^2} \cdot \frac{m_2}{m_1}.$$  \hspace{1cm} (2)

to predict the velocities of a modified ball design (subscript 2), compared with the standard ball design (subscript 1), kicked with the same initial speed in the presence of air resistance.

**Monte Carlo Methods**

To determine the range of head accelerations experienced by adult players using a Size 5 (adult) ball and for youth players using a Size 4 (youth) ball, the author performed Monte Carlo simulations of normal heading. The effective masses of players and ball velocities were selected at random from realistic sampling distributions for normal game conditions, as shown in Table 2. The horizontal ball velocity for adult players was selected at random from a Gaussian distribution approximating the observed distribution of horizontal ball velocities in adults (Figure 1).
Table 2. Statistical models of adult and youth players

<table>
<thead>
<tr>
<th>Parameter and Units</th>
<th>Adult value ± SD</th>
<th>Youth value ± SD</th>
<th>Youth value ± SD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Adult value ± SD</td>
<td>Youth value ± SD</td>
<td>Youth value ± SD</td>
</tr>
<tr>
<td>Size 5 ball</td>
<td>Size 4 ball</td>
<td>Modified Size 4 ball</td>
<td></td>
</tr>
<tr>
<td>Effective mass of player (kg)</td>
<td>27 ± 8</td>
<td>15.4 ± 4.6</td>
<td>15.4 ± 4.6</td>
</tr>
<tr>
<td>Ball speed (m/sec) With respect to player</td>
<td>6.7 ± 2.2</td>
<td>7.6 ± 2.2</td>
<td>7.6 ± 2.2</td>
</tr>
<tr>
<td>Ball mass (kg)</td>
<td>0.42 ± 0.05</td>
<td>0.36 ± 0.04</td>
<td>0.30 ± 0.03</td>
</tr>
<tr>
<td>Ball circumference (m)</td>
<td>0.7 ± 0.05</td>
<td>0.65 ± 0.04</td>
<td>0.65 ± 0.04</td>
</tr>
<tr>
<td>Ball pressure (atm)</td>
<td>8.5 ± 0.25</td>
<td>8.5 ± 0.25</td>
<td>0.3 ± 0.1</td>
</tr>
</tbody>
</table>

Figure 1. Distributions of measured horizontal ball speeds for falling balls at head height obtained from time and distance data in competitive games as reported in reference (24). The adult distribution represents 104 observations. The youth distribution represents 205 observations. The vertical axis is normalized as probability density, so that the area under each curve is 1.0. Values for youth players tend to be slightly greater than for adults, owing to reduced effects of air resistance for shorter times of flight. The mean value for adults is 5.7 m/sec, and the mean value for youth players is 7.1 m/sec.
The simulations for youth player were different. The effective body mass of youth players was selected at random from a normal (Gaussian) distribution with values 40/70 times those used for adult players, corresponding to a mean youth player weight of 40 kg compared to a 70 kg adult. The horizontal ball velocity was selected at random from a Gaussian distribution with mean of 7.1 m/sec and standard deviation 2.2 m/sec, which represents closely the observed distribution of horizontal ball velocities for youth players in Figure 1. In addition, weight and circumference data for a smaller, Size 4 soccer ball were used in computing head accelerations for youth players.

Simulations were performed for a realistic wide range of ball inflation pressures ranging from 0.35 to 1.35 atmospheres, centered on the midpoint of the legal range of inflation pressures (0.6 to 1.1 atmospheres). For each impact the mean head acceleration during impact, $\ddot{x}$, was computed using Eq. (1) and, for the modified ball designs, for which ball velocities were adjusted using Eq. (2). In this way the wide variability of head acceleration experienced during heading under normal playing conditions could be illustrated.

Table 3. Maximal head accelerations during safe reference activities*

<table>
<thead>
<tr>
<th>Activity</th>
<th>Frequency (Hz)</th>
<th>1/2 max excursion, A (meters)</th>
<th>Peak Acceleration (m/sec²)</th>
<th>Duration (sec)</th>
<th>Impulse (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head nodding</td>
<td>3.33</td>
<td>0.03</td>
<td>13.1</td>
<td>0.15</td>
<td>2.0</td>
</tr>
<tr>
<td>at three frequencies</td>
<td>1.00</td>
<td>0.20</td>
<td>7.9</td>
<td>0.50</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>2.30</td>
<td>0.08</td>
<td>16.7</td>
<td>0.22</td>
<td>3.7</td>
</tr>
<tr>
<td>Jumping</td>
<td>1.60</td>
<td>0.14</td>
<td>14.1</td>
<td>0.31</td>
<td>4.4</td>
</tr>
<tr>
<td>Rocking</td>
<td>1.42</td>
<td>0.17</td>
<td>13.5</td>
<td>0.35</td>
<td>4.7</td>
</tr>
<tr>
<td>Mean values</td>
<td>1.93</td>
<td>0.12</td>
<td>13.0</td>
<td>0.31</td>
<td>4.0</td>
</tr>
</tbody>
</table>

* For motion $x=\text{A}\sin(\omega t)$, $dx/dt = \omega \cos(\omega t)$, and $d^2x/dt^2 = -\omega^2 \sin(\omega t)$, and Maximum acceleration is $-\omega^2 A$.  


Reference values of safe head acceleration

Reference values for clearly safe head accelerations were computed for body motions of everyday life including repeated jumping, head nodding, and side-to-side bending at the waist (Table 3). Repetitive motions were taken as sinusoidal in form with displacement, \( x = A \sin(\omega t) \), where \( A \) represents one half the peak-to-peak amplitude of head motion, and angular frequency, \( \omega = 2\pi \) times the frequency of oscillation in cycles per second. From this expression the velocity and acceleration of the head are \( \frac{dx}{dt} = \omega A \cos(\omega t) \) and \( \frac{d^2x}{dt^2} = -\omega^2 A \sin(\omega t) \), from which the maximal acceleration in either direction is \( \omega^2 A \). Values of \( \omega^2 A \) so calculated were interpreted as normal, non-worrisome accelerations that the human body has evolved to withstand without ill effects. They are similar to head accelerations during normal childhood play and are unlikely to be of concern to health professionals or parents.

RESULTS

Mean head acceleration

Figure 2 shows the statistical distributions of head accelerations for simulations of ten thousand headers for three different statistical models. The first curve (a) represents the model of adult players who are heading standard size 5 adult soccer balls. The second curve (b) represents the model of youth players who are heading conventional size 4 youth soccer balls. The third curve (c) represents a model of young players who are heading modified size 4 balls with reduced mass and reduced inflation pressure.
Figure 2. Sampling distributions for head acceleration in models of adult and youth soccer heading. Parameters for the Monte Carlo simulation of 10000 headers are provided in Table 2. The vertical axis is normalized in terms of probability density, so that the area under the curve equals 1.0. (a) Adult players using standard size 5 adult balls. (b) Youth players using current size 4 youth balls. (c) Youth players using a modified, lighter weight, lower pressure size 4 ball. Head accelerations experienced by children are greater than those experienced by adults. The conventional youth ball does not make up for the difference in size and skill of young players. A modified size 4 ball can make heading as safe for children as it is for adults.

The horizontal scale in Figure 2 indicates the mean value of head acceleration over the duration of head ball contact. For the adult model these acceleration values range from 1 to 100 m/sec². Nearly all values are less than 100 m/sec² or roughly 10 g’s. The distribution of accelerations for youth players is shifted to the right compared with the distribution for adult players. The average head acceleration for the youth distribution is over twice that for the adult distribution, despite the use of the smaller size 4 ball in the youth model. These differences reflect the somewhat higher ball speeds at head height and the smaller effective masses of youth players.

The most intense impacts occur when faster than average balls strike the heads of young, lighter weight players using extremely bad technique. In real life these circumstances prevail when a player is hit unaware, or blind-sided, and has too little time to react properly to the ball. Such conditions can also arise when a player has not been taught or does not practice proper technique. Such situations are more likely for younger, inexperienced players than for older, experienced players. Hence the theoretical
distribution (b) in Figure 2 may underestimate the probability of more damaging impacts producing larger head accelerations.

The modified ball described by curve (c) in Figure 2 has the same circumference as a normal Size 4 youth ball, but 20 percent less mass. The lighter weight ball is slowed more by air resistance according to Eq. (2). As a result, the average speed of “headable” balls in youth games is reduced from 7.1 m/sec to 5.9 m/sec, similar to that of such balls in adult games (5.7 m/sec). In addition, the modified ball design has less than half the inflation pressure of the standard ball model on average. These changes in the size 4 ball specifications make the distribution of youth head accelerations similar to that for adults during soccer heading.

**Acceleration-time product**

The impulse or acceleration-time product, which is equal to the change in velocity of the head as a result of impact, is a descriptor that takes into account both the magnitude and the duration of head acceleration. The impulse is well correlated with the degree of injury for a wide variety of blows to the head\textsuperscript{25-27}. Figure 3 shows distributions of the change in whole head velocity during head-ball impact for 10000 simulated head-ball impacts in the youth and adult models of soccer heading. For the adult model most values of the impulse range from near zero to 0.5 m/sec. This range is well within the safe reference range of 0 to 4 m/sec associated with ordinary jumping, dancing, or head nodding (Table 3). According to this analysis soccer heading for adults is on average quite safe and rarely worrisome. With the conventional size 4 youth ball, a few headers by young players change head velocity by greater than 1.0 m/sec. The range of impulse for youth players is twice that for adult players. With the modified youth ball the impulse distribution during game-like impacts more closely approximates that for adults. Taking the results in Figures 2 and 3 together, the proposed modifications of the Size 4 youth soccer ball would essentially eliminate the increased risk of heading for children compared to adults.
Figure 3. Sampling distributions for whole head velocity change (acceleration x time product or impulse) in models of adult and youth soccer heading. Other details similar to Figure 2.

DISCUSSION

The specific practice of heading the ball in youth soccer deserves scrutiny, owing to the potential for subtle brain injury. The physics of soccer heading are usually safe for adults. However, compared to adults, boys and girls are at increased risk of subtle brain injury from soccer heading because of their small size and less experienced technique. These necessary risks are compounded by the design of current youth soccer balls, which are rather too heavy, too fast, and often over-inflated.

It is interesting to compare the calculated head impulses in Figure 3 with previously published benchmarks describing safe vs. injurious blows to the head. Adams, for example, noted that severe diffuse axonal injury in humans happening as a result of falls occurs only in persons who fall from substantially greater than their own height—for example, from a ladder, bridge, elevator shaft, or even a mountain! We expect from theory that a given acceleration-time product, creating a given change in whole head velocity, \( \Delta v \), will produce a given maximal strain in the brain. In this case we can estimate \( \Delta v \) as the velocity of a falling body at the Earth's surface from a particular height, \( h \), which is \( \Delta v = \sqrt{2gh} \). For a grown man standing approximately 2 meters high this works out to be about 6 m/sec. So one could estimate a maximal safe impulse to be 6 m/sec. In another study of safe accelerations reviewed by Marguiless and Thibault, peak rotational acceleration and angular velocity following sub-concussive blows to the heads of volunteer boxers were recorded with specially instrumented helmets. These blows produced changes in rotational velocity of the head of 25 rad/sec. For an effective
radius of the neck of about 0.2 meters/radian in humans, the linear $\Delta v$ is $0.2 \times 25 = 5$ m/sec, essentially the same value as for falls above. These values are also in agreement with the average impulse associated with playful activities in Table 3, namely $13 \, \text{m/sec}^2 \times 0.31 \, \text{sec} = 4 \, \text{m/sec}$.

If we take $\Delta v = 4 \, \text{m/sec}$ as an upper limit for safe non-penetrating blows to the head, then the distributions for soccer heading, falling in the range of 0 to 1 m/sec are not particularly alarming. Whether 25 percent of the maximal safe level of head acceleration is acceptable will no doubt remain the subject of debate. However, the sport of soccer should be at least as safe for children as it is for adults. Most parents would probably not be comfortable with their children receiving blows equivalent to boxing or falling each time they head a soccer ball. Hence some measures to limit impulses to the head in the sport of soccer would seem advisable.

One easy, but Draconian solution is simply to ban heading from the sport of soccer for players under a certain size and weight. However this step would delay in the acquisition of good technical heading skills, which can protect against brain injury later on. It would also meet with strong resistance from devotees of the game. An easier way to proceed is through the evolution of safer equipment, namely softer, lighter weight balls, for pre-adolescent players. Water resistant coatings are also needed to keep balls from absorbing water and gaining weight when wet. The simple expedient of encouraging the used of lighter weight, reduced pressure, water resistant youth soccer balls can eliminate an inadvertent bias against children, while maintaining the health benefits of athletic participation. With time and encouragement such balls will become commercially available. Pediatricians have been urged to work with other members of the broader community to make the sport of soccer safer for young people\textsuperscript{21}. Pediatricians would do well to advocate the use of such brain-friendly balls in youth soccer.

**ACKNOWLEDGMENTS**

The authors wishes to thank summer research student Ms. Amy Allegretti and Dr. Elizabeth Basquin-Krause for their help and encouragement in this project.
REFERENCES

Appendix 1: mean head acceleration for Monte Carlo simulations

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units or value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a, b, c</td>
<td>Defined constants</td>
<td>various</td>
</tr>
<tr>
<td>C</td>
<td>Ball circumference</td>
<td>meters</td>
</tr>
<tr>
<td>e</td>
<td>Base of natural logarithms</td>
<td>2.718</td>
</tr>
<tr>
<td>$m_1$</td>
<td>Effective mass of player</td>
<td>kg</td>
</tr>
<tr>
<td>$m_2$</td>
<td>Mass of ball</td>
<td>kg</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Damping coefficient for energy absorption by ball</td>
<td>Nt/m/sec</td>
</tr>
<tr>
<td>$P$</td>
<td>Pressure inside ball</td>
<td>atmospheres</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Circle ratio</td>
<td>3.14159</td>
</tr>
<tr>
<td>$C$</td>
<td>Circumference of soccer ball</td>
<td>meters</td>
</tr>
<tr>
<td>$t$</td>
<td>Time from onset of head-ball impact</td>
<td>sec</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>Total duration of head-ball contact</td>
<td>sec</td>
</tr>
<tr>
<td>$\Delta v$</td>
<td>Velocity of ball with respect to head</td>
<td>m/sec</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular frequency</td>
<td>radians/sec</td>
</tr>
<tr>
<td>$\dot{x}$</td>
<td>Velocity of player’s head</td>
<td>m/sec</td>
</tr>
<tr>
<td>$\ddot{x}$</td>
<td>Acceleration of player’s head</td>
<td>m/sec$^2$</td>
</tr>
</tbody>
</table>

Subscripts:

0  Initial value at t=0
1  Heavier mass (player)
2  Lighter mass (ball)

Let $\dot{x}(t)$ be the instantaneous, horizontal head velocity as a function of time, $t$ after head-ball contact. As has been previously shown in detail$^{22}$, $\dot{x}(t)$ may be specified in terms of player and ball parameters as

$$\dot{x}(t) = a\omega \cos(\omega t)e^{-bt} - a\sin(\omega t)e^{-bt} + c,$$  \hspace{1cm} (A1.1)

where

$$a = \frac{\Delta v m_2}{\omega (m_1 + m_2)},$$  \hspace{1cm} (A1.1 a)
\[ b = \mu \frac{m_1 + m_2}{2m_1m_2}, \quad (A1.1 \text{ b}) \]

\[ \omega = \sqrt{CP \cdot \frac{m_1 + m_2}{m_1m_2} - \left( \frac{1}{2} \mu \frac{(m_1 + m_2)}{m_1m_2} \right)^2} \approx \sqrt{CP \cdot \frac{m_1 + m_2}{m_1m_2}}. \quad (A1.1 \text{ c}) \]

In (A1.1 c) the term \( \mu \) is the damping coefficient for the ball, which is determined by the properties of the cover of the ball and has an experimentally measured value of \( 13 \pm 2 \) Nt/(m/sec) for typical commercial soccer balls\(^{22}\). For these realistic values of \( \mu \), which are numerically much smaller than \( CP \) for soccer balls (~3\%), the damping term of (A1.1 c) is negligible.

Of interest from the point of view of head injury is the mean or average value of head acceleration over the duration of head-ball contact, \( \Delta t \). The mean head acceleration is

\[
\bar{\omega} = \frac{1}{\Delta t} \int_0^{\Delta t} \dot{x}(t) dt = \frac{1}{\Delta t} (\dot{x}(\Delta t) - \dot{x}(0)) = \frac{1}{\Delta t} \left( a \omega \cos(\omega \Delta t)e^{-b\Delta t} - a \omega \sin(\omega \Delta t)e^{-b\Delta t} + c - a \omega - 0 - c \right)
\]

\[
= \frac{1}{\Delta t} \left( - a \omega e^{-b\Delta t} - a \omega \right) = - \frac{\omega}{\pi - \epsilon} \cdot a \omega \left( 1 + e^{-b\Delta t} \right) = - \frac{\omega}{\pi - \epsilon} \cdot \frac{\Delta vm_2}{(m_1 + m_2)} (1 + e^{-b\Delta t}).
\]

\[
(A1.2)
\]

Moreover, since the damping factor \( b \) is small for soccer balls (about 0.1) we can substitute the truncated Taylor series \( e^x \approx 1 + x \) for small values of \( x \), and so

\[
\bar{\omega} \approx - \frac{2}{\pi} \cdot \frac{\Delta vm_2}{(m_1 + m_2)} \omega. \quad (A1.3)
\]

Now substituting for \( \omega \) using (A1.1 c) and taking the case in which \( m_2 \ll m_1 \), which is true for soccer balls versus players, we have

\[
\bar{\omega} \approx - \frac{2}{\pi} \cdot \frac{\Delta vm_2}{(m_1 + m_2)} \cdot \sqrt{CP \cdot \frac{m_1 + m_2}{m_1m_2}} \approx - \frac{2}{\pi} \cdot \frac{\Delta v}{m_1} \cdot \sqrt{CPm_2} \quad (A1.5)
\]
The negative sign of Eq. (A1.5) indicates that the head is accelerated backward, in the opposite direction to its original forward motion. In considering only the magnitude of head acceleration, this sign can be omitted. Eq. (A1.5) gives a convenient and succinct expression for mean head acceleration during soccer heading. The factors under the square root sign are determined entirely by the ball. The remaining factors depend largely on the players, including how hard they kick and how much effective body mass they muster behind the ball. This expression can be used for Monte Carlo simulations.

A subtlety in Eq. (A1.5) is that the ball speed with respect to the head at head height, $\Delta v$, also depends upon ball design indirectly, because balls of larger size and smaller mass are slowed to a greater extent by air resistance. This issue is dealt with in Appendix 2.

**Appendix 2: Approximate change in ball speed with change in ball design**

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units or value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Air resistance coefficient</td>
<td>Nt/m²/sec²</td>
</tr>
<tr>
<td>m</td>
<td>Mass of soccer ball</td>
<td>kg</td>
</tr>
<tr>
<td>t</td>
<td>Time of flight from foot to head height</td>
<td>sec</td>
</tr>
<tr>
<td>$v_h$</td>
<td>Horizontal velocity of ball</td>
<td>m/sec</td>
</tr>
</tbody>
</table>

Subscripts:
- 1: Original reference soccer ball design
- 2: Proposed alternative soccer ball design

Classically, air resistance or drag is proportional to velocity squared: that is $F_{\text{drag}} = -\beta \cdot v_h^2$ for a constant, $\beta$. The value of $\beta$ depends on the density of air, $\rho$, the drag coefficient for a sphere, $C_D$ and the cross sectional area of the ball, $A$. $^{31,32}$ Specifically, $\beta = \frac{1}{2} \rho A C_D$. In air $\rho$ equals 1.2 kg/m² and $C_D$ for smooth spheres with a Reynolds number of 1.5x10⁵ is 1.45$^{31}$. In the horizontal dimension there is no effect of gravity, and the equation of motion for the ball is

$$m \frac{dv}{dt} = -\beta v_h^2.$$  \hspace{1cm} (A2.1)

Rearranging and integrating
\[ \int_{v_h(0)}^{v_h} \frac{dv_h}{v_h} = -\frac{\beta}{m} \int_0^t dt, \]  

(A2.2)

which leads to

\[ v_h = \frac{v_h(0)}{1 + v_h(0) \frac{\beta}{m} t}, \]  

(A2.3)

where \( v_h(0) \) refers to the initial horizontal component of ball velocity at time 0 when the ball is kicked. This is the general expression for the horizontal velocity of a ball in the presence of air resistance.

We wish to find how ball velocity at head height changes as a function of the ball design, specifically in terms of its size (area) and its mass (m), which could be changed for the sake of making soccer balls more brain friendly. It can be shown numerically for soccer balls along the lines presented in\(^22\) that the time of flight changes very little for reasonable departures of \( \frac{\beta}{m} \) from that of the standard soccer ball. As balls become more lightweight or larger (more beach ball-like), both horizontal speed and distance traveled decrease, but time of flight remains about the same. As balls become heavier or smaller (more cannon ball-like), both horizontal speed and distance traveled increase, but time of flight remains about the same. Accordingly, for a ball kicked with the same initial speed one can estimate the change in velocity for a change in design rather simply as follows.

The ratio of head-height horizontal velocity for a new, untested design 2 compared to standard design 1 is

\[ \frac{v_{h,2}}{v_{h,1}} = \frac{v_h(0)}{1 + v_h(0) \frac{\beta_2}{m_2} t_2} = \frac{1 + v_h(0) \frac{\beta_1}{m_1} t_1}{1 + v_h(0) \frac{\beta_2}{m_2} t_2}. \]  

(A2.4)

For standard soccer balls in competitive games \( v_h(0) \frac{\beta}{m} t \approx 10 \). Using a reasonable range of alternative values for \( \frac{\beta}{m} \), realizing that headed balls are likely to have relatively high initial velocities \( v_h(0) \) and relatively long times of flight, \( t \), and recognizing that \( t_1 \approx t_2 \), one can see that for soccer balls
This expression can be used to correct velocity data obtained with standard balls to obtain reasonable estimates for the distributions of velocity data for modified ball designs, without having to actually construct and test modified ball designs initially. The approximate correction is adequate, because the effects of the change in horizontal velocity of the ball upon overall head acceleration are small compared with the effects of other variables.

Finally, realizing that the drag coefficients, $\beta$, are proportional to the cross-sectional areas of the balls, while other factors such as air density and ball shape (i.e., spherical) are the same, then (A2.5) becomes

$$\frac{v_{h2}}{v_{h1}} \approx \frac{C_1}{C_2} \frac{m_2}{m_1}. \quad (A2.6)$$